The Proof of Collatz Conjecture

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Abstract: This paper redefines the Collatz conjecture and proposes the equivalence Collatz conjecture, which is a necessary and sufficient condition for the Collatz conjecture. The Collatz transform is divided into Collatz even transform and Collatz odd transform. The scale coefficient of Collatz even transform is 0.5, and the scale coefficient of Collatz odd transform is greater than 1.5, but less than 1.501. Furthermore, In the process of Collatz transforms, the probability of Collatz even transforms and that of Collatz odd transforms are equal, and both of them are 0.5. Through the above analysis of the characteristics of Collatz transforms, it can be concluded: Take any positive integer N greater than 1, perform Collatz transforms on N for m times, when m is large enough, the Collatz transform result N_m must be less than its initial value N. That is, the equivalent Collatz conjecture is true, then the Collatz conjecture must also be true. Based on binomial distribution and normal distribution, it is deduced that any positive integer N greater than 1, the number of equivalent Collatz transforms m_{ce} = 100 * (1+ log N), then the number of Collatz transforms m_c = (100 * (1+ log N)) * (N-1). Further analysis can be concluded that all the Collatz transform results must be unequal, so as to ensure that the transform results will not enter a dead loop during the Collatz transform process.

Keywords: Collatz conjecture, Equivalent Collatz conjecture, Collatz transform, Collatz even transform, Collatz odd transform, Number of Collatz transforms, Number of equivalent Collatz transforms, Collatz transform probability distribution theorem, binomial distribution, normal distribution

1. Introduction

Take any positive integer greater than 1. if it is even, divide it by 2. If it is odd, multiply it by 3 and add 1. Repeat this process, and the final result is 1. This is the Collatz conjecture which is quite possibly the simplest unsolved problem in mathematics.

In August 1999, mathematicians from all over the world gathered in Eichstadt, Germany, to hold a two-day international seminar on the Collatz conjecture. In 2010, as a special issue, the American Mathematical Society released the "Ultimate Challenge: 3x +1 Problem." [3]

“This is a really dangerous problem. People become obsessed with it and it really is impossible,” said Jeffrey Lagarias, a mathematician at the University of Michigan and an expert on the Collatz conjecture.

In September 2019, Dr. Terence Tao, one of the top mathematicians in the world, posted a proof[2] showing that 99% of starting values greater than 1 quadrillion eventually reach a value below 200. This was the most significant result on the Collatz conjecture in decades.

In September 2022, the author of this article posted a study[1], proposed the strong Collatz conjecture and constructed a non-negative integer inheritance decimal tree. With computer large numbers and big data calculation, it concluded that 99.9999999% of positive integers, but it is not a complete proof on the Collatz conjecture.

2. Redefinition of the Collatz conjecture

For the discussion, we set the following definition.

(Definition 2.1) Collatz transform: Take any positive integer N greater than 1, if N is even, divide N by 2, i.e., transform N / 2. If N is odd, multiply N by 3 plus 1 and divide it by 2, i.e., transform (3N + 1) / 2.

The Collatz transform is divided into the Collatz even transform and the Collatz odd transform.

(Definition 2.2) Collatz even transform: Take any positive integer N greater than 1, if N is even, divide N by 2, i.e., transform N / 2.

(Definition 2.3) Collatz odd transform: Take any positive integer N greater than 1, if N is odd, multiply N by 3 plus 1 and divide it by 2, i.e., transform (3N + 1) / 2.

(Definition 2.4) Collatz transform result sequence: Take any positive integer N greater than 1, and perform Collatz transforms on N, the list of integers you obtain as you repeat the process is defined as Collatz transform result sequence.

For example, the Collatz transform result sequence of the positive integer 8 is {8, 4, 2, 1}. The result sequence of the positive integer 11 is {11, 17, 26, 13, 20, 10, 5, 8, 4, 2, 1}.

Now, we redefine the above-mentioned Collatz conjecture as follows:

(Conjecture 2.5) Collatz conjecture: Take any positive integer N greater than 1, perform Collatz transforms on N, and repeat the process m_c times. The final result must be 1.

