Section - Abstract

This is a mathematical analysis of everything Collatz. I've come up with a revolutionary way of representing the natural counting numbers as an infinite set of equations. From these I am able to make some provable connections that not only show that all counting numbers are used once in the Collatz Tree structure; but where additional loops originate; the importance of 4x+1 and 2x+1; duality of even numbers; among others. I also show that there can only be one unbroken chain of continuous “3n+1 / 2” growing toward infinite number sizes approaching infinity but never actually getting there. This would be the 'only' counter-example that is possible and as odds would have it, it does not pan out. That only possible counter example is not to be.

Using the induction method where we show that x = 1 is true (elementary, since it is part of the initial loop); from there we assume that x from 1 to k are also true building on the x=1 being true; then k+1 is also true. That is a complicated way of saying that if we know and assume all numbers from 1 to k are true, then the very next number k+1 is also true in as much as we apply the two rules correctly so the number reduces to one that is already in the proven set!

The first three equations of my infinite set of equations are easy to apply this induction to and cover 87.5% of the counting number set. I change things up a bit for the upper level equations. I am able to prove through the same induction method that any number that is not a multiple of 3 (falling in these levels/equations) is also provable. Stepping outside the usual method of this proof I investigate the multiples of three separately to prove they are all following a similar induction proof. And they do. All said and done I am able to prove that 100% are provable. (4x+1) is important in this proof as well the application of (3x+1)/2. Read on to find out what I mean.

I've covered off on the loop issue part of the proof by showing how additional loops come about in the Collatz Tree structure. There is only one loop in Collatz (positive counting numbers) and that is the trivial {1 – 4 – 2} loop. No others are possible no matter how close to infinity one gets and all numbers will reduce to this trivial loop.

The detailed discussion of how I arrived at these different conclusions is outlined below. I apologize if some sections are difficult to follow. I am not a mathematician by nature or profession. I do love mathematics though. I hope you enjoy my approach of showing the 'self' enlightening process as I continued to explore. As my expertise improved, other intuitive aspects became readily useful in the proof. The reader will appreciate knowing why I went down each path I chose to pursue.

This is an updated version of my original document with a new section near the end that gets into the details of the proof. The remainder of the original report remains intact for the most part but does have additional details and concepts introduced and dispersed therein.
This is the third version where I have solved the outstanding subset of multiples of 3. I believe you will find that method eye-opening since it involves some under the sheets number manipulation by multiple applications of \((3x+1)/2\). I also introduce the 'duality' nature of even numbers that remain hidden in the Collatz tree structure... and that is that those even numbers can behave as if they were odd numbers; \((\text{Odd} \times 3) + 1 = \text{Even} \); \((\text{Even} \times 3) + 1 = \text{Odd} \); \((4 \times \text{Even}) + 1 = \text{Odd} \).

After arriving at this proof I go back to my original set of equations to see if they behave the same way and may provide an easier more condensed method to the final proof. Low and behold they do! It boils down to one chart. Now that I see it on paper I am impressed with my progress. It has taken just over 3 years to finalize the process.

In this, the forth version, I have looped back to my original infinite set of equations to formulate a single chart from whence a complete proof can be understood. I have also added a 'final' section that simplifies the induction method of all odd numbers. I think you'll be enlightened with that approach since it really simplifies the inductive proof. If I am right this simplified approach can be a much simplified proof in itself.

This detailed analysis has led me to two alternate methods for complete proofs. Enjoy.

**Section 1 - Introduction**

The Collatz conjecture is a sequence of numbers generated by applying two rules; if the number is Odd multiply it by 3 and add 1 ( \(3n+1\) ); if the number is even then divide by 2 ( \(n/2\) ). So the Collatz sequence is \(\{3n+1; n/2\}\).

The conjecture states that if you start at any number from 1 to infinity ( positive natural counting numbers) you will eventually end up in a \(\{1 - 4 - 2\}\) loop.

Sounds simple enough. The concept is, but proving that this is infact true over the entire set of natural counting numbers is quite difficult. Apparently, folks have been searching for a proof for close to 100 years.

My attempt is to approach the proof from a slightly different angle and look at the natural counting numbers in a more confined fashion. This will allow for the observation that something fundamental is occurring. That will become clear in the following sections.

I am not a mathematician per say... but a computer scientist… and we all know computers are just large computational devices that rely on mathematics and logic. I do not have access to a mathematical addon for publishing in the correct format so I will make due with what I can get off the keyboard ( symbol wise ). My terminology may also be lacking, but I am confident you will comprehend it just fine.

I've created this report in a fashion where you can follow my maturation process as I studied the Collatz Conjecture. I ask myself questions and then go about determining if they are something I can use towards a proof.

**Section 2 – Infinite Sequence of Equations to Create ALL Counting Numbers ( Primes )**

The basis of my observations and subsequent conclusion is the understanding that all the natural numbers ( 1 to infinity ) can be represented by the following infinite set of equations.
• 0 + 2x  { 0 + (2^1)x }  { (((2^1) / 2) -1) + (2^1)x }
• 1 + 4x  { 1 + (2^2)x }  { (((2^2) / 2) -1) + (2^2)x }
• 3 + 8x  { 3 + (2^3)x }  { (((2^3) / 2) -1) + (2^3)x }
• 7 + 16x  { 7 + (2^4)x }  { (((2^4) / 2) -1) + (2^4)x }
• ...  
• ((2^y) / 2) -1) + (2^y)x
• ...
• ((2^infinity) / 2) -1) + (2^infinity)x

As seen above this is an infinite sequence of equations and it will cover all the natural numbers (1 to infinity). Each individual counting number exists only ONCE in this set of equations. I've expanded out the first ten equations to show how they are formed. Note that 'powers of 2' play a very important role. Now, there is an unexpected reality to these equations in that 0 + 2x contains all the even numbers (a subset that contains exactly half (½) of the natural number set). For example { 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, ... }. The next equation 1 + 4x spawns the following subset: { 1, 5, 9, 13, 17, 21, ... } This subset contains exactly one quarter (¼) of the entire natural number set. So the first 2 equations account for (¾) of the natural number set. You will find that the next equation subset will contain only (1/8) of the natural numbers: { 3, 11, 19, 27, ... }. And the following equation has (1/16) of the natural numbers { 7 + 16x } { 7, 23, 39, 55, ... }. Do you see a pattern here? The subset for any equation contains (1/2^y): { (½) for 2^1; (¼) for 2^2; (1/8) for 2^3; ... }. As we approach the infinity power of 2 we find that the subset contains only (1/infinity) elements...a very tiny number. So just for kicks, let's calculate how what proportion of the natural number set are included with the first 10 equations (½) + (¼) + (1/8) + (1/16) + (1/32) + (1/64) + (1/128) + (1/256) + (1/512) + (1/1024) = (1023/1024). Interesting, indeed. The vast majority of all the natural numbers can be created using only the first 10 equations. We will come back to this point later. Here's the above discussion in the form of a chart for easier visualization:

<table>
<thead>
<tr>
<th>Equation</th>
<th>Proportion</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 + 2x</td>
<td>1/2</td>
</tr>
<tr>
<td>1 + 4x</td>
<td>1/4</td>
</tr>
<tr>
<td>3 + 8x</td>
<td>1/8</td>
</tr>
<tr>
<td>7 + 16x</td>
<td>1/16</td>
</tr>
<tr>
<td>15 + 32x</td>
<td>1/32</td>
</tr>
<tr>
<td>31 + 64x</td>
<td>1/64</td>
</tr>
<tr>
<td>63 + 128x</td>
<td>1/128</td>
</tr>
<tr>
<td>127 + 256x</td>
<td>1/256</td>
</tr>
<tr>
<td>255 + 512x</td>
<td>1/512</td>
</tr>
<tr>
<td>511 + 1024x</td>
<td>1/1024</td>
</tr>
<tr>
<td>1023 + 2048x</td>
<td>1/2048</td>
</tr>
</tbody>
</table>

Just so we are all on the same page I've listed the first several equations with the numbers they create:

{ 0 + 2x } → 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, ...
{ 1 + 4x } → 1, 5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 45, 49, 53, 57, 61, 65, 69, 73, 77, 81, 85, 89, ...
{ 3 + 8x } → 3, 11, 19, 27, 35, 43, 51, 59, 67, 75, 83, 91, 99, 107, 115, 123, 131, 139, 147, 155, ...
{ 7 + 16x } → 7, 23, 39, 55, 71, 87, 103, 119, 135, 151, 167, 183, 199, 215, ...
{ 15 + 32x } → 15, 47, 79, 111, 143, 175, 207, 239, 271, 303, 335, 367, ...
{ 31 + 64x } → 31, 95, 159, 223, 287, 351, 415, 479, ...
{ 63 + 128x } → 63, 191, 319, 447, 575, 703, 831, ...
{ 127 + 256x } → 127, 383, 639, 895, 1151, 1407, 1663, ...
{ 255 + 512x } → 255, 767, 1279, 1791, 2303, ...

Collatz Conjecture Explored – Complete Proof
This is likely as good a spot as any to show how primes work into my equations. The negative natural numbers shown is subsequent sections work in the same fashion. I'm going to list off the first 21 equations:

\[
\begin{align*}
\{0 + 2x\} & \rightarrow 0 + 2x \\
\{1 + 4x\} & \rightarrow 1 + 4x \\
\{3 + 8x\} & \rightarrow 3 + 8x \\
\{7 + 16x\} & \rightarrow 7 + 16x \\
\{15 + 32x\} & \rightarrow 5 \cdot 3 + 32x \\
\{31 + 64x\} & \rightarrow 31 + 64x \\
\{63 + 128x\} & \rightarrow 7 \cdot 3 \cdot 3 + 128x \\
\{127 + 256x\} & \rightarrow 127 + 256x \\
\{255 + 512x\} & \rightarrow 17 \cdot 5 \cdot 3 + 512x \\
\{511 + 1024x\} & \rightarrow 73 \cdot 7 \cdot 1 + 1024x \\
\{1023 + 2048x\} & \rightarrow 31 \cdot 11 \cdot 3 + 2048x \\
\{2047 + 4096x\} & \rightarrow 89 \cdot 23 + 4096x \\
\{4095 + 8192x\} & \rightarrow 13 \cdot 7 \cdot 5 \cdot 3 \cdot 3 + 8192x \\
\{8191 + 16384x\} & \rightarrow 8191 + 16384x \\
\{16383 + 32768x\} & \rightarrow 127 \cdot 43 \cdot 3 + 32768x \\
\{32767 + 65536x\} & \rightarrow 151 \cdot 31 \cdot 7 + 65536x \\
\{65535 + 131072x\} & \rightarrow 257 \cdot 17 \cdot 5 \cdot 3 + 131072x \\
\{131071 + 262144x\} & \rightarrow 131071 + 262144x \\
\{262143 + 524288x\} & \rightarrow 73 \cdot 19 \cdot 7 \cdot 3 \cdot 3 \cdot 3 + 524288x \\
\{524287 + 1048576x\} & \rightarrow 524287 + 1048576x \\
\{1048575 + 2097152x\} & \rightarrow 41 \cdot 31 \cdot 11 \cdot 5 \cdot 5 \cdot 3 + 2097152x
\end{align*}
\]

The important thing to notice here is that the first part of every equation is simply some \(2^x - 1\) and that each of them in turn is formed by nothing but PRIME factors. The ultra important realization is that starting at 3 every second equation after that is comprised of factors that contain at least one factor of 3. All the other equations do not include that factor of 3. This makes every second equation a 'multiple of 3' equation? At the bare minimum those equations start with a multiple of 3. All of the equations contain multiples of 3. This observation likely plays into the process but at this point I'm not convinced it can be used to formulate a proof.

We will see that any odd number that is a multiple of 3 can not form further branches; it is a 'dead-end' row. I love how primes have made an appearance, but anyone involved with number theory knows that any number is created by nothing but prime factors. Later we will see the appearance of 3\(^x\) = 2\(^y\) + 1 and how it can be used to explain the formation of additional loops. Again a connection with powers of 3 and powers of 2. Note there are only two cases where this is true; 3\(^1\) = 2\(^1\) + 1 and 3\(^2\) + 2\(^3\) + 1. The above primes discussion play with 2\(^x\) – 1. Quite a coincidence, isn't it? Every second equation is the same as saying add 3 multiplied by '4' or '2^2'. 0+(3*1)=3; 3+(3*4)=15; 15+(3*16)=63; 63+(3*64)=255;... Note that as we jump to the next equation we are multiplying by 4 more...3*4; 3*4*4; 3*4*4*4;... This is how we skip over every other equation and why we see branches separated by '4' or '2^2'. You obviously see this is not the complete picture. The other subset of equations do something very similar. 1+(3*2)=7; 7+(3*8)=31; 31+(3*32)=127; 127+(3*128)=511. Again we are multiplying by 4 (2*2). This allows us to skip over every other equation. Combining the two cover all my equations.

