About the Higgs particle mass, the neutron and the role of the Kerr metric

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ABSTRACT

A model for calculating the mass of a particle is presented which has the same mass as the Higgs particle. The model is based on the ratio of Newton’s and Coulomb’s laws. The model is used to calculate the charge radius of the proton, the neutron decay times(s), the magnetic moment of the neutron as well as the anomalous magnetic moment of the proton and the electron. Some of the results are obtained from analysis of the multiple resonances of coordinate or epicyclic oscillations, respectively, occurring at extreme values of the Kerr spin parameter and the Aschenbach effect.

Key words: gravitation – blackhole physics – astroparticle physics – atomic data

1 INTRODUCTION

The Higgs particle has a mass concluded from the LHC experiments, and the numerical value of the mass as measured has been published. Here I present a model which is based on combining quantum theory and the Kerr solution of general relativity. The model allows calculating the mass of a particle which is consistent with the Higgs particle as measured. The model has consequences as there are the prediction of the mass of the neutron and its magnetic moment as well as the neutron decay time(s). The numerical results are presented. It is shown that the anomalous magnetic moments of the proton and electron are connected to different but specific values of the Kerr angular momentum parameter \( a_K \), usually expressed by the letter \( a \). The value of the gravitational constant \( G \), which is the least precise known fundamental physical constant, is revisited delivering a model predicted value. The numerical results have been derived assuming dark matter and by analysing the anomalous orbital velocity effect in the Kerr metric which I discovered in 2004 (Aschenbach 2004). Sometimes the effect is referred to as Aschenbach effect (Stuchlík et al., 2005; Khodagholizadeh, 2020). The assumption of dark matter is not essential to the model. It is used to make the numerical results as precise as possible.

To avoid any misunderstanding or misinterpretation I stress that I do not claim that I have identified through this paper the Higgs particle, its properties and the Higgs mechanism. It concerns just a particle the mass of which is consistent with the mass measured and attributed to the Higgs particle. Based on this result conclusions are drawn and presented in this paper. Some of these conclusions, but not each one, are based on dimensionless quantities. This has happened in physics before, as there are Sommerfeld’s fine structure constant, Schwinger’s constant for the anomalous magnetic moment of the electron and its expansion in quantum electrodynamics, again a dimensionless quantity.
2 CALCULATION OF THE NUMERICAL VALUE OF THE HIGGS PARTICLE EQUIVALENT MASS AND THE GRAVITATIONAL CONSTANT

The ratio of Newton’s gravitational attractive force between two masses to Coulomb’s force between two electrons of rest mass $m_e$ and elementary charge $q = e$ is used for defining a coupling constant.

$$F_{gq} = 4\pi\varepsilon_0 G \left(\frac{m}{e}\right)^2$$

which can also be written as

$$F_{gq} = \left(\frac{m_e}{m_{pl}}\right)^2/\alpha$$

$m_{pl}$ is the Planck mass and $\alpha$ is Sommerfeld’s fine structure constant. The standard notation for the fundamental physical constants is used and their values in SI units listed in the 2018 CODATA tables (Tiesinga et al. 2021; https://doi.org/10.1103/RevModPhys.93.025010) are used. $F_{gq}$ is a dimensionless quantity including the product of two masses. I suggest that $\sqrt{F_{gq}}$ is an expression for the mass sum of two unknown ‘objects’ each of mass $m_u$, undergoing gravitational interaction, such that

$$\sqrt{F_{gq}} = m_u + m_u + E_b/c^2$$

$E_b$ is the gravitational binding energy between the two particles, i.e.,

$$E_b = G m_u m_u/r_b$$

and $r_b$ is some distance between them. In general relativity the radial coordinate $r$ is expressed in units of the gravitational radius $r_g = G m_u/c^2$ such that $r_b = x r_g$ and

$$E_b/c^2 = G m_u m_u/(x r_g)$$

$$E_b/c^2 = m_u/x$$

In order for the two particles to separate against gravity a minimum amount of energy is required. The amount of energy is defined by the condition that one particle reaches the most distant innermost stable circular orbit ISCO. This is an approximation because it is assumed that the two masses have the same value. The radius $r_{isco} = 6r_g$ in the Kerr metric for an angular momentum Kerr parameter $a_K = 0$, which is the result of the Schwarzschild solution. The Schwarzschild solution assumes that the two gravitationally interacting masses are point-like. But this is a mathematically based approximation. Assuming a physical extent, i.e., a volume housing the mass, the effective energy separating the two particles, though, is the energy lifting one mass from the surface of the counter mass such that $r_b = 5r_g$ or $x = 5$ and

$$\sqrt{F_{gq}} = 2m_u + m_u/5 = 11/5 m_u$$
and

\[ m_u = \frac{5}{11} \sqrt{F_{gq}} \]  