In the above definition, we define m_c as the **Number of Collatz transforms (Definition 2.6).**

From the Collatz transform result sequences of integers 8 and 11, the number of Collatz transforms for the integer 8 is 3, and that for the integer 11 is 10.

Suppose the Collatz conjecture is correct; for a positive integer N greater than 1, if all results in its Collatz transform sequence are even, then m_{cMin} is the minimum number of Collatz transforms, and we have

$$N = 2^{m_{cMin}}$$

Then, $$m_{cMin} = \log_2 N \qquad (2\text{-}1)$$

For positive integer 8, all results in its Collatz transform result sequence are even, so its number of Collatz transforms satisfies (2-1):

$$m_{cMin} = \log_2 8$$

$$= 3$$

The expression of the Collatz conjecture is quite possibly the simplest unsolved conjecture, and it is difficult to find a mathematical method to solve the problem. We will seek another expression of the Collatz conjecture to find a solution

(Conjecture 2.7) Equivalence Collatz conjecture: Take any positive integer N greater than 1; perform Collatz transforms on N; repeat the process m_{ec} times, and the transform result N_m < N.

(Definition 2.8) Number of Equivalence Collatz transforms: Take any positive integer N greater than 1, perform Collatz transforms on N, and repeat the process m_{ec} times; the transform result is N_m < N. If in the processes of the first m_{ec}–1 times, the transform results are all greater than integer N, then the number of transforms m_{sc} is defined as **the number of equivalence Collatz transforms.**

Equivalent Collatz conjecture is a necessary and sufficient condition for Collatz conjecture.

Because 1 is the smallest positive integer, if (Conjecture 2.5) the Collatz conjecture is true, (Conjecture 2.7) the equivalent Collatz conjecture must be true. So the equivalent Collatz conjecture is the necessary condition of Collatz conjecture.

Below, we will prove that if (Conjecture 2.7) the equivalent Collatz conjecture is true, (Conjecture 2.5) the Collatz conjecture must also be true. That is, the equivalent Collatz conjecture is the sufficient condition for Collatz conjecture.

According to the equivalent Collatz conjecture, take any positive integer N greater than 1, and perform m_n Collatz transforms on N; the transform result N_m < N. Because N_m <N, we can assume that N_m takes the largest possible integer N-1. Perform m_{n-1} Collatz transforms on N-1, and the transform result is N-2, and so on. We obtain the following Collatz transform result sequence:

$$\{N，N\text{-}1，N\text{-}2 \ldots N\text{-}k\text{+}1, N\text{-}k, N\text{-}k\text{-}1, \ldots 2, 1\}$$

The above sequence has N elements, and its corresponding sequence of the number of equivalent Collatz transforms is as follows:

$$\{m_{en}，m_{en-1}，m_{en-2} \ldots m_{en-k+1}，m_{en-k}，m_{en-k-1} \ldots m_3, m_2\}$$

This sequence has N -1 elements. The number of Collatz transforms from N to 1 is

$$m_c = m_{en} + m_{en-1} + \ldots + m_{en-k+1} + m_{en-k} + m_{en-k-1} + \ldots + m_3 + m_2$$

Let m_{ecMax} be the maximum value of the sequence of the number of equivalent Collatz transforms above. Then, the maximum number of Collatz transforms from N to 1 is

$$m_{cMax} < m_{ecMax} *（N-1） \quad (2\text{-}2)$$

Therefore, if the equivalent Collatz conjecture is true, the Collatz conjecture must also be true. The equivalent Collatz conjecture is a necessary and sufficient condition for the Collatz conjecture.

3. Collatz transform characteristics

The Collatz transform is divided into the Collatz even transform and the Collatz odd transform. Take any positive integer N greater than 1, if N is even, divide N by 2, i.e., transform N / 2. If N is odd, multiply N by 3 plus 1 and divide it by 2, i.e., transform (3N + 1) / 2.

3.1 Probability distribution characteristics of Collatz transform

All positive integers are divided into even and odd numbers. The integers with 0, 2, 4, 6 and 8 in one-digit are even numbers, and the integers with 1, 3, 5, 7 and 9 in one-digit are odd numbers. The probability of even and odd numbers in positive integers is 0.5. This is the same as the probability of front and back sides for randomly flipping coins.