Now, another item that may be important to explore here before going futher is the relationship between 3 and 2. This relationship fits in with how the Collatz tree propagates. If you multiply a number (say 1) by three and add one (3n+1) you are in effect doing 3+1=4. 4 is simply 2+2=4. 4 is an important transition point.
in the tree. Let's do another iteration of 3n+1 but not by multiplying but simply adding the effect. 3n+1+3n+1 = 3+3+2 = 8. Can we mirror this with 2? Yes, 2+2+2+2 or 4+4 = 8. 3, 6 and 2, 4 are all an important numbers when building tables for Collatz:

<table>
<thead>
<tr>
<th>Odd number</th>
<th>3n+1</th>
<th>n/2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>16</td>
<td>8</td>
</tr>
<tr>
<td>7</td>
<td>22</td>
<td>11</td>
</tr>
<tr>
<td>9</td>
<td>28</td>
<td>14</td>
</tr>
<tr>
<td>11</td>
<td>34</td>
<td>17</td>
</tr>
<tr>
<td>13</td>
<td>40</td>
<td>20</td>
</tr>
<tr>
<td>15</td>
<td>46</td>
<td>23</td>
</tr>
<tr>
<td>17</td>
<td>52</td>
<td>26</td>
</tr>
<tr>
<td>19</td>
<td>58</td>
<td>29</td>
</tr>
<tr>
<td>21</td>
<td>64</td>
<td>32</td>
</tr>
</tbody>
</table>

See that the Odd number column is separated by 2 in each step up (+2). 3n+1 is (+6) in each step up. And just for kicks, n/2 is (+3) in each step up. Interesting INDEED! Not really... (+6)/2=(+3)! So there is a definite link between 3n+1 and n/2; that is 3 and 6.

What happens on the third iteration is very important to note. This is an important transition step. 3n+1+3n+1+3n+1 = 3+3+3+3 = 12. So the excess I's give an even 3 after 3 iterations. That is important because it becomes evenly divisible by 3. And it's connection to 2 is 2+2+2+2+2+2 = 12 or 6+6. or 4+4+4. This is not needed for the proof I outline below. At least not in this fashion.

You are likely saying we can't use this and you are likely right but it was a stepping stone to show what I really intended. Again, suppose n=1 for ease of understanding. 3n+1 if n=1 is 4. Now apply 3n+1 to that and do it a second time ending up with 3(3n+1)+1 or 27n+13. This is just three iterations of 3n+1. Let's rearrange 27n+13 to 27n+9+4 and factor out 9 giving 9(3n+1)+4 and since 4 is actually 3n+1, replace the 4 giving 9(3n+1)+(3n+1). This is the case so long as we keep n=1. You can now note that we actually have 10(3n+1). This means that after 3 consecutive iterations of 3n+1 we should be able to divide out an extra 2 (n/2). BUT, actually what is happening is (3n+1)/2. So to complicate things a tad bit what happens if we add in the n/2 each iteration. Should be nothing, really. First yields (3n+1)/2. Next yields (3(3n+1)/2)+1)/2. And the third gives (3((3(3n+1)/2)+1)/2)+1)/2. Multiplied through we get (27n+19)/8. If we try to do like above to factor out 9 we get (9(3n+1)+10)/8. Separate out a 4 from the 10 to give (9(3n+1)+(3n+1)/6)/8 or (10(3n+1)+6)/8. And we can still mathematically strip out a 2 as follows: 2(5(3n+1)+3)/8. In essence we continue to get an extra n/2 every three iterations. This observation must provide statistical advantage to increase the overall number of (n/2). Something similar must be happening when n is other than 1. I am unable to make that leap at this point.

I will come back to this connection later in this discussion when I formulate the proof. It is very useful in proving a subset of multiples of 3.

Why have I discussed any of this in the first place. It was to show that all natural counting numbers are included in the tree structure. None are missed. As well, it is to show how powers of 2 and 3 play an important role in the construction of this tree. Since all odd numbers are in the tree implies that all even numbers are as well (since any even number can be formed by multiplying an odd number by two or another even number by 2). Again, this is a multiple of 2 (2^1).
Section 3 – Cascading Effect

You are likely asking why I am about to point out the cascading effect. That's where this gets very interesting. The structure of the tree is dictated by the odd number at any of the nodes; a 'node' being designated by it's location in the tree – in this case anywhere where you can go right by multiplying by two and up by multiplying by three and adding one. There are only two paths. Other nodes with two paths contain only two multiply by 2. So I call them connector nodes. I also call all other nodes with 3 paths connector nodes; they have a 'minus one and divide by three' and a 'divide by two' and a 'multiply by two'.

“node”
{even number = node * 3 + 1}
       |  
{ node } – { node * 2 }

“connector node”
{ connector node / 2 } – { connector node } – { connector node * 2 }

“connector node (all other nodes)”
{ connector node / 2 } – { connector node } – ( connector node * 2)

|      node     |
| 1 – 2 – 4 – 8 – 16 – …
| 05 – 10 – 20 – 40 – …
|           | 13 – 26 – 52 – …
| 17 – 34 – 68 – …
| 03 – 06 – 12 – …  11 – 22 – 44 – …
| 07 – 14 – 28 – …  09 – 18 – …

1, 5, 3, 13, 17, 11 – nodes ( 1 is included as a node because it loops back to 4 )
2, 4, 8, 16, 10, 20, 40, 6, 12, 26, 52, 34, 68, 22, 44, 14, 28, 18 – connector nodes

If a node contains an odd number from say { 7 + 16x } … the very next odd number will be (3n+1)/2
and will be a number contained in \( \{ 3 + 8x \} \) … with the very next odd number a further \((3n+1)/2\) and it will fall in \( \{ 1 +4x \} \) … till it finally falls into \( \{ 0 +2x \} \). This is the case no matter what subset you were to start at. If you started at \( \{511 + 1024x \} \) it would cascade uninterrupted through each prior equation one-by-one till it gets to \( \{ 0 + 2x \} \).

Note that any odd number that is a multiple of '3' is a starting node. No other node can migrate through it on its way back to the \( \{ 1 – 4 – 2 \} \) loop. 3 and 9 shown above in red are such nodes. Do you see why this is the case? Any multiple of 3 cannot be arrived at by applying \(3n+1\) to another odd.

A cascade is in the form:

\[
\begin{align*}
& E \quad E \\
| & O \quad E \quad E \\
| & O \quad E \quad E \\
| & O \quad E \quad E \\
| & O \quad E \quad E \\
& O \quad E \quad E \\
\end{align*}
\]

Using \(O=odd\) and \(E=even\) we see an unbroken \(O \quad E \quad O \quad E \quad O \quad E \quad O\)...

\(7 \quad 22 \quad 11 \quad 34 \quad 17 \quad 52 \quad 26\) is such a chain... 7 is level 4; 11 is level 3; 17 is level 2; and 26 is level 1. If there are two evens side by side in the chain is not a cascade. \(17 \quad 52 \quad 26 \quad 13 \quad 40 \quad 20 \quad 10\) ... \(O \quad E \quad E \quad O \quad E \quad E \). The following paragraphs point out other important features of the tree structure.

This may be an appropriate place to point out another mathematical oddity that occurs in this tree. A node (all nodes are Odd) will also cover itself times 4 plus 1. \(4*(\text{Odd Node}) + 1\)... and that new node will have the same applied to it and so forth all the way though the tree for all nodes with the exception of multiple of 3 nodes.

\[
\begin{align*}
1 & \quad 2 \quad 4 \quad 8 \quad 16 \quad 32 \quad 64 \quad 128 \\
| & \quad | \quad | \quad | \quad | \quad | \\
21 & \quad 42 \quad 84 \\
| & \quad | \quad | \quad | \quad | \quad | \\
05 & \quad 10 \quad 20 \quad 40 \quad ...
1 & \quad 13 \quad 26 \quad 52 \quad ...
| & \quad | \quad | \quad | \quad | \quad | \\
17 & \quad 34 \quad 68 \quad ...
03 & \quad 06 \quad 12 \quad ...
11 & \quad 22 \quad 44 \quad ...
07 & \quad 14 \quad 28 \quad ...
09 & \quad 18 \quad ...
\end{align*}
\]

Drawn slightly different with the '1' hanging where it should be you can see this... 1, 5, 21, ... or \(1*4+1 = 5\); \(5*4+1=21\); etc. 3, 13, 53, ... or \(3*4+1 =13\); \(13*4+1 = 53\); etc. All nodes display this feature. This occurs because of the way the tree is constructed and branches form...namely that after the first branch on any row is
formed, $2^2$ or multiply by 4 to get the next branch on the row. An example is 10 and 40 on that row. $10 \times 2 \times 2 = 40$. The branch at 10 gives a node of 3. The branch at 40 gives a node of 13. And the next branch at 160 ($40 \times 2 \times 2 = 160$) will give a node of 53 which is $53 \times 3 + 1 = 160!$ And 53 is $13 \times 4 + 1$. All rows that can have branches do this indefinitely.

You will notice that this $4x+1$ plays prominently in the Collatz structure. Every backbone (except those that start with multiples of 3) spawn limbs that have this $4x+1$ applied over and over...except of course those backbones that start with multiples of 3.

Looking a little deeper into this we see the following:

$$2 - 4 - 8 - 16 - 32 - 64 - 128 - 256 - 512 - 1024 - ...$$

| 1 | 5 | 21 | 85 | 341 |

The above is a snippet of the '2' backbone. It starts with the only even node in the entire tree which is a special case because it is actually the '1' backbone which is odd too. I've drawn the 1 hanging off of 4 to make this $4x+1$ easier to spot. All other nodes will follow this feature without question. Notice how each of 1, 5, 21, 85, 341, display this feature:

- $1 \rightarrow 4(1)+1=5$
- $5 \rightarrow 16(1)+5=21$
- $21 \rightarrow 64(1)+21=85$
- $85 \rightarrow 256(1)+85=341$

... Now you can see how multiplying by 4 (2 followed by 2) gives rise to these. It's convenient that $1+4=5; 5+16=21; 21+64=85;...$ This is exactly the same as saying $4x+1...4(1)+1=5; 4(5)+1=21; 4(21)+1=85; 4(85)+1=341...$

This exact same thing occurs with all the non-multiple of 3 nodes. For example let's show '5'.

$$5 - 10 - 20 - 40 - 80 - 160 - 320 - 640 - ...$$

| 3 | 13 | 53 | 213 |

- $3 \rightarrow 4(3)+1=13$
- $13 \rightarrow 16(3)+5=53$
- $53 \rightarrow 64(3)+21=213$

...

Let's do one more to hammer this point home; let's do '11':

$$11 - 22 - 44 - 88 - 176 - 352 - 704 - 1408 - ...$$

| 7 | 29 | 117 | 469 |

- $7 \rightarrow 4(7)+1=29$
- $29 \rightarrow 16(7)+5=117$
- $117 \rightarrow 64(7)+21=469$
So all nodes with the exception of the multiples of 3 will do this. This is the 4x+1 rule and will prove invaluable in the following proof.