(7)

Taking the values of the fundamental physical constants listed in the 2018 CODATA tables ([https://doi.org/10.1103/RevModPhys.93.025010](https://doi.org/10.1103/RevModPhys.93.025010)) a value for \( m_u \) is obtained which is \( m_u = 2.2270916 \times 10^{-22} \). This is some unidentified mass, but attaching a mass measuring unit of gram as gauge measure \( m_u = 2.2270916 \times 10^{-22} \text{ g} = 124.9306766 \text{ GeV/c}^2 \). Of course, this attachment is arbitrary. Nevertheless, this mass is close to the mass of the Higgs boson \( m(H^0) \) concluded from the measurements. The Atlas experiment (The ATLAS Collaboration 2018) recorded a mass of \( m(H^0) = 124.97 \pm 0.24 \text{ GeV/c}^2 \), whereas the CMS experiment (The CMS Collaboration, 2020) reported a mass of \( m(H^0) = 125.35\pm0.15 \text{ GeV/c}^2 \). With the identification of the gram mass gauge unit it can be implemented in equations (6) and (7) as \( m_g \). Then, equations (6) and (7) would read

\[ \sqrt{F_{gq}} = 2m(H^0)/m_g + 1/5m(H^0)/m_g = 11/5m(H^0)/m_g \]  

(6a)

\[ m(H^0)/m_g = 5/11\sqrt{F_{gq}} \]  

(7a)

If \( m(H^0) \) is measured in units of kg the SI choice for the mass measuring unit is \( m_g = 10^{-3} \text{ kg} \).

However, the measurements did not yield two particles identical in mass or energy, respectively. The two masses of the model need to differ from each other, and one mass is required to have escaped from detection in the Large Hadron Collider (LHC) experiments. It could represent some ‘dark’ mass, interacting just via its gravitational force. The coincidence of the mass derived with the mass of the Higgs particle may be accidentally, but may stimulate some further considerations.

The involvement of two particles of differing masses leads to a model of two groups of particles. One group is the neutron group (n-group) with mass \( m_{u1} \) and the other group is the proton-electron pair group (pe-group) with mass \( m_{u2} \). Again the association of the mass is arbitrary, but I follow this assumption. The n-group consists of 133 elements, each element representing the energy of one neutron. The pe-group consists of 133 elements as well, but each element represents the sum of the rest masses of the proton and the electron, which have no electric charge. The element \#133 of the n-group is assumed to not contain any rest mass energy, so that the symmetry with respect to the number of elements is maintained but the gravitational balance with respect to rest mass between the two groups is broken. The total mass or energy of the n-group amounts to \( (m_{u1} m_g) = 133 m_p = 124.9622009 \text{ GeV/c}^2 \), which is consistent with the mass measured for the Higgs particle. The total mass of the pe-group amounts to \( (m_{u2} m_g) = 133 (m_p + m_e) = 124.8581505 \text{ GeV/c}^2 \). This mass may be identified as the mass of a second Higgs particle interacting just by gravitation.

With two different masses \( m_{u1} \) and \( m_{u2} \) equation (6a) needs to be modified reading

\[ \sqrt{F_{gq}} = m_{u1} + 1/5 m_{u1} + m_{u2} \]  

(6b)
\[ \sqrt{F_{gg}} = \frac{6}{5} m_{ul} + m_{u2} \]  

(6c)

This leads to

\[ \left[ 4\pi\varepsilon_0 G \left( \frac{m_e}{e} \right)^2 \right]^{1/2} = 133 m_n / m_g \left[ \frac{6}{5} + \left( m_p + m_e \right) / m_n \right] \]  

(8)