The n-bit positive integer N_n can be expressed as follows:

$$N_n = d_n d_{n-1} \ldots d_{m+1} d_m d_{m-1} \ldots d_3 d_2 d_1 \tag{3-1}$$

If N_n is even, divide N_n by 2, the result of Collatz transform $N_n'' = N_n / 2$;

If N_n is odd, multiply N_n by 3 plus 1 and divide it by 2, the result of Collatz odd transform $N_n'' = (3N_n + 1) / 2$. Let the intermediate result of Collatz odd transform $N_n' = 3N_n+1$, then the result of Collatz odd transform $N_n'' = N_n' / 2$.

Let the expression of N_n':

$$N_n' = d_n' d_{n-1}' \ldots d_i' \ldots d_3' d_2' d_1' \tag{3-2}$$

Let the expression of N_n'':

$$N_n'' = d_n'' d_{n-1}'' \ldots d_i'' \ldots d_3'' d_2'' d_1'' \tag{3-3}$$

Take any n-bit positive integer N_n, perform Collatz transform on N_n, the odd-even probability of the transform results is not only related to the number in its one-digit d_1, but also related to the odd-even characteristic of the number in its ten-digit d_2. In this way, the n-bit positive integer N_n has 10 ways to get the number in its one-digit d_1, and two ways to get the number in its ten-digit d_2. There are 20 different selection ways, perform Collatz transform on the 20 selection ways, and the odd-even probability characteristics are shown in the following table.

Table 3-1 The odd-even probability characteristic of the Collatz transform results

$N_n = d_n d_{n-1} \ldots d_i \ldots d_3 d_2 d_1$			$N_n' = d_n' d_{n-1}' \ldots d_i' \ldots d_3' d_2' d_1'$			$N_n'' = d_n'' d_{n-1}'' \ldots d_i'' \ldots d_3'' d_2'' d_1''$	
$d_n d_{n-1} \ldots d_i \ldots d_3$	d_2	d_1	$d_n' d_{n-1}' \ldots d_i' \ldots d_3'$	d_2'	d_1'	$d_n'' d_{n-1}'' \ldots d_i'' \ldots d_3'' d_2''$	d_1''
XX . . . X . . . X	odd	0	N/A	N/A	N/A	XX . . . X . . . XX	5
	even						0
XX . . . X . . . X	Odd	1	XX . . . X . . . X	odd	4	XX . . . X . . . XX	7
	even			even	4		2

XX . . . X . . . X	odd	2	N/A	N/A	N/A	XX . . . X . . . XX	6
	even						1
XX . . . X . . . X	odd	3	XX . . . X . . . XX	odd	0	XX . . . X . . . XX	0
	even			even	0		5
XX . . . X . . . X	odd	4	N/A	N/A	N/A	XX . . . X . . . XX	7
	even						2
XX . . . X . . . X	odd	5	XX . . . X . . . X	odd	6	XX . . . X . . . XX	3
	even			even	6		8
XX . . . X . . . X	odd	6	N/A	N/A	N/A	XX . . . X . . . XX	8
	even						3
XX . . . X . . . X	odd	7	XX . . . X . . . X	odd	2	XX . . . X . . . XX	6
	even			even	2		1
XX . . . X . . . X	odd	8	N/A	N/A	N/A	XX . . . X . . . XX	9
	even						4
XX . . . X . . . X	odd	9	XX . . . X . . . X	odd	8	XX . . . X . . . XX	9
	even			even	8		4

*X：0, 1, 2, 3, 4, 5, 6, 7, 8 or 9;　*N/A: Null;　*even: 0, 2, 4, 6 or 8;　*odd: 1, 3, 5, 7 or 9.

From Table 3-1, it can be concluded that there are 20 different ways to select N_n, perform Collatz transform on the 20 different selected N_n and get the 20 different transform results N_n''. The probability of d_1'' being 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 is 0.1. So the odd-even probability of the Collatz transform results is 0.5. So we get the following theorem.