Something important to mention is that there are special occurrences where an even number (all even numbers) will give the same result as an odd multiplied by 3 and then add one. They behave like; but instead of giving an even number one gets an odd number. But the 4x+1 rule stands. Example:

\[
\begin{align*}
07 & \rightarrow 14 \rightarrow 28 \rightarrow 56 \rightarrow 112 \rightarrow 224 \rightarrow 448 \ldots \\
02 & \rightarrow 09 & \rightarrow 37 & \rightarrow 149
\end{align*}
\]

\[
2 \rightarrow 4(2)+1=9 \\
9 \rightarrow 16(2)+5=37 \\
37 \rightarrow 64(2)+21=149
\]

That's very cool...but it is invisible when drawing the tree. And all even numbers display this feature. Let's do a couple more to show this:

\[
\begin{align*}
13 & \rightarrow 26 \rightarrow 52 \rightarrow 104 \rightarrow 208 \rightarrow 416 \rightarrow 832 \rightarrow 1664 \ldots \\
4 & \rightarrow 17 & \rightarrow 69 & \rightarrow 277
\end{align*}
\]

\[
4 \rightarrow 4(4)+1=17 \\
17 \rightarrow 16(4)+5=69 \\
69 \rightarrow 64(4)+21=277
\]

And how about 6:

\[
\begin{align*}
19 & \rightarrow 38 \rightarrow 76 \rightarrow 152 \rightarrow 304 \rightarrow 608 \rightarrow 1216 \rightarrow 2432 \ldots \\
6 & \rightarrow 25 & \rightarrow 101 & \rightarrow 405
\end{align*}
\]

\[
6 \rightarrow 4(6)+1=25 \\
25 \rightarrow 16(6)+5=101 \\
101 \rightarrow 64(6)+21=405
\]

This is the case for 'all' even numbers. They will make an invisible presence in the tree.

The first snippet from the Collatz structure with 2 placed in there does show the point. The even number when multiplied by 4 and add one gives the odd 9. (2*4)+1=9. Also note that that same even number when multiplied by 3 and add one gives another odd number very closely related to 9. (2*3)+1=7.

\[
\begin{align*}
13 & \rightarrow 26 \rightarrow 52 \ldots \\
04 & \rightarrow 09 & \rightarrow 37 & \rightarrow 149
\end{align*}
\]

\[
07 \rightarrow 14 \rightarrow 28 \rightarrow 56 \rightarrow 112 \rightarrow 224 \rightarrow 448 \ldots
\]

This can only happen where you have that opening available. That appears to be where ever a level 2 (1+4x) starts. No other levels can do this because they collapse or cascade directly down to level 1 (0+2x). This is very important to remember. When we visit the proof later, we'll see situations where an odd number can be passed though reverse 4x+1 and give these evens. These do not mess up the Collatz structure and shows the
inter-connectivity between the different backbones. These special even number play dual roles, not only can they have the n/2 rule for being even; they also fit into the structure (invisibly) where they are also 3x+1 and 4x+1 rules.

Section 4 – Validating the Cascade Mathematically

Now I will take a moment to show how this works. Let's start with \( \{ 7 + 16x \} \). Any number created from this equation will be odd so one must apply the 3n+1 followed by n/2.

\[
\begin{align*}
(3\left(\{7 + 16x\}\right) + 1) / 2 \\
(21 + 48x + 1) / 2 \\
(22 + 48x) / 2 \\
11 + 24x \\
3 + 8 + 24x \\
3 + 8 (1 + 3x) & \text{ or } \{3 + 8x \text{ since } 1+3x \text{ is actually an 'x' after applying } 3n+1\} \\
\end{align*}
\]

So as you can see from the above the very next odd number will fall in the prior equation \( \{3 + 8x\} \). Since it falls in this subset it is automatically an odd and can't be further divided by 2. Replace 1+3x with the new x and run this new odd again:

\[
\begin{align*}
(3\left(\{3 + 8x\}\right) + 1) / 2 \\
(9 + 24x + 1) / 2 \\
(10 + 24x) / 2 \\
5 + 12x \\
1 + 4 + 12x \\
1+ 4 (1 + 3x) & \text{ or } \{1 + 4x \text{ since } 1+3x \text{ is actually an 'x' after applying } 3n+1\} \\
\end{align*}
\]

And this continues uninterupted until you get to the very first equation, which are the even numbers:

\[
\begin{align*}
(3\left(\{1 + 4x\}\right) + 1) / 2 \\
(3 + 12x + 1) / 2 \\
(4 + 12x) / 2 \\
2 + 6x \\
2 (1 + 3x) \\
2 (1 + 3x) & \text{ or } \{0 + 2x \text{ since } 1+3x \text{ is actually an 'x' after applying } 3n+1\} \\
\end{align*}
\]

Now this is an even number which can be divided at least once more by 2. Continually dividing by additional 2's will give us another odd number eventually. This odd number will fall into an upper equation but we have no way of knowing which one...we can not predetermine as far as I can tell. This will cause another uninterupted cascade down to the \( \{0 + 2x\} \) equation. All cascades behave in this fashion and since the tree is nothing but cascades, the entire tree is one giant cascade.

Section 5 – Observations from Cascading

This a good place to point out an obvious fact. Starting at any level equation, it must then continually and directly cascade to the first level \( \{0 + 2n\} \). So for each number in a given level it cascades directly to level \( \{0 + 2n\} \) through it's very own path. This implies that the same number of entries in the preceding cascade are accounted for. So if \( \{7 + 16x\} \) has a finite number of say 8 entries; and the preceding level \( \{3 + 8x\} \) has twice as many to start; 16; then 8 of those are automatically accounted for. If level \( \{1 + 4x\} \) has double that again;
32; and 8 of those are accounted for; leaving 24. And so on and so forth. But remember that all entries in the \{ 3 + 8x \} also cascade uninterrupted to first level...so only half of the prior levels entries are left in play...meaning that at level \{ 0 + 2x \} only half remain in play? (that means ½ of the entire natural counting numbers set). The rest fall on/within some predetermined path from higher levels? As seen the following chart level \{ 0+2x \} behaves a bit differently in that only 1/3 of it's members are part of upper level cascading stacks. Right? That's because each level spreads out in multiples of 3...and it's only when you reach level \( 0+2x \) that this becomes obvious.

Let's see if I can show this concept in a chart:

| \{0+2x\} | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 | 32 | 34 | 36 | 38 | 40 | 42 | 44 | 46 | 48 | 50 | 52 |
| \{1+4x\} | 1 | 5 | 9 | 13 | 17 | 21 | 25 | 29 | 33 |
| \{3+8x\} | 3 | 4 | 11 | 19 |
| \{7+16x\} | 7 | 6 |

Duality 2 (2*3+1=7) 8 → 25 14 → 43 20 → 61 26 → 79
4 (4*3+1=13) 10 → 31 16 → 49 22 → 67 28 → 85
6 (6*3+1=19) 12 → 37 18 → 55 24 → 73 30 → 91

So you can now see how all the odd numbers are covered and consumed in a stack that leads/cascades back to level \( 0+2x \). I've included dualities of even numbers to show that they do not impact our thoughts and only show up at the start of already existing cascade stacks. Only 1/3 of the even numbers are consumed. But remember the other rule \( n/2 \) allows us to consume any even that is double \((2*\text{odd})\); example 1*2=2; 3*2=6; 5*2=10; 7*2=14; 9*2=18; 11*2=22; 13*2=26... Shown in red below. Remember that the terminus of stacks accounts for 1/3 shown in blue. There is some overlap between the red and blue. You can begin to imagine how the entire tree is held together by those even numbers. The remainder of the even numbers are simply double some other even already covered (shown in green).

<table>
<thead>
<tr>
<th>{0+2x}</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
<th>16</th>
<th>18</th>
<th>20</th>
<th>22</th>
<th>24</th>
<th>26</th>
</tr>
</thead>
<tbody>
<tr>
<td>{1+4x}</td>
<td>1</td>
<td>5</td>
<td>9</td>
<td>13</td>
<td>17</td>
<td>21</td>
<td>25</td>
<td>29</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>{3+8x}</td>
<td>3</td>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>{7+16x}</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\{ 0 + 2x \} \( 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, \ldots \)
\{ 1 + 4x \} \( 1, 5, 9, 13, 17, 21, 25, 29, \ldots \)
\{ 3 + 8x \} \( 3, 11, 19, 27, 35, \ldots \)
\{ 7 + 16x \} \( 7, 23, 39, \ldots \)

So for the above all 3 number shown in subset for \( 7 + 16x \) cascade through each prior level consuming one number in each of those levels. And there's a pattern that forms. Taking 7; it translates to \((7*3+1)/2 = 11\). 23 translates to 35 in the prior level. So the first entry (smallest) ends up translating to the second entry in the prior level. The next translates to the third item past 11 in the prior level – 35; and the next to three items past 35; as so on. If we start in the prior level with that first item 3; it translates to 5 in the prior level...11 translates to to three past 5 or 17...and so on. When jumping to the first (evens) level it does not translate to the second but the first...so 1 translates to 2 which is the first item in \( 0 + 2x \). But each additional item hits 3 items higher after that; 5 translates to 8 – 9 translates to 14.

It may not be so obvious at this point but all the odd entries (all odd number in the natural number set) are accounted for. All the entries are already accounted for in all levels above \( 0 + 2n \). That implies that any of the evens when divided by the appropriate number of 2s will spill to an odd number in a higher level that has
already been accounted for. So without taking a leap of faith we can be confident that each and every natural number is included in the tree. Right?

**Section 6 – Trivial Loop Jumps Out**

This is a good place to point out the trivial loop and how it comes into being:

\[
\{ 0 + 2x \} \quad 2, 4, 6, 8, 10, \ldots \\
\{ 1 + 4x \} \quad 1, 5, 9, 13, \ldots
\]

See how this happens? With these equations it jumps right out the page.

**Section 7 – Putting it Together**

The next leap comes when you accept that no matter how much bouncing around it does, this process will eventually lead down to the trivial loop \( \{ 1 – 4 – 2 \} \). But I don't expect you to accept this blindly. If every third item in \( \{ 0 + 2x \} \) is accounted for; that is 2, 8, 14, 20, 26, … let's do some quick number crunching... 2 reduces to the trivial; 8 also reduces to the trivial loop; in fact all powers of 2 which are included in this subset will do just that. I call these the 'backbone collapse to trivial'. This is the obvious part. The final not so obvious is in the tree structure itself as I have drawn it. The power of 2s backbone is across the very top and the only possible direction in that row is left to '1' by dividing by 2 over and over. The next level down is where any possible backbone entry less a 1 is divisible by 3...example \((5*3)+1 = '16'\). Now 5 can grow to the right by multiplying by 2 consecutively – 10, 20, 40, … or it can go up if multiplied by 3 and one added.

\[
1 \quad 2 \quad 4 \quad 8 \quad 16 \quad \ldots \\
\downarrow \\
05 \quad 10 \quad 20 \quad 40 \quad \ldots \\
\downarrow \\
13 \quad 26 \quad 52 \quad \ldots \\
\downarrow \\
17 \quad 34 \quad 68 \quad \ldots \\
\downarrow \\
03 \quad 06 \quad 12 \quad \ldots \\
11 \quad 22 \quad 44 \quad \ldots \\
\downarrow \\
07 \quad 14 \quad 28 \quad \ldots \\
\downarrow \\
09 \quad 18 \quad \ldots
\]

Once the tree is built it should be obvious that you can only proceed left and up. Going left and up will eventually lead to the backbone. Right? So the rest of those evens that are not exact powers of 2 will be found somewhere else in this tree structure where it can only go up or left and approach the backbone. Maybe someone has a better way to explain this. I sure hope that made sense!

I'm not too worried about the rest of the structure because I am ultimately trying to show there is at least one case where this cascade will be infinite and hence unending (or continually growing). This is the only case where the tree can grow forever...it has to be an infinite cascade. So how does this play into it?

As one approaches infinity the ultimate number of steps in the cascade I discussed above approaches
infinity as well. At infinity the process breaks. Infinity will enter an infinite number of steps in this cascade. So is this not a counter example? To disprove the conjecture?

I am thinking NOT. Since this happens at the very endpoint we can likely use this to show that the only case where it can grow infinitely is at the endpoint of infinity and since we can never get to the endpoint of infinity; there are no other situations where it is possible so long as \( n < \infty \). All numbers from 1 up to but not including infinity will reduce to the ultimate loop \{ 1 – 4 – 2 \}.

This is an aside that may useful to point out at this time. And it is likley to play an important role in an inductive proof. Notice how going right and down will allow us to realize a smaller ending number than the beginning number in most cases. If you include the duality concept I will introduce later all of these will be able to do just that. Put this on the back burner for now.

**Section 8 – Exploring the Negative Numbers in the Sequence \{ 3n-1 ; n/2 \}**

I found it interesting in that if one uses the negative natural counting numbers from -1 to -infinity in the \{ 3n-1 ; n/2 \} instead of the above Collatz \{ 3n+1 ; n/2 \} one gets the exact same tree as outlined above...except it contains nothing but negative numbers; and instead of going left and up as seen in Collatz it goes right and up. It changes direction which is expected. The magnitude remains the same. The same trivial loop occur except it is \{-1 – -4 – -2 \}.

