The Time Dilation Conundrum and the Ageing of the Twins

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Abstract

In previous works we establish a mathematical expression like the time dilation expression, a time dilation-like (time dilation is valid in relation to a preferred frame) and considered by the standard formulation the time dilation expression, valid reciprocally. This conundrum has also been previously eliminated. Now we show how we can use this time dilation-like expression eliminating the twin paradox conundrum that standard formulation is unable to accomplish, since erroneously attribute the meaning of a reciprocal relation to the ageing of the twins. Therefore, we calculate the classic example of the twins whatever the frames considered. The twin that returns is the younger because the cumulative effect of the ageing is not reciprocal. Since the time dilation like exist and can be used, originating the idea of "seeing the other twin ageing slower"- the origin of the conundrum. This cannot subsist because the relation between ageing is a relation between proper times. The time dilation-like expression is a relation between proper times only for the preferred frame.

Introduction

In previous works [1-17] particularly in "The physical meaning of synchronization and simultaneity in Special Relativity" [1] it is criticized the approach of Einstein [18] based on the postulates of the isotropy of speed light in every frame and the equivalence of every frame. Several works, some very recent, point out the importance of this discussion about the foundations of Mathematics, Philosophy, Relativity, Quantum Mechanics, Cosmology and Biophysics [19-103]. The consequent Principle of Relativity has been also considered in the articles "On the Consistency between the Assumption of a Special System of Reference and Special Relativity" [10] and "The Principle of Relativity and the Indeterminacy of Special Relativity" [12]. In a more recent work "Speakable and Unspeakable in Special Relativity: time readings and clock rhythms" [14] it is referred the consequences of these analysis particularly the physical meaning of time dilation and Lorentz-FitzGerald contraction mathematical expressions.

Now we show how we can use a time dilation-like expression eliminating the twin paradox conundrum that standard formulation is unable to accomplish, since erroneously attribute the meaning of a reciprocal relation to the ageing of the twins.

Twin A'' is moving through the x' axis of S' with Einstein speed $|V_E'|$. At $x' = l_1$ the twin return with speed $|V_E'|$ to the origin of S'. The proper times of the twin A'', τ'' for the trip to and τ'' for the trip from, are calculated. The proper times of the twins located at S', τ', is also calculated between the same events. We show how the standard formulation misinterpret the relation of proper times, the ageing of the twins at S'' and S'. For that we calculate through the time dilation – like equation the proper times τ'' with the Lorentzian times. It is easy to show the misinterpretation of the standard formulation through the equality of the two-way trip result, that is consistent with the one-way result, as expected.

I. The Time Dilation Conundrum

Consider in reference frame *S´* a section of *x´* axis with proper length l_1 moving with speed v_1 in relation to *S* EF (Einstein frame EF is the frame where the one-way speed of light is isotropic with the two-way value c) where is located another *x* axis section of frame *S* with proper length *l* (Fig.1).

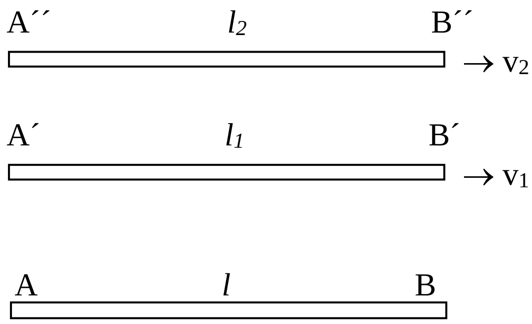

Fig. 1 Frame S´ represented by a rod with length l_1 is moving with speed v_1 in relation to frame *S*, EF, rod with length l. The extremities of the rods coincide simultaneously and therefore, can synchronize clocks at *A*, *A´*, *A´´* and *B*, *B´*, *B´´* [49, 50]. A twin located at *A´* we designate by twin *A´*. The same rule for the other positions.

Since rod *S´* is moving with speed v_1 twin *A´* is also moving with speed v_1. The time dilation is

$$d\tau' = dt\sqrt{1-\frac{{v_1}^2}{c^2}} \qquad (1)$$

τ' is the ageing of twin *A´*, the proper time. The symbol *t* is the synchronized time at *S*, EF. Therefore $dt = d\tau$. Therefore eq. 1 means the dilation of time. Twin *A´* or twin *B´* is ageing slower than twin *A*, or twin *B*. Similarly, we have

$$d\tau'' = dt\sqrt{1-\frac{{v_2}^2}{c^2}} \qquad (2)$$

Therefore, we have

$$d\tau'' = d\tau' \frac{\sqrt{1-\frac{{v_2}^2}{c^2}}}{\sqrt{1-\frac{{v_1}^2}{c^2}}} \qquad (3)$$