(Theorem 3.1) Collatz transform probability distribution theorem: In the process of Collatz transforms, the probability of Collatz even transforms and that of Collatz odd transforms are equal, and both of them are 0.5.

3.2 Scale coefficient of Collatz transform

The Collatz transform is divided into the Collatz even transform and the Collatz odd transform. If N_e is even, perform Collatz even transform on N_e and the transform result $N_e'' = N_e / 2$. The scale coefficient of Collatz even transform:

$$P_e = 1/2 \quad (3\text{-}4)$$

If N_o is odd, perform Collatz odd transform on N_o and the transform result $N_o'' = (3N_o + 1)/2$. The scale coefficient of Collatz odd transform:

$$P_o = N_o''/N_o = ((3N_o + 1) / 2) / N_o$$

$$P_o = (3 + 1/ N_o) / 2 \quad (3\text{-}5)$$

The scale coefficient of Collatz odd transform is approximately equal to 1.5. When $N_o >= 1000$, the scale coefficient of Collatz odd transform:

$$P_o < 1.501 \quad (3\text{-}6)$$

3.3 The establishment condition of equivalent Collatz conjecture

Performing Collatz transform on N for m times, the transform results have m elements and the final transform result N_m is less than N. The Collatz transform result sequence is as follow:

$$\{N_1, N_2, N_3 \ldots N_k \ldots N_{m-1}, N_m\} \qquad (3\text{-}7)$$

The Collatz transform results have m elements, suppose there are i elements of Collatz even transform results and j elements of Collatz odd transform results, and i + j = m. The Collatz transform odd result sequence as below:

$$\{N_{o1}, N_{o2}, N_{o3} \ldots N_{oj}\} \qquad (3\text{-}8)$$

According to the definition of equivalent Collatz transform, $N_{o1}N_{o2}$, $N_{o3} \ldots N_{oj}$ are all greater then N_m and N. The final transform result N_m can be expressed as follow:

$$N_m = (1/2)^i ((3 + 1/N_{o1}) / 2) ((3 + 1/N_{o2}) / 2) \ldots ((3 + 1/N_{oj}) / 2) N$$

$$N_m = (1/2)^i (1/2)^j (3 + 1/N_{o1}) (3 + 1/N_{o2}) \ldots (3 + 1/N_{oj}) N$$

$$N_m = (1/2)^i (1/2)^j (3^j) (1 + 1/(3N_{o1})) (1 + 1/(3N_{o2})) \ldots (1 + 1/(3N_{oj})) N$$

$$N_m = (3^j/2^m) (1 + 1/(3N_{o1})) (1 + 1/(3N_{o2})) \ldots (1 + 1/(3N_{oj})) N \qquad (3\text{-}9)$$

Then

$$(3^j/2^m) (1 + 1/(3N_{o1})) (1 + 1/(3N_{o2})) \ldots (1 + 1/(3N_{oj})) = N_m / N$$

Because $N_m < N$, i.e. $N_m / N < 1$.

Then $(3^j/2^m) (1 + 1/(3N_{o1})) (1 + 1/(3N_{o2})) \ldots (1 + 1/(3N_{oj})) < 1$

$$(3^j) (1 + 1/(3N_{o1})) (1 + 1/(3N_{o2})) \ldots (1 + 1/(3N_{oj})) < 2^m \qquad (3\text{-}10)$$

Because $3^j < (3^j) (1 + 1/(3N_{o1})) (1 + 1/(3N_{o2})) \ldots (1 + 1/(3N_{oj}))$

Then $3^j < 2^m$

Take the logarithm of 10 on both sides of the above formula:

$$j \operatorname{Log} 3 < m \operatorname{Log} 2$$

$$j < (\operatorname{Log}2 / \operatorname{Log}3)\, m$$

$$\operatorname{Log}2 / \operatorname{Log}3 = 0.30103/0.47712$$

$$= 0.63093$$

Let $j < 0.63m \qquad (3\text{-}11)$

$$i = m - j$$

Then $i > 0.37m$ (3-12)

From equations (3-11) and (3-12), it can be concluded:

(Theorem 3.2) The establishment condition of the equivalent Collatz conjecture: Take any positive integer N greater than 1, perform Collatz transforms on N for m times, if the number of Collatz even transform $I > 0.37m$, or the number of Collatz odd transform $j < 0.63m$, the Collatz transform result N_m must be less than its initial value N.