My special set of equations are slightly different but the same rules apply (Negatized).

- \(-0 + 2x\) \-
- \(-1 + 4x\) \-
- \(-3 + 8x\) \-
- \(-7 + 16x\) \-
- \(-(((2^y) / 2) -1) + (2^y)x\) \-
- \(-(((2^infinity) / 2) -1) + (2^infinity)x\)

\[
\begin{align*}
-0 + 2x & \rightarrow -2, -4, -6, -8, -10, -12, -14, -16, -18, -20, -22, -24, -26, \ldots \\
-1 + 4x & \rightarrow -1, -5, -9, -13, -17, -21, -25, -29, \ldots \\
-3 + 8x & \rightarrow -3, -11, -19, -27, -35, \ldots \\
-7 + 16x & \rightarrow -7, -23, -39, \ldots \\
\end{align*}
\]

See the same trivial loop \{-1 – -4 – -2 \} and it jumps out as well. The rest of the argument is exactly the same for the negative natural counting numbers in the sequence \{ 3n-1 ; n/2 \}.

Do my formulas show a convergence as well:

\[
\begin{align*}
(3 ( \{ -7 + 16x \} ) - 1) / 2 \\
(-21 + 48x - 1) / 2 \\
(-22 + 48x) / 2 \\
-11 + 24x \\
-3 - 8 + 24x
\end{align*}
\]
And this is the case for all these equations.

Let's try the cascade to \( \{ 0 + 2x \} \):

\[
\frac{3 \left( \{ -1 + 4x \} \right) - 1}{2} \\
\frac{-3 + 12x - 1}{2} \\
\frac{-4 + 12x}{2} \\
-2 + 6x \\
-0 - 2 + 6x \\
-0 + 2 \left( -1 + 3x \right) \text{ or } \{ 0 + 2x \text{ since } -1+3x \text{ is actually an 'x' after applying } 3n-1 \}
\]

They behave exactly the same way as the positives. So I will not bore you by showing more of them in detail. Once was quite enough to prove the point.

Here is the only tree with negative number in \( \{3n-1; n/2\} \)

\[
\begin{array}{c}
-1 \\
-2 \\
-4 \\
-8 \\
-16 \\
\vdots
\end{array}
\]

\[
\begin{array}{c}
-05 \\
-10 \\
-20 \\
-40 \\
\vdots
\end{array}
\]

\[
\begin{array}{c}
-13 \\
-26 \\
-52 \\
\vdots
\end{array}
\]

\[
\begin{array}{c}
-03 \\
-06 \\
-12 \\
\vdots
\end{array}
\]

\[
\begin{array}{c}
-11 \\
-22 \\
-44 \\
\vdots
\end{array}
\]

\[
\begin{array}{c}
-07 \\
-14 \\
-28 \\
\vdots
\end{array}
\]

\[
\begin{array}{c}
-09 \\
-18 \\
\vdots
\end{array}
\]

But, let's consider if \( 4x+1 \) holds true in this tree. No it does not. Here it must be altered to \( 4x-1 \). \((-1*4)-1 = -5, (-3*4)-1 = -13\). We will likely see the same thing occur in the following sections where 3 trees (loops) become possible.

Section 9 – Exploring the Negative Numbers in the Collatz Sequence \( \{ 3n+1: n/2 \} \)

Placing the negative numbers in my original equations (they have been negatized) yields the following:

- \( -0 + 2x \)  \( \{ -0 + (2^1)x \} \)  \( \{ -((2^1)/2) - 1) + (2^1)x \} \)
- \( -1 + 4x \)  \( \{ -1 + (2^2)x \} \)  \( \{ -((2^2)/2) - 1) + (2^2)x \} \)
- \( -3 + 8x \)  \( \{ -3 + (2^3)x \} \)  \( \{ -((2^3)/2) - 1) + (2^3)x \} \)
- \( -7 + 16x \)  \( \{ -7 + (2^4)x \} \)  \( \{ -((2^4)/2) - 1) + (2^4)x \} \)
- \( \ldots \)
I had to make a slight change to my equations to cover all the negative natural numbers but for all intents and purpose the same levels pop out valid.

Now what does the tree structure look like:

```


  -59 – -118 – -236 – …

  -79 – -158 – …

-3 – -6 – -12 – …
```

This first tree has the loop at the very top left before any branching begins. The loop is \{ -1 – -2 \}. Keep that in mind for the following two loops. Seems this tree does not include -5 so let's start a new tree with -5 as part of the loop:

```


-7 – -14 – -28 – -56 – …

-5
```

Seems this loop is \{ -5 – -14 – -7 – -20 – -10 \}. Also note that this being a loop for the second tree does in fact start at the top left and works its way down the second possible branch.

And finally there is yet a third tree with it's own loop that covers the remainder of \( \frac{1}{3} \) of the natural counting numbers. And I'm taking an educated guess that it is -16-1 = -17 because the last loop was -4-1 = -5 and the very first loop was just -1. So my thinking was -(0)-1 = -1 is the \{ -1 – -2 \} loop; -(2^2)-1 = -5 is the \{ -5 – -14 – -7 – -20 – -10 \} loop; -(2^4)-1 = -17 is the next loop. Interesting, eh? Also note that this loop as well begins at the upper left and proceeds down the second possible branch. I have not been able to show why this is the case but an educated guess would indicate it definitely has something to do with the 0; 2^2 and 2^4. It's also interesting that all three start numbers for each of the trees originates from \{ -1 + 4x \} = -1, -5, -9, -13, -17, -21,
...And it is not a coincidence. Another way to look at it is simply $1+0; 1+4; 1+16$ or $1+2^2; 1+2^4$. Powers of 2 still play an important role. It's going to take more work to determine exactly what is happening... the joy of number theory!

The following discussion is a fitting guess on what is happening and how these powers of 2 play into it. Directly following this I get into how to divide the natural counting numbers into 3 sets because $3n$ in the $3n+1$ dictates that much. It takes a little leap of faith to notice that in Collatz a power of three comes into play at two critical jump points ( to new separate trees ). Here is a table layout of the odd numbers applied to both $3n+1; n/2$ & $3n-1; n/2$:

Note that I have highlighted the odd numbers that can potentially jump off into their own tree which of course are given by $1; 1+2^2; 1+2^4 – 1, 5, 17$. See above. And because we are dealing with multiples of 3 and three groupings/sets where we have { multiples of 3 }; { multiples of $3 +1$}; { multiples of $3 +2$ }. Seems 3 plays a critical role.

So in Collatz we see what happens when we look at the three jump points 1, 5 and 17. 1 starts the natural loop { 1 – 4 – 2 }. At 5 we have the potential to jump off to a new tree but because 5 goes to $5 + 3 = 8$ it stays in the original tree. It's also interesting that $8 = 2^3$. Anything other than the addition of a power of 3 would have caused it to form it's own tree. Now with 17 we can see that again it goes to $17 + 3*3 = 26$. Now again there was the potential of jumping off to a new tree had this number been created using a power of 3. The power of 3 kept it in the original loop. So in the case of Collatz and 1, 5, 17 all three stay in the same $1 – 4 – 2$ loop.

Now see what happens when we look at the jump points 1, 5, 17 in the $3n-1; n/2$ sequence. { 1 – 2 – 1 } is the natural first base loop. In the case of 5 it gives $5 + 2 = 7$. This is adding a power of 2...not three. So 5 can break clean of the original loop because it has no way ( needed to add a power of 3 to fall into the original loop ) of entering the { 1 – 2 } loop.

The same thing happens with 17 in the $3n-1; n/2$ sequence. Instead of adding a multiple of 3 to enable it access to the original loop it has a multiple of 2 ( specifically $2 ^ 3 = 8$ ). Note as well that $3 = 2+1$ and $3*3 = 2*2*2+1$. I point this out because we are actually dealing with $3n-1$; so I would expect at these jump points to see a number that is one less than what it would’ve been in Collatz. Now I suspect that the jump points 5 and 17 are the only two points where we can have $3^{x+y} = 2^y + 1$. I've seen this at play elsewhere I think in the $a^x = b^y + 1$; where $x <> y$ ( not equal ).

How's that for some obscure reasoning?

<table>
<thead>
<tr>
<th>Odd number</th>
<th>$3n+1$</th>
<th>$n/2$</th>
<th>$3n-1$</th>
<th>$n/2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>5</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>16</td>
<td>8 ($5+3$)</td>
<td>14</td>
<td>7 ($5+2^2$)</td>
</tr>
<tr>
<td>7</td>
<td>22</td>
<td>11</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>9</td>
<td>28</td>
<td>14</td>
<td>26</td>
<td>13</td>
</tr>
<tr>
<td>11</td>
<td>34</td>
<td>17 ($11+3*2$)</td>
<td>32</td>
<td>16 ($11+5$)</td>
</tr>
<tr>
<td>13</td>
<td>40</td>
<td>20</td>
<td>38</td>
<td>19</td>
</tr>
<tr>
<td>15</td>
<td>46</td>
<td>23</td>
<td>44</td>
<td>22</td>
</tr>
<tr>
<td>17</td>
<td>52</td>
<td>26 ($17+3*3$)</td>
<td>50</td>
<td>25 ($17+2<em>2</em>2$)</td>
</tr>
<tr>
<td>19</td>
<td>58</td>
<td>29</td>
<td>56</td>
<td>28</td>
</tr>
<tr>
<td>21</td>
<td>64</td>
<td>32</td>
<td>62</td>
<td>31</td>
</tr>
</tbody>
</table>
Another interesting observation is that the set of all natural counting numbers can be subdivided into three distinct groupings. This provides ammunition and goes hand in hand with what I was discussing above regarding only three possible trees.

Let's look at the number line and logically break into three groups. This will make more sense as we look at it in detail.

\[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, \ldots\]

Starting at 0; add 3 consecutively to isolate all the multiples of 3. This is one third of the entire set:

\[0, 3, 6, 9, 12, 15, 18, 21, 24, \ldots\] and leaves:

\[1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20, 22, 23, \ldots\]

Next, starting at 1, add 3 consecutively and strip out that third. This is the subset that is any multiple of 3 plus 1.

\[1, 4, 7, 10, 13, 16, 19, 22, \ldots\] and leaves the final sub group:

\[2, 5, 8, 11, 14, 17, 20, 23, \ldots\]

So starting at 2 and adding 3 consecutively gives us all the remaining numbers of the final sub-group. This final sub-group is simply a multiple of 3 plus 2! There are no more multiples of 3 plus anything that will result in a fourth sub-grouping.

The three sub-groups are:

\[
\begin{align*}
\{1, 4, 7, 10, 13, 16, 19, 22, \ldots\} \\
\{2, 5, 8, 11, 14, 17, 20, 23, \ldots\} \\
\{3, 6, 9, 12, 15, 18, 21, 24, \ldots\}
\end{align*}
\]

This shows the three evenly distributed groupings that contain exactly 1/3 of the original natural counting numbers set. It also shows that even deeper than that, half of each of these 3 sub-groupings is even numbers. These 3 sub-groupings are integral in the Colatz tree as well. \(3n(+1)\) dictates that. Right?