\( G \) can be calculated from equation (8) to \( G = 6.672614972 \times 10^{-11} \, m^3 \, kg^{-1} \, s^{-2} \), which is vaguely consistent with the value recommended by CODATA, but it is among the various, widely spread measurement results, which range from \( 6.671 \times 10^{-11} \, m^3 \, kg^{-1} \, s^{-2} \) up to \( 6.676 \times 10^{-11} \, m^3 \, kg^{-1} \, s^{-2} \). Outstanding is the measurement of \( G = 6.67260 \pm 0.00025 \times 10^{-11} \, m^3 \, kg^{-1} \, s^{-2} \) reported by Parks & Faller (2019) because of its fairly low uncertainty. This measurement agrees with the theoretical value to \( <0.06\sigma \).

Of course, the assignment of \( m_g = 1g \) or \( 10^{-3} \, kg \) appears to be arbitrary, but this admittedly pure assumption appears to be justified as it has consequences and predictions as outlined in the sections below. Of course, there is no physical justification so far that \( m(H^0) = 133 m_n \), but I think the numerical coincidence is worth to have a deeper look.

### 3 CALCULATION OF THE CHARGE RADIUS OF THE PROTON

There is the difference \( \Delta m = m_n - (m_p + m_e) \) between the mass of the neutron and the sum of the rest mass of the proton and the electron. The equation

\[ \Delta m c^2 = \frac{e^2}{4\pi\varepsilon_0 r} \]  

(9)

defines a radius \( r \). Following the idea that physically existing sources have to have a finite extent and are not point-like as outlined in section 2 above, the value of \( r = 1.841228385 \, fm \) obtained from equation (9) can be split in two parts \( r_p \) and \( \Delta r \), by which the charge radius of the proton \( r_p \) and the separation \( \Delta r \) of the electron particle from the surface of the proton particle can be defined, neglecting any extent of the electron particle. Therefore any distance in a magnetic or electrical field can be expressed in units of the proton charge radius, i.e., \( r = k^* r_p \). There is a commonality between Newton’s and Coulomb’s forces, which is their \( r^{-2} \) dependence. Each one of the forces has a mathematical singularity at \( r \to 0 \), which disappears in their ratio. Equation (8), which is based on the idea that physical sources are extended, displays one factor which is reiterating this concept and may be considered as representing both Newton and Coulomb forces, defining a constant

\[ k^* = \left[ \frac{6}{5} + \left( m_p + m_e \right) / m_n \right] = 2.199167344 \]  

(10)

With this value for \( k^* \) the proton charge radius would be \( r_p = 0.837238871 \, fm \). Experimental results range from \( r_p = 0.877 \, fm \) to \( r_p = 0.831 \, fm \) with individually estimated uncertainties that make the results incompatible with each other. This is, experimentally, still a problem and is known as proton radius puzzle. Theoretically, the quantum chromodynamical calculations of Belushkin et al. (2007) reveal a value of \( r_p = 0.830 \, fm \) with an uncertainty range of \( (0.822 - 0.835) \, fm \).

Equation (9) implies that the energy \( \Delta m c^2 \) is potential energy without any contribution of rest mass energy. Therefore the electron antineutrino should have zero rest mass. With \( r_p, \Delta r, \) and \( m_p + m_e \)
known the mass of the neutron is given.

4 INTERPRETATION OF THE FINE STRUCTURE CONSTANT

The relation $\Delta m c^2 = e^2/(4\pi \varepsilon_0 r)$ can be written as $2\pi \Delta m rc = \alpha h$, or $\Delta m rc/\hbar = \alpha$. This means that Sommerfeld’s fine structure constant can be interpreted as a dimensionless angular momentum, although no rest mass is involved. If Bohr’s approach for explaining stable electron orbits in a hydrogen atom is a universal law the neutron is subject to decaying by some radiation, the electron antineutrino, as $\alpha$ is not a natural number.