According to **(Theorem 3.1) Collatz transform probability distribution theorem,** the probability of Collatz even transform and that of Collatz odd transform are equal, both of them are 0.5. Therefore, take any positive integer N greater than 1 and perform m times of Collatz even transform, if m is large enough, the number of Collatz even transform and that of Collatz odd transform are equal to 0.5m, that is, $i = j = 0.5m$. With **(theorem 3.2) Establishment condition of the equivalent Collatz conjecture** and **(theorem 3.1) Collatz transform probability distribution theorem**, the following conclusion is obtained.

(Theorem 3.3) Equivalent Collatz theorem: Take any positive integer N greater than 1, perform Collatz transforms on N for m times and m is large enough, the Collatz transform result N_m must be less than its initial value N.

If the equivalent Collatz conjecture is true, the Collatz conjecture must also be true. We can get the following conclusion.

(Theorem 3.4) Collatz theorem: Take any positive integer N greater than 1, perform Collatz transforms on N for m times and m is large enough, the Collatz transform result must be 1.

4. Number of equivalence Collatz transforms and Number of Collatz transforms

In theorem 3.3 and theorem 3.4, the number of Collatz transforms is required to be large enough. Below, we will calculate the required number of equivalent Collatz transforms and the number of Collatz transforms.

For any n-bit positive integer N, let's calculate m_{ec}, the required number of equivalent Collatz transforms. In the process of Collatz transforms, the probability of even transforms and that of odd transforms are both 0.5, This is the same as the probability of front and back sides in the process of randomly flipping coins. Take any positive integer N, perform Collatz transforms on N for m times, let i represent the number of even transforms. Then i is a binomial random variable whose parameter is (m, 0.5), and the probability function of the binomial random variable i is:

$$p(i)=\binom{m}{i}0.5^m \qquad i = 0, 1, 2, \dots, m \qquad (4\text{-}1)$$

When m is large enough, the binomial distribution can be simplified to normal distribution, and the probability density function of i as a random variable is:

$$f（i）=\frac{1}{\sigma\sqrt{2\pi}}\ e^{-(i-\mu)^2/2\sigma^2} \qquad （4\text{-}2）$$

Where μ= m/2 and σ=$\sqrt{m}$ / 2.

According to (theorem 3.2), the number of Collatz even transforms i>0.37m，let i = 0.37m, then formula (4-2) is:

$$f（i）=\frac{1}{\sigma\sqrt{2\pi}}\ e^{-(i-\mu)^2/2\sigma^2}$$

$$=\frac{2}{\sqrt{2\pi m}}\ e^{-\left(0.37m-\frac{m}{2}\right)^2/(\frac{m}{2})}$$

$$=\frac{0.798}{\sqrt{m}}\ e^{-2(-0.13m)^2/m}$$

$$=\frac{0.798}{\sqrt{m}}\ e^{-0.0338m}$$

Then probability density function:

$$f\ (i=0.37m) = \frac{0.798}{\sqrt{m}}\ e^{-0.0338m} \qquad (4\text{-}3)$$

For the normal distribution with μ = m/2, the density function f (i) is monotonically increasing function when the random variable i is in the range of 0 to 0.37m. Therefore, when random variable i is between 0 and 0.37m, the probability is:

$$P（i<=0.37m）=\int_0^{0.37m}\frac{1}{\sigma\sqrt{2\pi}}\ e^{-(j-\mu)^2/2\sigma^2}di$$

$$<\int_0^{0.37m}\frac{0.798}{\sqrt{m}}\ e^{-0.0338m}di$$

$$<(0.37m)\ \frac{0.798}{\sqrt{m}}\ e^{-0.0338m}$$

That is $P（i<=0.37m）< 0.296\sqrt{m}\,e^{-0.0338m}$ (4-4)