I wonder if there is a connection to my original group of equations:

\[
\begin{align*}
\{0 + 2x\} & \quad 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, \ldots \\
\{1 + 4x\} & \quad 1, 5, 9, 13, 17, 21, 25, 29, \ldots \\
\{3 + 8x\} & \quad 3, 11, 19, 27, 35 \ldots \\
\{7 + 16x\} & \quad 7, 23, 39, \ldots
\end{align*}
\]

And there is! Let's start with \(\{0 + 2x\}\):

\[
\begin{align*}
\{1, 4, 7, 10, 13, 16, 19, 22, \ldots\} \\
\{2, 5, 8, 11, 14, 17, 20, 23, \ldots\} \\
\{3, 6, 9, 12, 15, 18, 21, 24, \ldots\}
\end{align*}
\]

What about \(\{1 + 4x\}\):
And \{ 3 + 8x \}:

\{ 1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, \ldots \}
\{ 2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38, \ldots \}
\{ 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, \ldots \}

And last for us to make the point \{ 7 + 16x \}:

\{ 1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, \ldots \}
\{ 2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38, \ldots \}
\{ 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, \ldots \}

My original equations hit each of these three sub-groupings evenly and in a defined pattern. Notice that in \{ 0 + 2x \} each entry in the subgroup is separated by 6; 4 + 6 = 10 + 6 = 16, \ldots In the next \{ 1 + 4x \} it is a separation of 12; 1 + 12 = 13 + 12 = 25, \ldots And if I was a betting man I would wage a guess that the next \{ 3 + 8x \} is separated by 24; 11 + 24 = 35,\ldots with each equation we multiply the difference by an additional 2 (double it)\ldots 6, 12, 24, 48, 96, \ldots The original 6 in that sequence is the result of 3 * 2. Hmmm, multiples of 3 and powers of 2! 3 * 2^1; 3 * 2^2; 3 * 2^3; \ldots

So the third loop looks like this:

\begin{align*}
-17 & -34 - 68 - 136 - 272 \\
| & | -91 - 182 - 364 - \ldots \\
| & | -61 - 122 - 244 - \ldots \\
-23 & -46 - 92 - \ldots -41 - 82 - 164 - \ldots \\
| & | -31 - 62 - 124 - \ldots -55 - 110 - \ldots \\
| & | -21 - 42 - 84 - \ldots -37 - 74 - \ldots \\
| & | -25 - 50 - 100 - \ldots \\
| & | -17
\end{align*}

This loop is a little more involved: \{ -17 - 50 - 25 - 74 - 37 - 110 - 55 - -164 - 82 - 41 - 122 - 61 - 182 - 91 - 272 - 136 - 68 - 34 \}

Also of some interest is the length of these loops and how they appear to relate to the jump points they start from:

\{ -1 - 2 \} - two steps
The first loop begins at -1; but you need at least two steps to form a loop so voila you have a two step loop. The second loop starting with -5 requires exactly five steps. And the third loop starting -17 requires exactly eighteen steps. Now remember the way these trees work, powers of 2 and branching. The first and the third loops require one more step than the starting numbers. The second loop only requires the original five steps. This seems very coincidental, doesn't it? Too convenient! Now if I consider that in this case we are dealing with negative numbers (treat the negative sign as direction only; the actual magnitude of the numbers are same no matter the sign) then instead of adding '1' to the step count for the first and third loops I should've indicated that we are actually adding '-1'. -1 + -1 = -2; -17 + -1 = -18.

Generally, I would say since 'three' is prominent in the way this sequence works, we will only find the three separate trees with their own single loop. And I would expect that the numbers are distributed evenly among the three; with half of that third evenly split between even and odd.

Someone else has already done the statistics that show this to be the case; there are only the three trees and they each contain a 1/3 of the entire natural number set. So I'm not going to rehash that here and simply accept it.

Do my formulas show a cascading convergence as well:

\[
\frac{3 \left(\{ -7 + 16x \} \right) + 1}{2} \\
\frac{-21 + 48x + 1}{2} \\
\frac{-20 + 48x}{2} \\
-10 + 24x \\
-3 + 1 - 8 + 24x \\
1 - 3 - 8 + 24x \\
1 - 3 + 8 \left( -1 + 3x \right) \\
-3 + 8 \left( -1 + 3x \right) + 1
\]

It does cascade to an odd number in the prior level but has 1 added to make it even (or it ultimately jumps to \{ 0 + 2x \}).

It's a little difficult to explain. Suffice to say we do in fact cascade back to the prior level but instead of the number remaining odd it has one added to make it even again and thus divisible once more by 2...but this actually brings us directly back to the first level \{ 0 + 2x \}. This is holding true for all three of those loops. It does appear to be the case in other two trees with the other two loops? I'm going to have to investigate this further to see if I can determine what is happening there and explain it in mathematical terms. I will show in later sections how I was able to arrive at this conclusion which is true for all three loops.

So, No, they break down and can not show a step by step cascade! In the case of the first tree with the \{ -1 - 2 \} loop the cascade is directly to level \{ -0 + 2x \}. The other two trees do the same thing at least mathematically as we have shown by working these equations through 3n-1.

I Think we need to look specifically at what is happening at \{ -1 + 4x \} level. It's likely buried but doing the same cascade to \{ 0 + 2n \} level.
$$\frac{3 \{ -1 + 4x \} + 1}{2} \quad \frac{-3 + 12x + 1}{2} \quad \frac{-2 + 12x}{2} \quad -1 + 6x \quad 1 + 2 \{ -1 + 3x \} \quad -0 + 2 \{ -1 + 3x \} + 1$$

It is doing the same thing. There is a hidden cascade to the prior level but it gets lost in translation and is overridden to first even level \(\{ -0 + 2x \}\). So what this is ultimately saying is that all levels over \(\{ -0 + 2x \}\) have all their elements cascade directly to level \(\{ -0 + 2x \}\). Luckily there are enough elements in \(\{ -0 + 2x \}\) for a one-to-one match with all the elements combined from upper levels. Right?

So we can likely build on that fact like we did before. In this case all levels cascade directly to \(\{ -0 + 2n \}\). So yes, all odd numbers will be accounted for and as a result all evens. Likewise, if magically have three evenly \(\{ 1/3 \}\) distributed trees; that is 1/3 of all the natural number set falls in each of trees. The same odd and even as shown above will hold in each of these three trees as well.

Needless to say it is much easier to show with these three smaller trees that as \(n\) approaches infinity it is not creating a multi-level cascade that could reach infinity in steps...but instead have only a single cascade directly to level \(\{ 0 + 2n \}\). So, there is NO situation where this sequence can grow indefinately and no quasi-counter to use to prove by contradiction like we did above in earlier discussion. I don't think we need to.

It is easily shown after all this that there is one and only one loop for each of the three individual trees. The structure dictates that.

The Collatz trees each hold the 4x+1 rule we've seen in the above discussion. \(-3*4+1 = -11; -23*4+1 = -91\).

Let's go back to these three subsets outlined above:

\[
\{ 1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, \ldots \} \\
\{ 2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38, \ldots \} \\
\{ 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, \ldots \}
\]

The three loops occur and contain only numbers from the first two subsets... the ones that are not a multiple of 3 ( the third subset ). So the first two subsets only. The \(\{-1 - 2\}\) loop:

\[
\{ 1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, \ldots \} \\
\{ 2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38, \ldots \} \\
\{ 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, \ldots \}
\]

And the \(\{-5 - 14 - 7 - 20 - 10\}\) loop:

\[
\{ 1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, \ldots \} \\
\{ 2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38, \ldots \} \\
\{ 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, \ldots \}
\]

This makes sense since any row in the Collatz tree that starts with a multiple of 3 is a dead end row that can't spawn new branches so the loop items must not venture into that subset.

I wonder if there's a pattern here that me might pick up on if we overlay the three loops each in a different color:

I wonder what these loops look like in my equations:

That's interesting but doesn't tell us much except that the loops are confined to elements from \{ 0 + 2x \}, \{ 1 + 4x \}, \{ 3 + 8x \} and \{ 7 + 16x \} only; with each loop starting on an element in \{ 1 + 4x \} ONLY.

If I display the above observation in a slightly different fashion I'll be able to point out more easily some items I mentioned above.

I would like you to note right here that level 2 \((1+4x)\) equation items are not divisible by 4 after subtracting 1 (none of them); however all upper levels members are divisible by 4 after subtracting 1. This is the complete opposite of what I'll show you later for the positive numbers in Collatz where only the members of level 2 \((1+4x)\) are 4x+1 rule (all of them) with no other upper levels having such members.

You can see from the above table that all upper levels (levels 2 and up) immediately jump to level 1 (being even and all). No cascading appears in these trees.

All three trees (loops) can be built using the jump points identified and the 4x+1 rule to glue the backbones together. I'm not going to go into any further detail on how all that works. It does though. Be sure to...
use the dual even numbers as explained above for cohesion. They are invisible in the structures presented but can be drawn in for connectivity.

Section 10 – Exploring the Positive Numbers in the Sequence \{ 3n-1 ; n/2 \}

Much like the previous section, placing the positive numbers in the \{ 3n-1 ; n/2 \} sequence will generate the exact same three loops only in this case all the numbers are positive and the direction of travel is left and up instead of right and up.

We would use the original set of equation that have not been negatized.

- \(0 + 2x\) \{ 0 + (2^1)x \} \{ (((2^1) / 2) -1) + (2^1)x \}
- \(1 + 4x\) \{ 1 + (2^2)x \} \{ (((2^2) / 2) -1) + (2^2)x \}
- \(3 + 8x\) \{ 3 + (2^3)x \} \{ (((2^3) / 2) -1) + (2^3)x \}
- \(7 + 16x\) \{ 7 + (2^4)x \} \{ (((2^4) / 2) -1) + (2^4)x \}
- \cdots
- (((2^y) / 2) -1) + (2^y)x
- \cdots
- (((2^\text{infinity}) / 2) -1) + (2^{\text{infinity}})x

\{ 0 + 2x \} 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, ...
\{ 1 + 4x \} 1, 5, 9, 13, 17, 21, ...
\{ 3 + 8x \} 3, 11, 19, 27, 35 ...
\{ 7 + 16x \} 7, 23, 39, ...

Now what does the tree structure look like:

1 – 2 – 4 – 8 – 16 – 32 – …
    |    |    |    |    |    |    
    |    |    |    |    |    |    
   59 – 118 – 236 – …
    |    |    |    |    |    |    
   79 – 158 – …
    |    |    |    |    |    |    
  15 – 30 – 60 – …
    |    |    |    |    |    |    
  3 – 6 – 12 – …

This first tree has the loop at the very top left before any branching begins. Keep that in mind for the following two loops. Seems this tree does not include 5 so lets start a new tree with 5 as part of the loop:

    |    |    |    |    |    |    
  27 – 54 – 108 – …
    |    |    |    |    |    |    

Collatz Conjecture Explored – Complete Proof
Seems this loop is \{ 5 – 14 – 7 – 20 – 10 \}. Also note that this being a loop for the second tree does in fact start at the top left and works its way down the first possible branch.

And finally there is yet a third tree with its own loop that covers the remainder of \( \frac{1}{3} \) of the natural counting numbers. And I'm taking an educated guess that it is \( 16+1 = 17 \) because the last loop was \( 4+1 = 5 \) and the very first loop was just 1. So my thinking was \( 0+1 = 1 \) is the \{ 1 – 2 \} loop; \( 2^2+1 = 5 \) is the \{ 5 – 14 – 7 – 20 – 10 \} loop; \( 2^4+1 = 17 \) is the next loop. Interesting, ehh? Also note that this loop as well begins at the upper left and proceeds down the second possible branch. I have not been able to show why this is the case but an educated guess would indicate it definitely has something to do with the 0; \( 2^2 \) and \( 2^4 \). See above for further observations on this coincidence.

So the third loop looks like this:

\[
\begin{align*}
17 &- 34 - 68 - 136 - 272 \\
&\quad \quad | \quad | \quad | \quad | \\
&\quad \quad 91 - 182 - 364 - \cdots \\
&\quad \quad | \quad | \quad | \\
&\quad \quad 61 - 122 - 244 - \cdots \\
&\quad \quad | \\
&23 - 46 - 92 - \cdots \\
&\quad \quad | \\
&\quad \quad 41 - 82 - 164 - \cdots \\
&\quad \quad | \\
&\quad \quad 31 - 62 - 124 - \cdots \\
&\quad \quad | \\
&\quad \quad 55 - 110 - \cdots \\
&\quad \quad | \\
&\quad \quad 21 - 42 - 84 - \cdots \\
&\quad \quad | \\
&\quad \quad 37 - 74 - \cdots \\
&\quad \quad | \\
&\quad \quad 25 - 50 - 100 - \cdots \\
&\quad \quad | \\
&\quad \quad 17
\end{align*}
\]


Generally, I would say since 'three' is prominent in the way this sequence works, we will only find the three separate trees with their own single loop. And I would expect that the numbers are distributed evenly among the three; with half of that third evenly split between even and odd.

Again, someone else has already done the statistics that show this to be the case; there are only the three trees and they each contain a \( \frac{1}{3} \) of the entire natural number set. So I'm not going to rehash that here and accept it.

All of the exact same discussion remain true here for a proof as we have shown above in earlier sections.