5 ASCHENBACH EFFECT

The radius $r$ is the length of a vector which may point at any direction ($\theta, \varphi$) at any time. For some time a lingering state exists such that oscillations between the coordinates lead to time limited parametric resonances. Such resonances have been discovered in the Kerr metric by searching the dimensionless eigenfrequencies ($\Omega_r, \Omega_\theta, \Omega_\varphi$) for resonances in a sense that mutual ratios of the frequencies come as ratios of natural numbers, e.g., 3:1 or 3:2. The dimensionless frequencies are a function of distance $r$, measured in units of $r_g$, and dimensionless angular momentum quantified by the Kerr parameter $a_K$. The analytic expressions for the $\Omega$’s as function of $r$ and $a_K$ are well known. They are re-presented in Aschenbach (2004) with references to earlier sources for the origin of the equations.

Equations 11 to 13 show the relations describing the epicyclic frequencies $\Omega$ in standard notation of general relativity, i.e., $c = G = M = 1$. Whereas $c$ and $G$ are well defined physical constants, the case of $M$ is less obvious as $M$ is a symbol of energy, according to Einstein.

$$\Omega_\varphi = (r^{3/2} + a_K)^{-1}$$

(11)

$$\Omega_\theta^2 = \Omega_\varphi^2 (1 - 4a_K/r^{3/2} + 3a_K^2/r^2)$$

(12)

$$\Omega_r^2 = \Omega_\varphi^2 (1 - 6/r + 8a_K/r^{3/2} - 3a_K^2/r^2)$$

(13)

One resonance appears at $a_{K1} = 264/265 \approx 0.996226094$ with $\Omega_\varphi/\Omega_\theta = 3:1$ at $r = r_{31*}$ with $r_{31*}=1.394211672$. A second resonance appears at $a_{K2} \approx 14999a_{K1}/15000 = 0.99616$ with $\Omega_\theta/\Omega_r = 3:1$ at $r = r_{31}$, with $r_{31} = 1.54507896845$. At $a_{K2}$, a second resonance of $\Omega_\theta/\Omega_r$ appears at $r = r_{32}$, with $\Omega_\theta/\Omega_r = 3:2$. The radii $r_{31}$ and $r_{32}$ of these $\Omega_\theta/\Omega_r$ resonances are commensurable orbits in the classical astronomical (Kepler) sense as $\Omega_\varphi(r_{31}, a_{K2})/\Omega_\varphi(r_{32}, a_{K2}) = 3:1$. The frequency ratio $\Omega_\theta/\Omega_r = 3:2$ at $r_{32}$ explains quantitatively the twin-peak quasi-periodic oscillation frequencies (QPO’s) observed in the time power density spectra of black hole objects (Aschenbach 2004; Smith et al. 2013).

A third angular momentum parameter $a_K = a_c \approx 0.9953$ exists in the Kerr metric which marks a
dividing line. For values of $a_K > a_c$ the Kepler potential, defined by the orbital velocity $v_\phi$ as function of the orbit radius $r$ changes from $\frac{\partial v_\phi}{\partial r} < 0$ to $\frac{\partial v_\phi}{\partial r} \geq 0$ with a minimum-maximum structure of $v_\phi(r)$. This anomalous orbit velocity effect has been confirmed and named Aschenbach effect by Stuchlík et al. (2005). $v_\phi(r)$ shows with increasing $r$ the signature of a marginally stable equilibrium, an indirect equilibrium indicated by $\frac{\partial^2 v_\phi}{\partial r^2} = 0$ with a maximum of $\frac{\partial v_\phi}{\partial r}$, $\frac{\partial v_\phi}{\partial r}|_{max}$ at $r= r_{max}$ and an unstable equilibrium ordered along increasing $r$. The mean value of the orbital velocity is unexpectedly low with $v_\phi \approx 0.55$. The graph Fig. 5 (Aschenbach 2004) of $v_\phi$ versus $r$ shows a sinusoidal shape with $\frac{\partial v_\phi}{\partial r}>0$ for $r<r_{max}$ and $\frac{\partial v_\phi}{\partial r}<0$ for $r>r_{max}$ suggesting a wave-function behaviour, the shape of which changes with changing $a_K$ in the range $a_K > a_c$. The unexpectedly low value of $v_\phi$ suggests that kinetic energy is likely to be missed but stored as potential energy. A likely explanation of this energy transformation is the transport of angular momentum because of the alternating sign of $\frac{\partial v_\phi}{\partial r}$ along $r$, which induces associated oscillations with time limited, transient resonances between $a_K = 1$, $a_{K1}$, $a_{K2}$ and $a_c$, like in a time limited alternating spin-down spin-up process, but an eventually continuous spin-down. This action is suggested to be responsible for the decay time scale of the neutron. For the calculation of the velocity $v_\phi$ the description of the Kerr metric in spherical Boyer-Lindquist coordinates has been used. Approximate numerically calculated values of the characteristic parameters ($\Omega$, $r$)’s for $a_{K1}$ and $a_{K2}$ have been tabulated (Aschenbach 2004; Aschenbach 2019).

