Title: A Resolution of Bell's Spaceship Paradox

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Abstract:

There is not unanimous agreement about the outcome of Bell's Spaceship Paradox. Some say the thread breaks, and some say it doesn't. Who is correct? Does the string break or not? The answer depends on how the acceleration is determined. If the acceleration is determined by calculations and measurements made by various inertial observers, the string WILL break. But if the acceleration is what accelerometers attached to the spaceships display, the string will NOT break.

Two perpetually inertial observers (IO1 and IO2), perpetually mutually stationary with one another, are initially co-located with two separated observers (AO1 and AO2), with separation "L". AO1 and AO2 are about to begin a constant (according to them) acceleration "A" (with the separation in the direction of their acceleration). AO1 and AO2 KNOW that their acceleration is "A", because they each are carrying an accelerometer that confirms it. IO1 and IO2 will conclude that AO1 and AO2 maintain the separation "L" during the accelerations. And AO1 and AO2 will agree with that: AO1 and AO2 conclude that their separation remains constant at "L" during the acceleration. But two other inertial observers, IO3 and IO4, who are momentarily co-located with AO1 and AO2 at any time later in the trip, will NOT agree that the separation "L" is constant: they will say that it has increased since the start of the trip.

The separation of the two people undergoing the acceleration (AO1 and AO2), ACCORDING TO THOSE TWO PEOPLE THEMSELVES, can be referred to as the "PROPER" separation.

The REFERENCE FRAME of an accelerating observer, say AO1's reference frame, is constructed in the same manner as an inertial observer constructs his reference frame (as Einstein explained to us). AO1's helpers just lay out yardsticks, end-to-end, in the direction of the acceleration. To keep the yardsticks in place, they each are attached to an accelerometer-controlled rocket, so they each are accelerating at "A" lightyears/year/year. And between every pair of yardsticks, there is a clock. The clocks were initially synchronized before the constant acceleration started. Once the acceleration starts, the clocks don't remain synchronized. Clocks located farther in the direction of the acceleration tic faster, by the rate ratio "R". Einstein gave an exponential equation for "R" (https://einsteinpapers.press.princeton.edu/vol2-trans/319) but I have previously shown that his exponential equation is incorrect (https://vixra.org/abs/2109.0076). I later gave the corrected equation for the rate ratio "R" (https://vixra.org/abs/2201.0015).

Although Einstein’s exponential equation is incorrect, he was correct in his belief that the separation “L” is constant during the acceleration (according to the people actually undergoing the acceleration). Einstein clearly believed that "L" is constant, because he didn't give an equation for how "L" varies with time. If he had thought "L" varied with time, he would have needed to include an L(t) equation as part of his solution. He did NOT do that.

Each accelerometer directs its attached rocket to accelerate at exactly "A" lightyears per year per year. NEWTONIAN physics would say that the velocity of AO1 and AO2 would increase linearly with time, forever:

$$v = A * t. \quad \text{(incorrect)}$$

That equation means that "v" would go to infinity as "t" goes to infinity, which we know can't be true in special relativity. So the above equation is clearly wrong. Special relativity says the quantity "(A * t)", which it calls the "rapidity" (denoted by the variable "theta"), is related to the velocity "v" by

$$\text{velocity} = v = \tanh(\text{rapidity}) = \tanh(\text{theta}) = \tanh(A * t). \quad \text{(correct)}$$

That says that with a constant acceleration "A", "v" approaches (but never equals) the speed of light, "c", as "t" goes to infinity.

The distance "D" each rocket moves, according to AO1 and AO2, is

$$D = \text{integral } \{\text{from 0 to tau}\} [v\, dt] .$$

$$= \text{integral } \{\text{from 0 to tau}\} [\tanh(A*t)\, dt] .$$

The integral of tanh(x) is equal to log[cosh(x)], so

$$D = \log[\cosh(\text{tau})] / A - \log[\cosh(0)] / A .$$

$$D = \log[\cosh(\text{tau})] / A .$$

So "D" grows forever, but it's RATE of growth decreases as tau increases.

The distance "D" each rocket moves during the acceleration is EXACTLY the same, so the separation "L" between AO1 and AO2, according to THEM, can't change during the acceleration.

The INERTIAL observers (IO1 and IO2) also conclude that the separation "L" between AO1 and AO2 stays constant during the acceleration. But two inertial observers (IO3 and IO4) who are momentarily co-located with AO1 and AO2 at some later instant in the trip will conclude that the separation between AO1 and AO2 is LARGER than it was when the acceleration started. And that larger separation continues to increase as the trip progresses, according to the INERTIAL observers momentarily co-located with AO1 and AO2 later in the trip.

So the accelerating observers (AO1 and AO2) say that their separation is CONSTANT during their trip. The inertial observers (IO3 and IO4) say that the separation of AO1 and AO2 INCREASES during the trip. Those two groups of observers DISAGREE. That's just the way special relativity IS.

But it's normal in special relativity for an accelerating observer to agree with the inertial observer who is momentarily co-located with him at some instant ... that's what the Co-Moving-Inertial-Frame (CMIF) simultaneity method IS. The inertial observer IO3 is momentarily co-located with AO1, and IO3 tells AO1 that the separation between AO1 and AO2 is larger than it was when the separation began. Does that contradict my above argument? No, it doesn't, because the scenarios are themselves different: the actual accelerations are slightly different. How do the people producing the scenario with all the inertial observers achieve the acceleration "A"? It's based on the CALCULATIONS by the inertial people: they measure positions of AO1 and AO2 versus the time on their own watches, and COMPUTE the acceleration. It is NOT based on any accelerometer, and it differs from what AO1 and AO2 read on their accelerometers.

Note that the above description is relevant to the well-known (and much misunderstood) Bell's Spaceship Paradox:

https://en.wikipedia.org/wiki/Bell%27s_spaceship_paradox .

The above Wiki article should be read in its entirety, including its references. There have been MANY papers written on Bell's paradox over the years, and different conclusions have been drawn. There has definitely not been a unanimous conclusion.

Does the string break or not? The answer depends on how the acceleration is measured. If the acceleration is determined by calculations and measurements made by various inertial observers, the string WILL break. But if the acceleration is what accelerometers attached to the spaceships display, the string will NOT break.