Lets try a couple of the equations to make sure:
(3(7+16x) -1 ) / 2
21 + 48x - 1 ) / 2
( 20 + 48x ) / 2
10 + 24x
3 - 1 + 8 + 24x
-1 + 3 + 8 + 24x
-1 + 3 + 8 ( 1 + 3x )
3 + 8 ( 1 + 3x ) -1

and:

(3(1+4x ) -1 ) / 2
3 + 12x - 1 ) / 2
( 2 + 12x ) / 2
1 + 6x
-1 + 2 + 6x
-1 + 2 ( 1 + 3x )
0 + 2 ( 1 + 3x ) -1

As expected, instead of adding one to get even we subtract 1 to get even and back to level { 0 + 2x }. The mechanics are the same.

Also, the 4x+1 rule does NOT hold here as expected. This is the other situation where we need to use 4x-1. 3*4-1 = 11; 15*4-1 = 59. Mirror images. Think about that.

1 – 2 – 4 – 8 – 16 – 32 – …
   |                        | 59 – 118 – 236 – …
   |                        | 20 79 – 158 – …
   | 04 15 – 30 – 60 – …
3 – 6 – 12 – …

Remember how I pointed out even numbers could play a dual role in these trees. I've shown two examples above. Only in this case it makes use of 4x-1 and 3x-1 rules. 15+1=16/4=4; 4*3=12-1=11. Again, this is the mollasses that holds the tree together.

Section 11 – Understanding the 'NOT so Random Jumps' Within the Collatz Tree

What appears to be random jumps is actually constrained. Let's explore what is happening at each of my equations starting with { 0 + 2x }.

{ 0 + 2x } 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, 48, 50, …
In the above illustration I have highlighted in different colors the sequences where you take the first x=2 and multiply by 2 successively. 2, 4, 8, 16, 32,... I left the very first number in this sequence un-hilighted which will come in play later. The next available number is x=6 giving 6, 12, 24, 48, 96,... The next available number is x=10 giving 10, 20, 40, 80, 160,... And the next is x=14 giving 14, 28, 56,... Then it's x=18 giving 18, 36, 72,... Obviously there is a distinct pattern here and that is after rooting out all numbers that are multiples of 2 of a prior lower number we end up having every second number starting at 6 available for this operation... 6, 10, 14, 18, 22,... So obviously, every number in this equation will end up in the Collatz Tree. Where it is in that tree is unimportant. Half of this set is divisible by at least 4. The other half is only divisible by 2 leading to an odd number that will fall somewhere else in the tree. I hope you can accept that.

Let me show the next few equations expanded out:

\[
\begin{align*}
\{ 1 + 4x \} & \quad 1, 5, 9, 13, 17, 21, 25, 29, 33, 37, \ldots \\
\{ 3 + 8x \} & \quad 3, 11, 19, 27, 35, 43, 51, 59, 67, 75, \ldots \\
\{ 7 + 16x \} & \quad 7, 23, 39, 55, \ldots \\
\{ 15 + 32x \} & \quad 15, 47, 79, \ldots \\
\{ 31 + 64x \} & \quad 31, 95, \ldots
\end{align*}
\]

There is a pattern to how every second base even number in \{ 0 + 2x \} jumps to upper level equations. So for the sequence 2, 6, 10, 14, 18, 22, 26, 30, 34, 38, 42,... do the division by 2 and you get 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21,... Obviously 1, 5, 9, 13, 17, 21,... of this list all fall in the \{ 1 + 4x \} equation. Note that this list is formed by adding 4 consecutively; 1+4=5+4=9+4=13... I'll be willing to bet that starting at 3 and adding 8 consecutively will give us a list that in the \{ 3 + 8x \}... 3+8=11+8=19+8=27... 3, 11, 19, 27,... Then if we take 7 which is the next available starting sequence you would add 16 consecutively giving 7, 23, 39, 55, ... which is the \{ 7 + 16x \} equation. The pattern should now be obvious.

Let's explore the cascading level effect starting with the \{ 3 + 8x \} equation. If you pick 3 you will pass through to the prior level \{ 1 + 4x \} and that is so. 3*3+1=10/2=5. The same happens to 11...3*11+1=34/2=17. And the next 19 does it as well 3*19+1=58/2=29. And it just so happens 5, 17, 29,... are separated by 12 (3 * 4 or 3 * 2 * 2 ). This covers every number in \{ 3 + 8x \}. The exact same thing happens if we investigate \{ 7 + 16x \}... 3*7+1=22/2=11; 3*23+1=70/2=35; 3*39+1=118/2=59; or 11, 35, 59,... separated by 24 (3 * 8 or 3 * 2 * 2 * 2 ). Looking at \{ 15 + 32x \} we see similar 3*15+1=46/2=23; 3*47+1=142/2=71; 3*79+1=238/2=119; 23, 71, 119 are separated by 48 (3 * 16 or 3 * 2 * 2 * 2 * 2 ). Pattern has been established. Finally let's look at what happens with level \{ 1 + 4x \}. We can see from the above that only 5, 17, 29,... are pass through from upper levels. All other points in this equation remain untouched from upper levels leaving 1, 9, 13, 21, 25, 33, 37, ... Note that all those that are passed through from upper levels reduce to an odd number that is smaller than it started at. 5 reduces to 1; 17 reduces to 13; 29 reduces to 11; 41 reduces to 31; 53 reduces to 5; 65 reduces to 49, and so on. This is good because we can prove that given all numbers up to k are proven, then k+1 = 5 ends in a number that is less than 5 ( actually 1 ) and this is the case for all of these.

Let's continue on with this trend of thought. 1, 13, 25, 37,... is another sequence separated by 12 in \{ 1 + 4x \} that has not been touched from pass through from upper levels. These behave the same way as the pass throughs seen above. They all reduce to a number smaller than the starting number; 1 reduces to 1 ( trivial ); 13 reduces to 5; 25 reduces to 19; 37 reduces to 7; 49 reduces to 37; 61 reduces to 23. So with the same assumption that for k all lower assume true; k+1 = some number from this list results in a number smaller than k that has already been proven.

This leaves the final multiple of 3 sequence ( again separated by 12 ) 9, 21, 33, 45, 57, 69,... And once again for the same agreement above all these reduce to numbers smaller than the original. 9 reduces to 7; 21 reduces to 1; 33 reduces to 25; 45 reduces to 17; 57 reduces to 43; 69 reduces to 13; so if up to k assumed true;
it is obvious that k+1 ends up smaller than k so it is true as well.

This may be an awkward way to prove all numbers are included and reduce to the trivial loop in the Collatz tree. It does seem to work though. That will all become apparent in the below discussion when I start to use these building blocks to formulate the proof.

**Section 12 – Putting It All Together (Formulation of a Proof)**

Expanding upon the first few sections in this report, I will show where my set of equations originated and this is an important observation in showing that all the natural numbers are contained in the union of these subsets.

```
2 4 6 8 10 12 14 16 18 20 22 24 26
1 5 9 13 17 19
3 7 11 15
```

Let me draw the above in a format that is a little cleaner to follow, noting that I will skip over all the even number not formed by successively adding 6 to 2...

```
2 8 14 20 26 32 38 44 50 56 62 68 74 80
1 5 9 13 17 21 25 29 33 37 41 45 49 53
3 7 11 19 23 27 31
```

There is a very unique pattern that makes it very easy to see that 'ALL' the natural numbers will be included. I've taken away any even number that does not grow 'stacks' back to smaller upper level members. This shows the cascading effect I've tried to explain above in other sections. Note how each upper level injects it's first member onto the stack resulting from the second member of the previous level. It's next member is injected onto the third stack from the previous level...so each new member skips two prior level stacks before being injected. That is why I dropped two even numbers before creating stacks. It really jumps out now! Once you accept this you can see where my sequence of equations originated. And just in case you don't realize it, the lowest number in a stack multiplied by 3 and add 1 then divide by 2 give the next up... continue (*3+1)/2 to next level up and so on. These are the basic rules for odd/even numbers in Collatz conjecture.

This realization also brought me to the idea that if this goes on toward infinity there should be '1' stack approaching infinity! Right? The farther right one goes the longer the stacks can grow. But no prior stack less than infinity can be in the same state...the next closest one is one level smaller half way back from infinity ( infinity/2 ). Think about that for a moment. Remember that each upper level equation has half the members the previous one did... hence my halving infinity. This should be enough to show all numbers are infact included; it's a complete set.

The very first row of even numbers is 50% of the total natural number set. The second row is an additional 25% (¼) of the natural number set... the third is 12.5% (1/8) and so on and so forth. You can also see several patterns when written in this fashion. Each set contains only half as many members as the previous set.
You will also note that starting at row 1; the first available odd numbers missed in prior levels (even numbers row) start those sets. So the second row uses 1 as its starting number with successive members formed by adding 4 over and over. The third row would begin with 3 since it was not already used in the two prior rows...and it's members are given by adding 8 successively over and over. The next row begins with 7 and it's members are separated by 16. And this continues on. As you can see every number will be used and only ONCE. I'm also going to point out that if you pick the first member of any row greater than 1 (the even numbers row) and apply the \((3n+1)/2\) rules you will go up one row and to the right! For example \((1*3+1)/2=2\). The next row is \((3*3+1)/2=5\). The next row starting with 7... \((7*3+1)/2=11\); \((11*3+1)/2=17\). The next is \((15*3+1)/2=23\); \((23*3+1)/2=35\); \((35*3+1)/2=53\). That was the important stuff to take forward...

I've shown above that only 3 loops can occur in the negative counting numbers under \(3x+1; x/2\) and only 1 loop using the positive counting numbers under \(3x+1; x/2\). So the existence of a second loop is not possible if following the original conjecture using only positive counting numbers under \(3x+1; x/2\). Two additional loops become possible only when using the negative counting numbers under \(3x+1; x/2\). There are two breakaway points, one at -5 and an additional one at -17. The reasoning as shown above plays with the \(-3+1=-2 & -3*3+1=-2*2*2\) observation. The original loop as unstated would be \(-1*3+1=-2\). As you can obviously see this gives rise to \(-1*3+1=-1*2 & -1*3*3+1=-1*2*2*2\). I probably did a better job of showing this above. Needless to say the 3 jumping points (or three loops) start at -1; -5; and -17. You'll also note that \(-1+-2*2=-5\) and \(-1+-2*2*2=-17\) or \(-1+-2*2=-3*2+1\) and \(-1+-2*2*2=-3*3*2+1\). This special state cannot occur in the positive counting numbers so there is only one loop starting at 1. No other loops can exist. So part one of the proof is confirmed...only the main loop exists.

Now I can build the other part of the proof from above observations. I noted that these counting numbers can be created using an infinite set of sequences; \(0+2x; 1+4x; 3+8x; 7+16x; \ldots\) The first sequence forms all the even numbers. The second sequence has half as many members all of which are odd and separated by 4. The third sequence has half as many members as the second sequence with these being separated by 8...and so on and so forth.

I also noted that any number you start at would fit in one of the sequences and that as you apply the rules you end in the previous sequence stepping through each all the way back to the first. So if you started in the \(7^{th}\) sequence you end up in the \(6^{th}\), then the \(5^{th}\), \(4^{th}\), \(3^{rd}\), \(2^{nd}\), and finally \(1^{st}\). But the \(1^{st}\) may not and usually does not end there and this brings up to another sequence greater than or equal to 1! And that process continues until one reaches the main loop 4-2-1. And this observation is VERY important. No matter what the starting number it will cascade down through the second sequence \((1+4x)\) a number of times on it's way to the first sequence \((0+2x)\) where it'll make another jump wherever.

Lets take a closer look at just the even numbers. We know that there's a pattern here too. Check out the following:

\[
\begin{align*}
2 & - 1 \\
4 & - 2 & - 1 \\
6 & - 3 \\
8 & - 4 & - 2 & - 1 \\
10 & - 5 \\
12 & - 6 & - 3 \\
14 & - 7 \\
16 & - 8 & - 4 & - 2 & - 1 \\
18 & - 9 \\
20 & - 10 & - 5 \\
22 & - 11 \\
\end{align*}
\]
As clearly seen above only powers of 2 'even' numbers can reduce directly to 1. example 2^1, 2^2; 2^3;... or 2, 4, 8, 16, 32, 64, … Now looking at the remainder of this may be critical if you are playing the stats game. As you can see from the way I have it drawn... half the even numbers are divisible by 2 only once. That's 50% of them. Of the 50% that remain a further 50% of them are divisible by an additional 2. So 25% of the total natural even numbers are divisible by 4 (2*2). And if you take the remaining 25%, half of them are divisible by another 2... 12.5% are divisible by 8 (2*2*2). 6.25% are divisible by 16; and so on and so forth. I do not need any of this even number stuff for my proof though.