6 THE LIFETIME(S) OF THE NEUTRON

The 3:1 resonances at $a_{K2}$ and $a_{K1}$ both involve $Q_\theta$ which is a dimensionless frequency. Its inverse is a dimensionless time interval. Thus, the ratio $\frac{\Omega_\theta(a_{K1}, r_{31})}{\Omega_\theta(a_{K2}, r_{31})}$ is a dimensionless time interval as well. Their ratio $\tau = \frac{\Omega_\theta(a_{K1}, r_{31})}{\Omega_\theta(a_{K2}, r_{31})} = 0.886389951$. Attributing a time measuring gauge unit of 1 ks or 1000 s, $\tau_n = 1000\tau = 886.389951$ s. I suggest that this is the mean lifetime of the neutron. The value may be compared with the experimental value of $\tau_{n,beam} = 888.1\pm2$ s measured in the ‘beam’ experiments.

Out of the 133 elements of the n-group only 132 elements are leaving protons and electrons. Counting particles by number, which is done in experiments, i.e., neutrons vs. protons, the decay rate counting neutrons appears to be faster than the rate of counting the number of protons, which are among the remains of the decay. Accordingly the corresponding, associated lifetimes differ by a factor of 133/132. With $\tau_n = 886.389951$ s = $\tau_{n,beam}$ attributed to the ‘beam’ experiment, the lifetime of the neutron inferred from the ‘magnetic bottle’ experiments is expected to be $\tau_{n,bottle} = 879.725365$ s which may be compared with the experimental value of $\tau_{n,bottle} = 879.45\pm0.58$ s measured in the ‘magnetic bottle’ experiments.

The model presented here appears to explain quantititatively the discrepancy between the measurement results of the ‘beam’ experiments and the ‘magnetic bottle’ experiments.

The element #133 of the n-group does not contain any rest mass but possesses the total energy of one neutron. I suggest that it represents a spinning Dirac magnetic monopole, which implemented in the set of Maxwell’s equations would result in $\text{rot } E = -\frac{\partial B}{\partial t} + j_m$ and $\text{div } B = \rho_m \neq 0$ with $E$ an electric field and $B$ the corresponding magnetic inductance. Therefore the rotating magnetic

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monopole is the source of electrical charging of the proton and the electron, and indicates that the proton and the electron are being electrically charged in the course of the neutron oscillatory decay and that their electric charge is created by a magnetic field. This would explain the absence of an electrically charged proton and an electrically charged electron in the neutron.

7  MAGNETIC MOMENTS AND THEIR RELATION TO THE KERR ANGULAR MOMENTUM PARAMETER

The dimensionless magnetic moment of the neutron is \( \frac{\mu_n}{\mu_N} = 2(P_n - 1) \). \( P_n \) is the anomalous part of the magnetic moment and \( \mu_N \) is the nuclear magneton. The ‘1’ represents the maximum numerical value of the Kerr parameter \( a_K \). The CODATA group is recommending a value for \( \frac{\mu_n}{\mu_N} = -1.91304273(45) \). The number in brackets is the estimated one standard uncertainty wrt to the preceeding two digits 73. The model describes two groups of elements which contain 132 rest-mass elements and 133 rest-mass elements, respectively. This produces a gravitational imbalance. The elements \#133 may be considered as exchange elements. Therefore, the sum of individual states is neither 264 nor 265, but \( S = (264+265)/2 \). The elements are partitioned on 11 levels or shells such that the sum of states is \( \{132 = \Sigma(2n), n = 1,11\} \) and \( \{133 = 1+\Sigma(2n), n = 1,11\} \). For the two groups the degree of freedom \( f \) varies between 11 and 12 around an average of \( f = (11+12)/2 \). \( P_n \) is the probability for the appearance of the angular momentum associated with either the n-group or the pe-group which is determined by an oscillation process between the two states mediated by the exchange elements. The relevant \( P_n = [(11+12)/2]/[(264+265)/2] = 23/[265(a_K+1)] = 1/23 \). The model magnetic moment of the neutron is \( \frac{\mu_n}{\mu_N} = 2(P_n - 1) = -44/23 \approx -1.913043478 \), which agrees with the measurements to within 1.66\( \sigma \) of the estimated 1\( \sigma \) uncertainty of the measurements.

