Confirmation of Collatz conjecture correctness eliminating looping and divergence

Tsuneaki Takahashi

Abstract

Investigation is tried about correctness of Collatz conjecture. There reverse procedure of Collatz conjecture procedure is used. Possibility of looping and divergence is eliminated.

1. Introduction

Considering reverse procedure of Collatz conjecture procedure, the original conjecture is recognized same as following conjecture.

'Series of positive odd integers created by the reverse procedure are starting from integer 1 to every positive odd integer.'

2. Expansion and Contraction

Collatz conjecture procedure one step calculation is defined as Contraction, its reverse procedure calculation is defined as Expansion.

A. Expansion

Positive odd integer NOI is represented as follows on some condition of n and P.

$$NOI = 3n + P$$

n; positive integer

P; 0,1,2

NOO is calculated on following formula.

$$NOO = \frac{NOI \times 2^m - 1}{3} = \frac{(3n + P) \times 2^m - 1}{3}$$

Following conditions are required.

a) NOI, NOO are positive odd integer

b) Dividing by 3 in the formula has no remainder.

For these conditions, there are also other conditions regarding to n, m for each P value. Then calculation of NOO is done.

In the case of P=0:

In this case, calculation cannot be done based on b) for all n because the formula has always remainder, therefore there is no NOO for this NOI.

$$NOO = \frac{(3n + 0) \times 2^m - 1}{3} = \frac{3n \cdot 2^m - 1}{3}$$

Sample 1

n=25, m=4

$$NOO = \frac{(3 \times 25 + 0) \times 2^4 - 1}{3} = \frac{3 \times 25 \times 16 - 1}{3} = 399 \cdots 2$$

In the case of P=1:

m=even integer from 2: 2,4,6,8,10・・・ based on required condition b)

n=even integer from 0: 0,2,4,6・・・ based on required condition a)

$$NOO = \frac{(3n + 1) \times 2^m - 1}{3} = \frac{3n \cdot 2^m + (2^m - 1)}{3} = \frac{3n \cdot 2^m + (2 + 1)(\,) \cdots}{3}$$

Sample 2

n=26, m=4

$$NOO = \frac{(3 \times 26 + 1) \times 2^4 - 1}{3} = \frac{3 \times 26 \times 2^4 + 2^4 - 1}{3} = 421$$

Sample 3

n=26, m=8

$$NOO = \frac{(3 \times 26 + 1) \times 2^8 - 1}{3} = \frac{3 \times 26 \times 2^8 + 2^8 - 1}{3} = 6741$$

In the case of P=2:

m=odd integer from 1: 1,3,5,7,9・・・ based on required condition b)

n=odd integer from 1: 1,3,5,7・・・ based on required condition a)

$$NOO = \frac{(3n + 2) \times 2^m - 1}{3} = \frac{3n \cdot 2^m + (2^{m+1} - 1)}{3} = \frac{3n \cdot 2^m + (2 + 1)(\,) \cdots}{3}$$

Sample 4

n=27, m=3

$$NOO = \frac{(3 \times 27 + 2) \times 2^3 - 1}{3} = \frac{3 \times 27 \times 2^3 + 2^4 - 1}{3} = 221$$

B. Contraction

NOI is calculated on following formula from NOO.

$$NOI = \frac{NOO \times 3 + 1}{2^m}$$

Sample 5

NOO=221

$NOI = \frac{NOO \times 3+1}{2^m} = \frac{221 \times 3+1}{2^m} = \frac{664}{2^m} = 83$ m=3

These procedures have following characteristics.

- Multiple NOOs can be created from an NOI by Expansion.
- Same NOI is determined from multiple NOOs by Contraction.

Sample 6; from sample 2, 3,

421(NOO)－79(NOI)

6741(NOO)－79(NOI)

3. Characteristics of Expansion and Contraction

We can have following recognitions about Expansion and Contraction.

Expansion view point;

All positive odd integers have following five elements.

Regarding to the series of the integer from Expansion, there are three integers,

Before (member) number, Current (member) number (NOI),

Next (member) number (NOO).

Regarding to calculation of Next number, two numbers are used,

Base number (=Current number), Up level (2^m).

Contraction view point;

All positive odd integers have following five elements.

Regarding to the series of the integer from Contraction, there are three integers,

Before number, Current number (NOO), Next number (NOI).

Regarding to calculation of Next number, two numbers are used,

Base number (=Next number), Up level (2^m).

Expansion is Assembling Next number from Base number and Up level (2^m)

Base number =Current number → Next number

Up level (2^m)

Sample; Base number=5, Up level=2, Next number=3

Contraction is Disassembling Next number (NOI) and Up level (2^m) from Current number (NOO).

Current number (NOO) → Next number (NOI)

→ Up level (2^m)

Sample; Current number=3, Next number=5, Up level=2

Regarding to Contraction;

When Current number is all positive odd integer except multiple of 3, Before number actually exists.

When Current number is all positive odd integer, Next number actually exists.

Regarding to Expansion;

When Current number is all positive odd integer, Before number actually exists. (1)

When Current number is all positive odd integer except multiple of 3, Next number actually exists.

Specialty of integer 1;

Positive odd integer which satisfies $n = \frac{n \times 3+1}{2^m}$ is only n=1.

Based on this, we can say following.

For Expansion, when Current number is integer 1, Before number is also 1.

Therefore, only integer 1 can be starting odd integer of Expansion because other odd integer would have different Before number from Current number. (2)

For Contraction, when Current number is integer 1, Next number is also 1.