Because of the way the the Collatz Tree forms I've noted that the starting odds on the successive limbs of any backbone branch are formed by applying (odd*4)+1 to each upper limb. Example 1, 5, 21, ... and
So this leads to the obvious next step that I overlooked in my original report and that is that any odd number where you can subtract 1 and have it evenly divisible by 4 is automatically collapsible to the 4-2-1 loop. For example; 21 – 1 = 20 /4 =5. Note that you may be able to continue subtracting another 1 and still have it divisible by a further 4. But this is not the norm. So if we know 1 to x (assumed) are true, then x+1 being an odd number where x+1 subtract 1 is evenly divisible by 4 is also true. So the 25% of natural numbers in the 1+4x series are all true as well. Like the even numbers; if we know 1 to x (are assumed to be true) then x+1 as long as it is even is also true because (x+1)/2 is in the set we already assumed true...that's 1 to x.

So, we can easily show that all even numbers can reduced to the main 4-2-1 loop knowing that if you have already proven 1 to x; then x+1 if it happens to be even has the rule x/2 applied and the result is a proven x! We can now bring the above discussion (for odd numbers) about what happens if you can subtract 1 and have it divisible by 4...and that this will result in a number that falls in the 1 to x already proven. And this is good because the original sequences I used to create the counting number sets has a special feature. The second sequence 1+4x has all of it's elements being evenly divisible by 4 after subtracting 1. For example ((1+4x)-1)/4 =x That is the set 1, 5, 9, 13, 17, 21, … None of the other sequences will ever have an element that can do this. So the fact that we cascade through all sequences on the way down to the first sequence means we will go through the 1+4x sequence...and all elements in that set will automatically bring one to a number that is in the proven 1 to x! But this is only true if you start in 1+4x sequence. If you cascade from a higher level through 1+4x you are by no means proven. In some cases you may have a number that is smaller than the starting number and in the assume 1 to x true set, but this is not the norm.

Now, any odd number that falls in (is a member of) 1+4x sequence means that it starts proven. So we have been able to prove all even numbers (50%) & all odd numbers where x-1 is evenly divisible by 4 (25%) are Proven. That's 75% total.

If we take the third sequence 3+8x we can show that when it cascades into 1+4x it is close enough that it will be automatically proven.

\[
\frac{(3(3+8x)+1)}{2} \\
\frac{(10+24x)}{2} \\
\frac{(4+6+24x)}{2} \\
\frac{(4+6(1+4x))}{2}
\]
2\(+3(1+4x)\) now see if it is evenly divisible by 4 after subtracting 1...

\[
\frac{(2+3(1+4x)-1)}{4} = \frac{(1+3(1+4x))}{4} = (4+12x)/4 = 1+3x.
\]

So any odd number that falls in 3+8x sequence will automatically be smaller or in the 1 to x assumed. 1+3x is smaller than the original 3+8x.

So as seen above any number that falls in 3+8x sequence (level 3) will cascade directly to level 2 (1+4x) where it automatically becomes true! The resulting number is smaller than the starting odd and in the 1 to x assumed. So that’s an additional 12.5% which gives us a 87.5% of natural numbers proven.

Let’s try doing the same thing to the next two sequences to see if they are close enough as well. That’s the 7+16x and 15+32x. I’m going to try 31+64x as well because I know that’s where it begins to fail. The math shows they both are... however 31+64x is not! Nor are any above that.

\[
\frac{(3(7+16x)+1)}{2} = \frac{(3(15+32x)+1)}{2} = \frac{(3(31+64x)+1)}{2} = (34+72x)/2 = (70+144x)/2 = (94+288x)/2
\]

\[
\frac{(11+24x)}{2} = (23+48x)/2 = (46+96x)/2 = 47+96x
\]

\[
\frac{(3(11+24x)+1)}{2} = \frac{(3(23+48x)+1)}{2} = \frac{(3(47+96x)+1)}{2} = (35+72x)/2 = (70+144x)/2 = (142+288x)/2
\]

\[
\frac{(17+36x)}{2} = (35+72x)/2 = (71+144x)/2
\]

\[
\frac{(17+36x-1)}{4} = \frac{(3(35+72x)+1)}{2} = \frac{(3(71+144x)+1)}{2} = (106+216x)/2 = (214+432x)/2
\]

\[
\frac{(16+36x)}{4} = (53+108x)/2 = 107+216x
\]

\[
\frac{(4+9x)}{4} = (53+108x)/4 = (3(107+216x)+1)/2 = (52+108x)/4 = (322+648x)/2
\]

\[
\frac{(13+27x)}{2} = 161+324x
\]

\[
\frac{4+9x<7+16x!}{13+27x<15+32x!}
\]

\[
\frac{(13+27x-1)}{4} = \frac{(161+324x-1)}{4} = \frac{(160+324x)}{4} = \frac{40+81x}{4} = 40+81x > 31+64x!
\]

I’m going to apply a twist to all levels greater than the third (3+8x). Let’s go in the opposite direction. First let’s look at something special that occurs with a number of the upper level sequences...

Level 1 (0+2x) starts with 2 (even numbers)
Level 2 (1+4x) starts with 1
Level 3 (3+8x) starts with 3 (starts with a multiple of 3!)
Level 4 (7+16x) starts with 7
Level 5 (15+32x) starts with 15 (starts with an amultiple of 3!)
Level 6 (31+64x) starts with 31
Level 7 (63+128x) starts with 63 (starts with a multiple of 3!)
Level 8 (127+256x) starts with 127
Level 9 (255+512x) starts with 255 (starts with a multiple of 3!)
Level 10 (511+1024x) starts with 511

Any backbone row starting with an odd number that is divisible by 3 (multiple of 3) can not spawn new backbones. That exactly half of the remaining levels which is clearly the case as seen above. However, just
because the first member in that level is a multiple of 3 does not mean the others are multiples of 3 too; quite
the opposite. As we'll see below all upper level equations display the same properties.

Let's start with an odd number from the sequence 7+16x...say 23! Now let's multiply it by 2 and see if
the result subtract 1 is evenly divisible by 3. If it is, the number is proven because it falls in the 1 to x assumed
proved and is smaller than the original.

So the sequence starting with 7 has an even division of members into three groups; one where after you
multiply by 2 you can subtract 1 and have it evenly divisible by 3; one where you must multiply by 4 then
subtract 1 and it will be evenly divisible by 3 ( but the resulting number is not smaller than the starting! It is
however evenly divisible by 4 after subtracting 1! This then makes it smaller than the starting.); and a final
group that is evenly divisible by 3 ( a multiple of 3 – dead end backbone ) which I can not handle at this time.
So each group is exacly 1/3 ( 33% ). I can prove 2 of these subgroups meaning 66% are provable.

7 – 14 – 28        ( 28 – 1 = 27 / 3 = 9 ) 9 – 1 = 8 / 4 = 2!
or directly using even number 'duality' 7-1 = 6 / 3 = 2!
23 – 46            ( 46 – 1 = 45 / 3 = 15! )
39 – 78 – 156     ( Multiple of 3; I can't do anything with this yet )
55 – 110 – 220    ( 220 – 1 =219 / 3 = 73 ) 73 – 1 = 72 / 4 = 18!
or duality again 55-1 = 54 / 3 = 18!
71 – 142          ( 142 – 1 = 141 / 3 = 47! )
87 – 174 – 348    ( Multiple of 3!)
103                 103-1=102 / 3 = 34 (duality)
119 - 238         238-1=237 / 3 = 79
135               (Multiple of 3)
151              151-1=150 / 3 = 50 (duality)
167 – 334        334-1=333 / 3 = 111
183              (Multiple of 3)

And this pattern in the above listing continues to infinity. I've highlighted the ones I consider 'duality'
evens and you will note that they increase by exactly 16. The next subset are separated by exactly 32. As you
can see this goes on toward infinity. This sequence ( 7+16x ) is where 16 and 32 come from. In the first subset
our node is the key and being the first it reflects exactly 16. The second subset must have the node multiplied by
2 exactly once to give an even number and those are separated by exactly 32 (16*2). All other levels display
these same features.

Let's see if the next two levels ( 15+32x ) and ( 31+64x ) do the same thing:

15              (Multiple of 3)
47 – 94         94-1=93 / 3 = 31
79                79-1=78 / 3 = 26 (Duality)
111              (Multiple of 3)
143 – 286       286-1 = 285 / 3 = 95
175              175-1 = 174 / 3 = 58 (Duality)
207              (Multiple of 3)
239 – 478       478-1= 477 / 3 = 159
271              271-1 = 270 / 3 = 90 (Duality)
303              (Multiple of 3)
335 – 670       670-1= 669 / 3 = 223
367              367-1= 366 / 3 = 122 (Duality)
31  \quad 31-1 = 30 / 3 = 10 \text{ (Duality)}
95 – 190 \quad 190-1= 189 / 3 = 63
159 \quad \text{(Multiple of 3)}
223 \quad 223-1= 222 / 3 = 74 \text{ (Duality)}
287 – 574 \quad 574-1 = 573 / 3 = 191
351 \quad \text{(Multiple of 3)}
415 \quad 415-1 = 414 / 3 = 138 \text{ (Duality)}
479 – 958 \quad 958-1 = 957 / 3 = 319
543 \quad \text{(Multiple of 3)}
607 \quad 607-1= 606 / 3 = 202 \text{ (Duality)}
671 – 1342 \quad 1342-1 = 1341 / 3 = 447
735 \quad \text{(Multiple of 3)}

They both do and display the exact same attributes. So it is safe to assume that all the other levels whether or not they start with multiples of 3, behave in the same fashion. Let's look at the level starting with 127 (127+256x).

127 – 254 – 508 \quad ( 508 – 1 ) / 3 = 169 \quad (169 – 1) / 4 = 42!
\quad \text{or duality 127-1 = 126 / 3 = 42.}
383 – 766 \quad ( 766 – 1 ) / 3 = 255!
639 – 1278 – 2556 \quad \text{(Multiple of 3)}
895 – 1790 – 3580 \quad ( 3580 – 1 ) / 3 = 1139 \quad (1139 – 1) / 4 = 298!
\quad \text{duality 895-1 = 894 / 3 = 298.}
1151 – 2302 \quad ( 2302 – 1 ) / 3 = 767!
1407 – 2814 – 5628 \quad \text{(Multiple of 3)}
1663 \quad 1663-1=1662 / 3 = 554 \text{ (Duality)}
1919 – 3838 \quad 3838-1=3837 / 3 = 1279
2175 \quad \text{(Multiple of 3)}
2431 \quad 2431-1=2430 / 3 = 810 \text{ (Duality)}
2687 – 5374 \quad 5374-1=5373 / 3 = 1791
2943 \quad \text{(Multiple of 3)}

In this sequence/level (127+256x) we see the first subset separated by exactly 256 with the second set separated by exactly 512 (2*256). Cool.

Just for kicks, let's look at another multiple of 3 level (63+128x) to see if it displays the same properties.

63 – 126 – 252 \quad \text{(Multiple of 3)}
191 – 382 \quad ( 382 – 1 ) / 3 = 127!
319 – 638 – 1276 \quad ( 1276 – 1 ) / 3 = 425 \quad (425-1) / 4 = 106!
\quad \text{duality 319-1 = 318 / 3 = 106.}
447 – 894 – 1788 \quad \text{(Multiple of 3)}
575 – 1150 \quad (1150 – 1 ) /3 = 383!
703 – 1406 – 2812 \quad (2812 – 1 )/3 = 937 \quad (937-1) / 4 = 234!
\quad \text{duality 703-1 = 702 / 3 = 234.}
831 \quad \text{(Multiple of 3)}
959 – 1918 \quad 1918-1= 1917 / 3 = 639
A multiple of 3 equation behaves in exactly the same fashion...they are simply ordered otherwise. 66% are easily provable by the same techniques. Immediately above is level (63+128x) with one subset separated by exactly 128 and the other by exactly 256. Not a coincidence!