Introducing Lorentzian time through a desynchronization [1, 10, 12, 37]

$$t_L' = t' - \frac{v_1}{c^2}x' \qquad (4)$$

and from

$$dx' = V_E' dt_L' = V' dt' = V' d\tau' \quad (5)$$

with

$$V_E' = \frac{v_2 - v_1}{1 - \frac{v_1 v_2}{c^2}} \qquad (6)$$

and

$$V' = \frac{v_2 - v_1}{1 - v_1^2} \qquad (7)$$

we obtain

$$d\tau' = dt' = dt_L'\left(1 + V_E' \frac{v_1}{c^2}\right) \quad (8)$$

We have from Lorentz Transformation

$$t_L'' = \frac{t_L' - \frac{V_E'}{c^2}x'}{\sqrt{\left(1 - \frac{{V_E'}^2}{c^2}\right)}} \qquad (9)$$

and

$$dt_L'' = \frac{dt_L' - \frac{V_E'}{c^2}dx'}{\sqrt{\left(1 - \frac{{V_E'}^2}{c^2}\right)}} = \frac{dt_L' - \frac{V_E'}{c^2}V_E' dt_L'}{\sqrt{\left(1 - \frac{{V_E'}^2}{c^2}\right)}} = d\tau'' = dt_L'\sqrt{\left(1 - \frac{{V_E'}^2}{c^2}\right)} \quad (10)$$

And from (3)

$$d\tau'' = d\tau' \frac{\sqrt{1 - \frac{{v_2}^2}{c^2}}}{\sqrt{1 - \frac{{v_1}^2}{c^2}}} = dt_L^{'} \sqrt{\left(1 - \frac{V_E^{'2}}{c^2}\right)} \quad (11)$$

From (8) and (11)

$$d\tau'' = d\tau' \frac{\sqrt{\left(1 - \frac{V_E^{'2}}{c^2}\right)}}{1 + V_E^{'} \frac{v_1}{c^2}} = dt_L^{'} \sqrt{\left(1 - \frac{V_E^{'2}}{c^2}\right)} \quad (12)$$

$$\frac{d\tau''}{d\tau'} = \frac{\sqrt{1 - \frac{{v_2}^2}{c^2}}}{\sqrt{1 - \frac{{v_1}^2}{c^2}}} = \frac{\sqrt{1 - \frac{V_E^{'2}}{c^2}}}{1 + V_E^{'} \frac{v_1}{c^2}} \qquad (13)$$

The standard formulation consider

$$d\tau'' = dt_L^{'} \sqrt{\left(1 - \frac{V_E^{'2}}{c^2}\right)} \qquad (14)$$

the relation of the ageing of the twins [97] – this is the origin of the conundrum:

"The twin paradox is the consequence of the following thought experiment. System *O* is at rest and system *O´* is moving. Therefore, the clock in *O´* ticks slower than that in *O*. Thus, for example, if the two clocks are initially synchronized to read *t=t´=0*, after a while they may show *t=10* (some arbitrary unit of time) but *t´=6*. Therefore, an observer moving with system *O´* will be younger than that in system *O*. However, as seen by the observer in *O´*, she is at rest and system *O* is moving away from her. Therefore, according to the observer in *O´*, the observer in *O* should be younger. This is the foundation of the twin paradox, wich is stated as follows: Twin *A* is on Earth and twin *B* travels to a distant star with a speed close to the speed of light. Afterward, she returns to Earth with the same speed. When they reunite, according to twin *A*, twin *B* must be younger, but according to twin *B*, twin *A* must be younger "[97].

II. The Ageing of the Twins

Twin A'' travel from A' to B' (see fig.1) with $v_2 > v_1$. Therefore, from (14) we obtain

$$\tau''(A'B') = \Delta t_L'(A'B')\sqrt{1-\frac{V_E'^2}{c^2}} = \frac{l_1}{|V_E'|}\sqrt{1-\frac{V_E'^2}{c^2}} \qquad (15)$$

and return from B' to A' with the same Einstein speed (however now $v_2 < v_1$) we have

$$\tau''(B'A') = \Delta t_L'(B'A')\sqrt{1-\frac{V_E'^2}{c^2}} = \frac{l_1}{|V_E'|}\sqrt{1-\frac{V_E'^2}{c^2}} \qquad (16)$$

Therefore

$$\tau''(A'B') + \tau''(B'A') = [\Delta t_L'(A'B') + \Delta t_L'(B'A')]\sqrt{1-\frac{V_E'^2}{c^2}} \qquad (17)$$

$$\tau''(A'B') + \tau''(B'A') = \frac{2l_1}{|V_E'|}\sqrt{1-\frac{V_E'^2}{c^2}} \qquad (18)$$

for trips $A'B'$ and $B'A'$ with the same Einstein speed $|V_E'|$.