Therefore, only integer 1 can be terminal odd integer of Contraction because other odd integer would have different Next number from Current number.

4. Series of NOOs

Based on (1), all NOO of Extraction have Before number. On (2), only integer 1 can be starting point of series of NOO. Therefore, all positive odd integers are on any series of NOO from 1. (3)

If a NOO (integer nx, for example) of Expansion is not on the series of NOO from 1, its possible cases may be following two situations.

A: If the series of NOO from integer nx returns to nx, it makes looping. In this case, relevant series of NOI from Contraction never reaches to 1. Therefore, members of the looping are not on the series of NOO from 1

B: About the series of NOO, if it has no looping also it doesn't start from 1, relevant series of Contraction does not stop at integer 1. Therefore, there is no place other than going toward infinity as a limiting value.

5. Divergence

In the case of 4. B, starting point of NOO series for Expansion goes forward to infinite large number as a limiting value.

On the view of Contraction, this means the series of NOI divergent far away to large integer without reaching to integer 1.

On the view of Expansion, this means series of NOO comes from far away to Current number.

But in this case, Expansion calculation is impossible for infinity large integer because, for example, infinity+1=infinity, infinity×1=infinity.

Therefore, this situation NOO coming from far away should not exist because it is contradicted with (1).

6. Looping

. Collatz conjecture procedure is represented as follow.

$$(3((3((3\times n+1)/2^{m_1})+1)/2^{m_2})+1)2^{m_3}\cdots$$

$$=\frac{3^i n}{2^{m_1+m_2+\cdots+m_i}}+\frac{3^{i-1}}{2^{m_1+m_2+\cdots+m_i}}+\frac{3^{i-2}}{2^{m_2+\cdots+m_i}}+\cdots+\frac{3^1}{2^{m_{i-1}+m_i}}+\frac{3^0}{2^{m_i}} \qquad (4)$$

n: positive odd integer

If this procedure has looping, following equation is satisfied.

$$\frac{3^i n}{2^{m_1+m_2+\cdots+m_i}}+\frac{3^{i-1}}{2^{m_1+m_2+\cdots+m_i}}+\frac{3^{i-2}}{2^{m_2+\cdots+m_i}}+\cdots+\frac{3^1}{2^{m_{i-1}+m_i}}+\frac{3^0}{2^{m_i}}=n.$$

$$\left(2^{m_1+m_2+\cdots+m_i}-3^i\right)n=3^{i-1}+2^{m_1}\cdot 3^{i-2}+\cdots+2^{m_1+m_2+\cdots+m_{i-2}}\cdot 3^1+2^{m_1+m_2+\cdots+m_{i-1}}\cdot 3^0. \qquad (5)$$

Characteristics of right side (5) are followings.

It is expanded using power of 3 from 3^0 to 3^{i-1} term. (a)

Coefficient for each power of 3 is represented as one bit of binary (ex. $2^{m_1+m_2+\cdots+m_{i-2}}$), also each power of 3 and its coefficient has following relation.

$2^{m_1+m_2+\cdots+m_{i-j}}\cdot 3^{j-1}$ j: integer, $1,2,3\cdots i$, $m_0=0$. (b)

Existence of positive odd integer n solution for (5) means that there is looping.

We investigate whether such n solution could exist or not.

Left side of (5) could be expanded and become same format as right side. (6) defines m for it.

$$\mathrm{m}=(m_1+m_2+\cdots+m_i)/i \qquad (6)$$

Left side of (5) becomes (7).

$$\left(2^{mi}-3^i\right)n=(2^m-3)\left(3^{i-1}+2^m\cdot 3^{i-2}+\cdots+2^{m(i-2)}\cdot 3^1+2^{m(i-1)}\cdot 3^0\right)n. \qquad (7)$$

Then (5) becomes (8).

$$(2^m - 3)(3^{i-1} + 2^m \cdot 3^{i-2} + \cdots + 2^{m(i-2)} \cdot 3^1 + 2^{m(i-1)} \cdot 3^0)n$$
$$= 3^{i-1} + 2^{m_1} \cdot 3^{i-2} + \cdots + 2^{m_1+m_2+\cdots+m_{i-2}} \cdot 3^1 + 2^{m_1+m_2+\cdots+m_{i-1}} \cdot 3^0 \quad (8)$$

Because both sides are same format, every coefficient should be equal. It makes following equations.

$$(2^m - 3)n = 1 \quad (9)$$
$$(2^m - 3)n \cdot 2^m = 2^{m_1}$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$
$$(2^m - 3)n \cdot 2^{m(i-1)} = 2^{m_1+m_2+\cdots+m_{i-1}}$$

n is positive odd integer. m is positive integer comparing both sides. Therefore, on (9),

$$m = 2, n = 1$$

Resolving all equations sequentially, we can get result (10).

$$m_1 = m_2 = \cdots = m_i = m = 2 \quad (10)$$

This means that this looping is only one member looping or self-looping when n=1. Therefore, this is same as the result of 3., that is, n=1 can be terminal point of Contraction or starting point of Expansion.

Considering above process, we can find no other looping.

7. Conclusion

On (3), all positive odd integers are on any series of NOO from integer 1 if there is no looping or divergence. On above 5 and 6, there is no divergence and no looping (except integer 1). Therefore, all positive odd integers are on any series of NOO from integer 1. On the view of Contraction which is reverse procedure of Expansion and same procedure as Collatz conjecture process, this means every series of NOI reach to integer 1. Therefore, Collatz conjecture should be correct.