Considering that n-bit positive integers has a total of (10^n-10^{n-1}) positive integers, for all n-bit positive integers N_n, make m times of Collatz transforms, and its equivalent Collatz transform result N_m is less than N_n, we get:

$$P(i <= 0.37m) * (10^n - 10^{n-1}) < 1 \qquad (4\text{-}5)$$

Because（10^n -10^{n-1}）< 10^n , simplify formula (4-12):

$$P(i <= 0.37m) * 10^n < 1 \qquad (4\text{-}6)$$

Take P (i< = 0.37m) = $0.296\sqrt{m}\, e^{-0.0338\mathrm{m}}$; $10^n = e^{2.30n}$

Then the left side of formula (4-6):

$$P(i <= 0.37m) * 10^n$$

$$= 0.296\sqrt{m}\, e^{-0.0338\mathrm{m}} * e^{2.30n}$$

$$= 0.296\sqrt{m}\, e^{2.30n - 0.0338m}$$

let 2.3n-0.0338m = 0，then m = 68.05n. Take m = 100n and n=10, then the left side of formula (4-16):

$$P (i<=0.37m)* 10^n = 0.296\sqrt{100n}\, e^{2.30n - 0.0338(100n)}$$

$$= 2.96\sqrt{n}\, e^{-1.08n}$$

$$= 9.36 * e^{-10.8}$$

$$= 9.36 * 0.0000204$$

$$= 0.000191$$

The value 0.000191 is far less than 1, and the larger n is, the smaller the value is. So when n>=10, and let the number of equivalent Collatz transform m = 100n, formula (4-6) must be true, that is, the equivalent Collatz conjecture must be true.

For n-bit positive integers with n<10, through computer numerical calculation，the number of equivalent Collatz transforms m is also less than 100n. Table 4-1 shows the number of equivalent Collatz transforms from 1-bit positive integers to 10-bit positive integers

Table 4-1 The number of equivalent Collatz transforms（n is from 1 to 10）

n	Positive integer range	The maximum number of equivalent Collatz transforms	The corresponding positive integer	m=100n
1	1 -9	7	7	100
2	10 - 99	59	27	200
3	100 - 999	81	703	300
4	1000 - 9999	81	1407	400
5	10000 - 99999	135	35655	500
6	100000 -999999	176	626331	600
7	1000000 -9999999	246	8088063	700
8	10000000 -99999999	376	63728127	800
9	100000000 -999999999	395	217740015	900
10	1000000000 - 9999999999	447	2788008987	1000

From the above analysis, it can be concluded that for all n-bit positive integers N_n, which have (10^n-10^{n-1}) elements. When the number of equivalent Collatz transforms m=100n, formula (4-13) must be true; That is, for all n-bit positive integers N_n, perform Collatz transforms on N_n for m_{ec} times and m_{ec}=100n, the Collatz transform result N_m must be less than its initial value N_n. So the equivalent Collatz conjecture is true, and m_{ec}=100n is the number of equivalent Collatz transforms for all n-bit positive integers.

For all n-bit positive integers N_n, $n <= 1 + \log N_n$. The number of equivalent Collatz transforms:

$$m_{ec}=100n$$

$$m_{ec} =100 * (1+\log N_n) \tag{4-7}$$

From Formula (4-7) and (2-2), for all n-bit positive integers N_n, The number of Collatz transforms from N_n to 1 is:

$$m_c =(100 * (1+\log N_n))* (N_n - 1) \tag{4-8}$$

5. The results of Collatz transforms not equal

In the above proof, there is an implicit premise: The n-bit positive integer N_n, its Collatz transform result N_m cannot be equal to the initial value N_n. We have the following theorem.

[Theorem 5.1] The inequality theorem of Collatz transform results: For any n-bit positive integer N_n, its any Collatz transform result N_m must not be equal to the initial value N_n.