The ageing of the twins at reference frame S´ is given by (eq. (8))
$d\tau' = dt' = dt_L'\left(1 + V_E'\frac{v_1}{c^2}\right)$

Therefore

$$\Delta t'(A'B') = \frac{L_1}{V_E'}\left(1 + V_E'\frac{v_1}{c^2}\right) \qquad (19)$$

and

$$\Delta t'(B'A') = \frac{L_1}{V_E'}\left(1 - |V_E'|\frac{v_1}{c^2}\right) \qquad (20)$$

Therefore

$$\Delta t'(A'B') + \Delta t'(B'A') = \frac{2L_1}{V_E'} \qquad (21)$$

Therefore from (17) and (21)

$$\tau''(A'B') + \tau''(B'A') = [\Delta t'(A'B') + \Delta t'(B'A')]\sqrt{1-\frac{V_E'^2}{c^2}} \quad (22)$$

but

$$\tau''(A'B') \neq \Delta t'(A'B')\sqrt{1-\frac{V_E'^2}{c^2}} \quad (23)$$

or

$$\tau''(A'B') \neq \tau'(A'B')\sqrt{1-\frac{V_E'^2}{c^2}} \quad (24)$$

and

$$\tau''(B'A') \neq \Delta t'(B'A')\sqrt{1-\frac{V_E'^2}{c^2}} \quad (25)$$

or

$$\tau''(B'A') \neq \tau'(B'A')\sqrt{1-\frac{V_E'^2}{c^2}} \quad (26)$$

Another way to calculate the ageing of the twins at *S´* is (see fig.1, consider that twin *B´*, at rest in *S´*, is moving from *B´´* to *A´´*)

$$\tau'(B''A'') = \Delta t_L''(B''A'')\sqrt{1-\frac{V_E'^2}{c^2}} = \frac{l_2}{V_E'}\sqrt{1-\frac{V_E'^2}{c^2}} \quad (27)$$

From the Lorentz-FitzGerald contraction we have

$$l_1\sqrt{1-\frac{v_1^2}{c^2}} = l_2\sqrt{1-\frac{v_2^2}{c^2}} = l \quad (28)$$

and from (13) we obtain consistently

$$l_2 = \frac{l_1(1 + V_E^{\prime} \frac{v_1}{c^2})}{\sqrt{1 - \frac{V_E^{\prime\,2}}{c^2}}} \qquad (29)$$

and from (27) and (29)

$$\tau^{\prime}(B^{\prime\prime}A^{\prime\prime}) = \frac{l_1}{V_E^{\prime}}\left(1 + V_E^{\prime} \frac{v_1}{c^2}\right) \qquad (30)$$

When $A^{\prime\prime}$ return we have

$$\tau^{\prime}(A^{\prime\prime}B^{\prime\prime}) = \frac{l_1}{V_E^{\prime}}\left(1 - \left|V_E^{\prime}\right| \frac{v_1}{c^2}\right) \qquad (31)$$

since when twin return change v_2 and therefore (with the same Einstein speed $\left|V_E^{\prime}\right|$)

$$l_2 = \frac{l_1(1 - \left|V_E^{\prime}\right| \frac{v_1}{c^2})}{\sqrt{1 - \frac{V_E^{\prime\,2}}{c^2}}} \qquad (32)$$

Therefore, we obtain the same value for the proper time of the twins at $S^{\prime}$ for the $A^{\prime\prime}$ trip, to and from the star, as in the classic case, independently of the frames chosen. The twin that is younger is Twin $A^{\prime\prime}$. But Twin $A^{\prime\prime}$ age more than twin $A^{\prime}$ in the returning trip.

Conclusion

From the presentation of the broader view of special relativity proposed [1-17], with the distinction between clock rhythms and clock time readings, together with the indeterminacy of special relativity [12, 99], which does not allow to know in which inertial frame clocks are actually running slower, the meaningful solution to the paradox is already clear. The symmetry in the description of the outward trip between Andrew and Bob when we refer to Lorentzian times, repeated also for the return trip, does not correspond to a symmetry in the ageing (proper times) of the twins. Regarding proper times, we know that during the outward trip either Andrew or Bob is ageing slower and, without a reference to the rest system, we do not know which of them. And it may even happen that both are ageing at the same rhythm. The same occurs during the return trip. It is possible that it is always one of the twins who is ageing slower, both on the onward and on the return trip, or that one of the twins ages slower during the onward trip and the other during the return trip, or even that in one of the trips they are ageing at the same rate. However, we do know that when they meet it is Bob who is younger, and by which factor. Of course, that all calculations can be made both with Lorentzian times and with synchronized times and from the point of view of each of the twins: all these calculations must give the same result.

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