The Collatz tree nodes as rooms in a Hilbert hotel: disproof the Collatz conjecture

Wiroj Homsup and Nathawut Homsup

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Abstract

Consider an infinite perfect binary tree as an infinite stories Hilbert hotel , we transform an infinite perfect binary tree with node 1 as its root node to have the Collatz tree as its subtree, and show that there is at least one node (room) in a perfect binary tree (Hilbert hotel) that is not in the Collatz tree, supporting the conclusion that the Collatz conjecture is false.

1. Introduction

Define the iterating function introduced by R. Terras[1] :

$$a_{n+1} = (3^b a_n + b)/2 \qquad (1)$$

where $b = 1$ when a_n is odd, and $b = 0$ when a_n is even. The Collatz conjecture asserts that by starting with any positive integer a_0, there exists a natural number k such that $a_k = 1$. In [2], by analyzing the stopping times of various Collatz sequence, a pattern emerges that indicates the existence of non-empty sets of integers with stopping times greater than any given integer, supporting the conclusion that the Collatz conjecture is false.

2. The Collatz tree

The Collatz tree is defined as follows:

Start with number 1 as a root node , with each node as a parent node it can have one or two children nodes. If node $i \equiv 2 \bmod 3$, then its two children nodes are $2i$ and $(2i-1)/3$. If node $i \equiv 0 \bmod 3$ or $i \equiv 1 \bmod 3$, then its children node is $2i$. A 5- levels Collatz tree is shown in Figure 1.

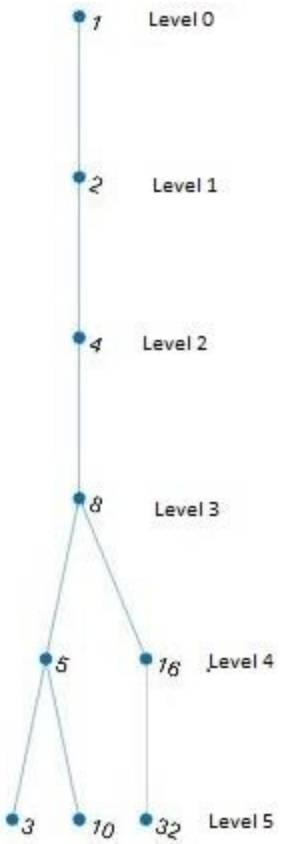

Figure 1. 5- levels Collatz tree

3. **A Perfect binary tree**

A perfect binary tree is a binary tree in which all interior nodes have two children and all leaves have the same depth. Let number 1 as a root node , two children nodes of a node i are 2i and 2i+1. An example of a 5- levels perfect binary tree is shown in Figure 2.

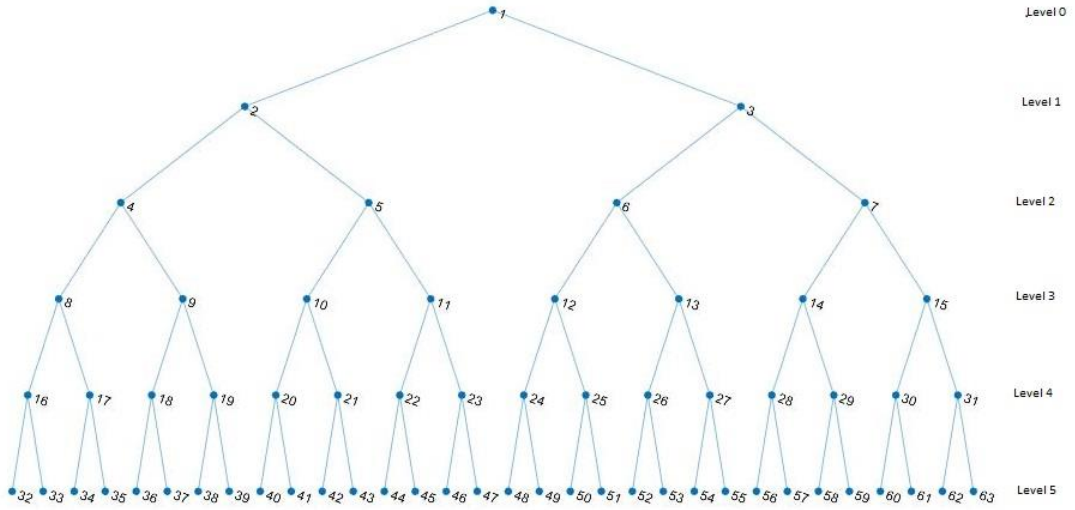

Figure 2 5- levels perfect binary tree (5 stories of a Hilbert Hotel).

By interchanging a pair of nodes in a perfect binary tree, the Collatz tree can be obtained as a subtree of a perfect binary tree. Since each node in the Collatz tree can have one or two children nodes, then there are nodes in a perfect binary tree that are not in the Collatz tree. Let an infinite binary tree as a infinite stories Hilbert hotel with guest i stay in room number i, interchanging a pair of nodes in binary tree is equivalent to interchange quests between pairs of rooms. As an example, the transformed 5- levels perfect binary tree that has the Collatz tree as its subtree is shown in Figure 3.

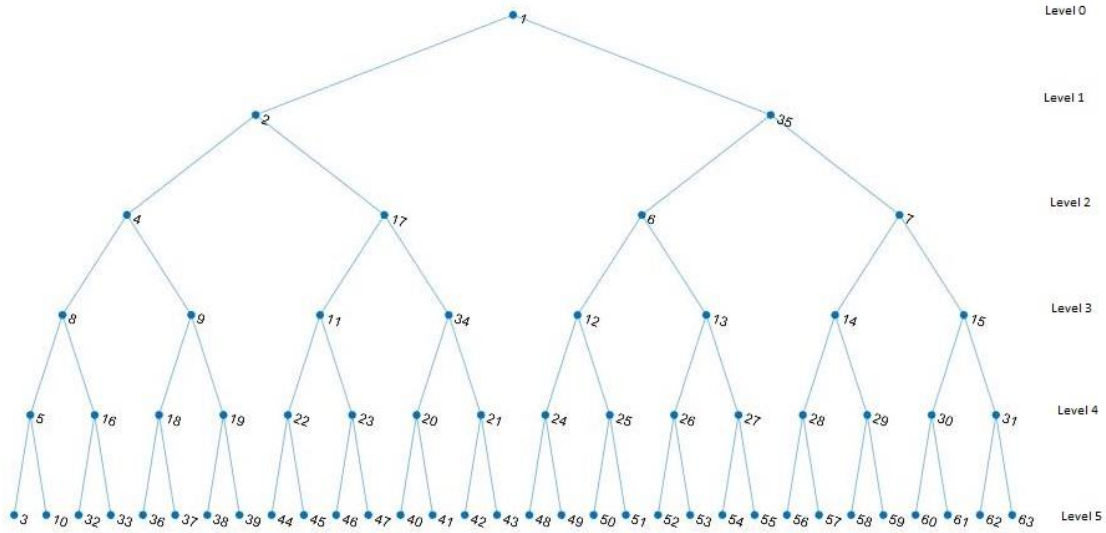

Figure 3. A transformed 5- levels perfect binary tree (5 stories of a Hilbert hotel after interchanging quests between pairs of rooms).

Clearly, at least one node in a perfect binary tree is not in the Collatz subtree , supporting the conclusion that the Collatz conjecture is false.

Reference

[1] R . Terras, (1976). " A stopping time problem on the positive integers". Acta Arithmetica, 30(3), 241-2

[2] J. A. Perez,(2017), " Collatz Conjecture: Is it false ?", arXiv:1708.04615[math.GM]