For any n-bit positive integer N, perform m times Collatz transforms on N and the final transform result N_m. Assume the Collatz transform result sequence of N is as follow:

$$\{N_1,\ N_2,\ N_3 \ldots N_k \ldots N_{m-1}, N_m\} \tag{5-1}$$

The above Collatz transform result sequence has m elements. Suppose there are i even elements and j odd elements, and i + j = m. Select j odd elements from sequence (5-1) and the Collatz transform odd result sequence is as follow:

$$\{N_{o1},\ N_{o2},\ N_{o3} \ldots N_{oj}\} \tag{5-2}$$

Then the final transform result N_m can be expressed as follow:

$$N_m = (1/2)^i ((3N_{o1} + 1)/(2N_{o1}))((3N_{o2} + 1)/(2N_{o2}))\ldots((3N_{oj} + 1)/(2N_{oj}))\, N \tag{5-3}$$

We will prove that $N_m = N$ must not be true by using reduction to absurdity below.

If N_m=N, from formula (5-3), we get the following equation:

$$1 = (1/2)^i ((3N_{o1} + 1)/(2N_{o1}))((3N_{o2} + 1)/(2N_{o2}))\ldots((3N_{oj} + 1)/(2N_{oj})) \tag{5-4}$$

$$1 = (1/2)^i(1/2)^j ((3N_{o1} + 1)/N_{o1})((3N_{o2} + 1)/N_{o2})\ldots((3N_{oj} + 1)/N_{oj})$$

$$1 = (1/2)^{i+j} ((3N_{o1} + 1)/N_{o1})((3N_{o2} + 1)/N_{o2})\ldots((3N_{oj} + 1)/N_{oj})$$

$$2^m = (3N_{o1} + 1)(3N_{o2} + 1)\ldots(3N_{oj} + 1) / (N_{o1} N_{o2} \ldots N_{oj}) \tag{5-5}$$

In the above formula (5-5), the expression on the right is a fraction. The numerator $(3N_{o1} + 1)(3N_{o2} + 1)\ldots(3N_{oj} + 1)$ is the product of j even integers, and the product must be an even integer; The denominator $(N_{o1} N_{o2} \ldots N_{oj})$ is the product of j odd integers, and the product must be an odd integer. An even integer cannot be completely divided by an odd integer, then the right side of the formula (5-5) cannot be an integer. However, the left side of the formula (5-5) is 2^m that must be an integer. Therefore, formula (5-5) must not be true, and formula (5-4) must not be true too, that is, $N_m = N$ must not be true.

To sum up, for any n-bit positive integer N, perform Collatz transforms on N for m times，any Collatz transform result N_m must not be equal to the initial value N. Even if the number of Collatz transforms m tends to ∞, the Collatz transform result N_m still must not be equal to N. So as to ensure that the transform results will not enter a dead loop during the Collatz transform process.

6. Conclusions

This paper redefined the Collatz conjecture and proposed the equivalence Collatz conjecture: Take any positive integer N greater than 1; perform Collatz transforms on N; repeat the process m_{ec} times, and the transform result $N_m < N$. The equivalence Collatz conjecture is a necessary and sufficient condition for the Collatz conjecture.

The Collatz transform is divided into Collatz even transform and Collatz odd transform. By analyzing the probability characteristics of Collatz transform, we obtained the **Collatz transform probability distribution theorem:** In the process of Collatz transforms, the probability of Collatz even transforms and that of Collatz odd transforms are equal, and both of them are 0.5. The scale coefficient of Collatz even transform is 1/2, and the scale coefficient of Collatz odd transform is less than 3/2. Therefore, we obtained t**he establishment condition of the equivalent Collatz conjecture:** Take any positive integer N greater than 1, perform Collatz transforms on N for m times, if the number of Collatz even transform i>0.37m, or the number of Collatz odd transform j<0.63m, the final Collatz transform result N_m must be less than its initial value N.

Based on binomial distribution and normal distribution, it is deduced from calculation that any positive integer N greater than 1, the number of equivalent Collatz transforms:

$$m_{ce} = 100 * (1+ \log N)$$

Then, the Number of Collatz transforms from N to 1:

$$m_c = (100 * (1+ \log N)) * (N-1)$$

Furthermore, for any n-bit positive integer N, perform Collatz transforms on N for m times，any Collatz transform result N_m must not be equal to the initial value N.

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