I suggest that the anomalous orbital velocity effect is quantitatively related to the anomalous magnetic moment of both the electron and the proton via the Kerr angular momentum \( a_K \). According to the 2018 CODATA group recommendation the anomalous magnetic momentum of the electron is \( a_e = -1.15965218128(18) \times 10^{-3} \) (Tiesinga et al. 2021). The model relation connecting the anomalous angular momentum for the electron is calculated to

\[
a_e = (a_e - 1)/4
\]

(14)

\( a_c \approx 0.9953 \) marks the transition from the non-Keplerian potential to the Kepler potential. The exact value of \( a_c \) could be calculated from the condition \( \lim(\partial v_r / \partial r)_{\max} = 0 \) for \( a_K \rightarrow a_c \). A grid search with a resolution of \( \Delta r = 10^{-5} \) shows a change in sign for \( \Delta v_r \) around \( a_c \approx 0.9953 \). For \( a_e = -1.15965218128 \times 10^{-3} \) the corresponding value for \( a_c \) would be \( a_c = 0.99536139149 \).

The 2018 CODATA dimensionless magnetic moment of the proton is \( \frac{\mu_p}{\mu_N} = +2.79284734463 \) (Tiesinga et al. 2021). The definition of the anomalous magnetic moment of the proton is \( a_p = \frac{\mu_p}{\mu_N} - 2 \). The ratio \( a_p/a_e \) is suggested to be

\[
a_p/a_e = 1000 [133 + 135/3 + 136/3000] (a_K^2 - 1)
\]

(15)

Then, \( a_p = +0.79284734463 \), which is the CODATA recommended value. This value does not apply
for $a_{K2} = 0.99616$, but for $a_{K2} = 0.996159999935$.

8 APPLICATION TO BLACK HOLES

The model, which describes processes between elementary particles down to the rest mass of the proton and electron has initially been derived from observations of astrophysical accreting black holes involving masses from a few solar masses up to $10^7$ or even more solar masses, including galactic microquasars, the Galactic Centre black hole and extragalactic active galactic nuclei. The model seems to be quite universal. It is scale invariant because of using ratios between forces ($F_{gg}$), lengths ($r´s$) and times ($Ω´s$).

The explanation of the decay of the neutron suggests an alternative interpretation of the term black hole insofar as it is not an ‘object’ of ever increasing mass. What is commonly called a black hole may be viewed as a volume of space for storage of potential (dark) energy, or, using Einstein’s $E = mc^2$, (dark) mass and angular momentum but of limited storage capacity. The limit has been reached when $a_K > 264/265$. The system which consists of the black hole and its surrounding accretion disk is going to collapse. This collapse mimics the collision of two black holes. The model predicts the masses (energy) of the two black holes to come in a ratio of $≈(6/5):1$ (equation 8). ‘Viewing’ the energy of the gravitational wave radiation as dark energy I suggest that the total energy (mass) involved is split into three parts which come in fractions of 12:9:1 attributed to the two black hole masses and the gravitational wave mass (energy). This is consistent with the results of the analysis of the measurements of the so far recorded gravitational wave events.

9 SOME REMARKS

Given the success of quantum mechanics, quantum electrodynamics and quantum chromodynamics it sounds absurd if not even ridiculous to involve classical mechanics, classical electrodynamics, classical thermodynamics and general relativity to calculate at least some fundamental constants like the gravitational constant, the charge radius of the proton, the magnetic moment of the neutron, the neutron decay times(s). I hope that future highly precise measurements will show whether the predictions made here will last. I emphasize that this paper is not meant to be a confrontation with quantum theory but just supplementary. I note that some of the concepts proposed in this paper had been advocated already by Arthur Eddington, see Kragh 2015.

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