So like I mentioned above 7+16x and 15+32x can be proven in the same fashion as 3+8x because they are within a distance that will allow for it. I do however use both those sequences above to show what happens in all upper levels and how three distinct groupings/sets become possible. The numbers are smaller to deal with to show this point. Looking at sequence 127+256x you can see how quickly the numbers grow.

So as stated above we've shown that 66% of the members in each upper level sequences (the ones that start with multiples of 3 are simply ordered differently) by simply applying the rules as shown above; one third are simply multiplied by 2 then divisible by 3 after subtracting 1; another third by multiplying by 4 then divisible by 3 after subtracting 1...but can be further reduced by subtracting 1 and have it divisible by 4; the remaining third are multiples of 3 and no proof yet.

I now realize that the approach I'm taking by backward traversing to prove by induction can be used to prove all numbers that are not multiples of 3; example is multiply by 2 and/or subtract 1 and then divisible by 3. You will notice that all odd numbers (except multiples of 3) display this feature. We can use this as a second method that compliments my first method. As to speak they work hand in hand and prop up one another as an even stronger proof concept. Using duality makes this doable and easier to spot.

Snippet one...

\[
\begin{array}{c}
5 - 10 - 20 - 40 \\ | \\
| 13 - 26 \\ | \\
3 - 6 \\
\end{array}
\]

Snippet two...

\[
\begin{array}{c}
31 - 62 - 124 - 248 \\ | \\
10 - 41 - 82 \\ | \\
27 - 54 \\
\end{array}
\]

The above two snippets show this concept clearly. By working backwards we have a result number smaller than the beginning number. Induction! 5 can easily be reduced to 3. 13 easily reduces to 4 (duality). 31 easily reduces to 10. I can't believe this has been staring me in the face all this time. My discussion on duality made it a reality for me.

Doing the math we have 12.5% remaining to cover off the upper levels but remember that as we go up levels the members included are halved. So the levels have the following associated percentages:
So continuing on with the math we can prove 50% + 25% + 12.5% + 6.25% + 3.125% + 1.0417% +
0.516% + 0.2604167% + 0.1256% + 0.0628 % giving a grand total of 98.88%. So I am able to prove slightly
more than 98% of all the natural counting numbers set are provable.

My quandry now is that I can not fashion a method to handle those multiples of 3 instances (the
remainder and only case yet to be proven) which account for less than 2%. Wow, that's close. I wonder if
anyone else has come this close?

I haven't abandoned hope of solving the 'multiple of 3' issue and wish to share what I do know so far.
The following table contains all the multiples of 3 up to 207. Take note of how I reduce them to provable. I
have not included any even multiples of 3...examples 6, 12, ...:

| Level | Formula 1 | Formula 2 | Formula 3 | Formula 4 | Provable
|-------|-----------|-----------|-----------|-----------|---------
| 3     | (3*3+1)/2 = 5 | (5-1)/4 = 1 | Provable |
| 9     | (9-1)/4 = 2 | Provable |
| 15    | (3*15+1)/2 = 23 | (3*23+1)/2 = 35 | (3*35+1)/2 = 53 | (53-1)/4 = 13 | Provable |
| 21    | (21-1)/4 = 5 | Provable |
| 27    | (3*27+1)/2 = 41 | (41-1)/4 = 10 | Provable |
| 33    | (3*33+1)/2 = 59 | (3*59+1)/2 = 89 | (81-1)/4 = 20 | Provable |
| 39    | (3*39+1)/2 = 77 | (77-1)/4 = 19 | Provable |
| 45    | (3*45+1)/2 = 113 | (113-1)/4 = 28 | Provable |
| 51    | (3*51+1)/2 = 167 | (167-1)/4 = 46 | Provable |
| 57    | (3*57+1)/2 = 221 | (221-1)/4 = 55 | Provable |
| 63    | (3*63+1)/2 = 285 | (285-1)/4 = 71 | Provable |
| 69    | (3*69+1)/2 = 341 | (341-1)/4 = 85 | Provable |
| 75    | (3*75+1)/2 = 409 | (409-1)/4 = 102 | Provable |
| 81    | (3*81+1)/2 = 477 | (477-1)/4 = 119 | Provable |
| 87    | (3*87+1)/2 = 545 | (545-1)/4 = 136 | Provable |
| 93    | (3*93+1)/2 = 613 | (613-1)/4 = 153 | Provable |
| 99    | (3*99+1)/2 = 681 | (681-1)/4 = 170 | Provable |
| 105   | (3*105+1)/2 = 749 | (749-1)/4 = 187 | Provable |
| 111   | (3*111+1)/2 = 817 | (817-1)/4 = 204 | Provable |
| 117   | (3*117+1)/2 = 885 | (885-1)/4 = 221 | Provable |
| 123   | (3*123+1)/2 = 953 | (953-1)/4 = 238 | Provable |
| 129   | (3*129+1)/2 = 1021 | (1021-1)/4 = 255 | Provable |
| 135   | (3*135+1)/2 = 1089 | (1089-1)/4 = 272 | Provable |
| 141   | (3*141+1)/2 = 1157 | (1157-1)/4 = 289 | Provable |
| 147   | (3*147+1)/2 = 1225 | (1225-1)/4 = 306 | Provable |
| 153   | (3*153+1)/2 = 1293 | (1293-1)/4 = 323 | Provable |

Collatz Conjecture Explored – Complete Proof
As seen in table above (green rows) which account for 50% of the multiples of 3 are immediately provable \((x-1)/4\). You should come to realize that this 50% are contained in my level 2 equation. All the even multiples of 3 which I excluded from the above list are with the level 1 equation. Another 25% (yellow) of the multiples of 3 first have to go through 1 iteration of \(3x+1\) which will immediately be reducable because less 1 is divisible by 4. These coincide with my level 3 equation. Another 12.5% (blue) must run through two iterations of \(3x+1\) before becoming candidates for less 1 divisible by 4 evenly. This is my level 4 equation. And finally another 6.25% after 3 iterations of \(3x+1\) become evenly divisible by 4 after subtracting 1 (purple). These are my level 5 equation. Now that totals to 93.75% of the odd multiples of 3 are provable. If we include the even multiples in the overall calculation it turns out to be 50% + 25% + 12.5% + 6.25% + 3.125% for a total of 96.88% are easily provable by the techniques already outlined above.

Now I list the non-provables in a table where one can note they are separated by 96:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>63</td>
<td>159</td>
<td>255</td>
</tr>
<tr>
<td>351</td>
<td>447</td>
<td>543</td>
</tr>
<tr>
<td>639</td>
<td>735</td>
<td>831</td>
</tr>
<tr>
<td>927</td>
<td>1023</td>
<td>1119</td>
</tr>
</tbody>
</table>

What's important to note here are the items I marked red. These are the very first members of my equations for those levels that are multiples of 3. Imagine that. You can see that 127 and 511 which are not multiples of 3 are not included in this list.

Taking a look at my equation that starts with 31 will proceed with the following members 95, 159, 223, 287, 351, 415, 479, 543, 607, 671, 735, 799, 863, 927, 991, 1055, 1119. If we look at another equation starting with 127 we have the following sequence of members 383, 639, 895, 1151, 1407. Notice how the multiples of 3 entries found in the non-multiple of 3 equations in upper levels are found in this above list. This is also the case for those equations that start with multiples of 3...they are ordered differently but each multiple of 3 appears in this list too! Example 53, 191, 319, 447, 575, 703, 831, 959, 1087.

We can safely concluded that all the easily provable multiples of 3 fall in those levels 1 to 5 equations. And that the remainder of those multiples of 3 are found in upper levels and not easily provable; not proven so far.
I've also found another connection that I will point out here (remember the duality of even numbers):

Level starting with 31:

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>31</td>
<td>95</td>
<td>159</td>
<td>223</td>
<td>287</td>
</tr>
<tr>
<td>(31-1)/3</td>
<td>(95*2-1)/3</td>
<td>Mult 3</td>
<td>(223-1)/3</td>
<td>(287*2-1)/3</td>
</tr>
<tr>
<td>10</td>
<td>63</td>
<td>74</td>
<td>191</td>
<td></td>
</tr>
</tbody>
</table>

Level starting with 63. This clearly brings us back to the cascading effect.

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>63</td>
<td>191</td>
<td>319</td>
<td>447</td>
<td></td>
</tr>
<tr>
<td>Mult 3</td>
<td>(191*2-1)/3</td>
<td>(319-1)/3</td>
<td>Mult 3</td>
<td>(447*2-1)/3</td>
</tr>
<tr>
<td>127</td>
<td>106</td>
<td>575</td>
<td>703</td>
<td></td>
</tr>
</tbody>
</table>

Level starting with 127:

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>127</td>
<td>383</td>
<td>639</td>
<td>895</td>
<td>1151</td>
</tr>
<tr>
<td>(127-1)/3</td>
<td>(383*2-1)/3</td>
<td>Mult 3</td>
<td>(895-1)/3</td>
<td>(1151*2-1)/3</td>
</tr>
<tr>
<td>42</td>
<td>255</td>
<td>298</td>
<td>767</td>
<td></td>
</tr>
</tbody>
</table>

Does the same thing as above 3 levels with the second member pointing to the first item of the next level up. There are other observations that I don't think will play a role in the proof. The fifth item points to the second of the next level. I wonder if the eighth item will point to the 3rd in the next level. Let's check:

Level starting with 255:

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>415</td>
<td>479</td>
<td>543</td>
<td>607</td>
<td>671</td>
</tr>
<tr>
<td>(415-1)/3</td>
<td>(479*2-1)/3</td>
<td>Mult 3</td>
<td>(607-1)/3</td>
<td>(671*2-1)/3</td>
</tr>
<tr>
<td>138</td>
<td>319</td>
<td>202</td>
<td>447</td>
<td></td>
</tr>
</tbody>
</table>

It does and if one continues this the pattern becomes obvious and holds true in all upper levels. I find that very interesting, indeed. You can likely see other connections as I do but nothing that will help me with the multiple of 3 dilemma I have.

I wouldn't be surprised if we see this same pattern all the way from the level that starts with 3 (3+8x). I'll leave that to you to investigate. I do not believe I need it to prove those lower levels since I already have a method to do just that. And my quick inquiry does indicate it is! There's all kinds of patterns and connectivity.

With some further pondering, I've decided to reconsider the multiples of 3 in a their own light. The do cover 1/3rd of the entire natural counting number set. First, if I look at just the multiples of 3 the following chart becomes obvious. These multiples of 3 account for 1/3 of the entire counting number set. Right?

<table>
<thead>
<tr>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
<th>15</th>
<th>18</th>
<th>21</th>
<th>24</th>
<th>27</th>
<th>30</th>
<th>33</th>
<th>36</th>
<th>39</th>
<th>(+3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>18</td>
<td>21</td>
<td>24</td>
<td>27</td>
<td>30</td>
<td>33</td>
<td>36</td>
<td>39</td>
<td>(x/2)</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Dividing by 2 will eliminate 50% half of these as automatically provable. These coincidentally coincide with my level 1 equation (0+2x). You'll also note that all these are separated by exactly 6; 6+6=12+6=18+6=24. Half of the remaining are divisible by 4 after subtracting 1. That's another 25%. This is my level 2 equation (1+4x). These are separated by 12; 9+12=21+12=33. I might also point out that results all seem to be spaced out by exactly 3. For example, after dividing through by 2 we get 3+3=6+3=9+3=12+3=15+3=18... After doing (x-1)/4 we get 2+3=5+3=8+3=11...
For easier viewing I'm going to eliminate that 75% from my next chart as provable.

<table>
<thead>
<tr>
<th></th>
<th>3</th>
<th>15</th>
<th>27</th>
<th>39</th>
<th>51</th>
<th>63</th>
<th>75</th>
<th>87</th>
<th>99</th>
<th>111</th>
<th>123</th>
<th>135</th>
<th>(+12)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>41</td>
<td>77</td>
<td>113</td>
<td>149</td>
<td>185</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(3x+1)/1</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>19</td>
<td>28</td>
<td>37</td>
<td>46</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(x-1)/4</td>
</tr>
</tbody>
</table>

You can clearly see that 50% of the remainder are level 3 equation (3+8x) and are separated by 12. As seen elsewhere these can be reduced to provable after one iteration of (3x+1)/2 then apply (x-1)/4. The end row is separated by 9; 1+9=10+9=19+9=28...

Let's redraw the remainder:

<table>
<thead>
<tr>
<th></th>
<th>15</th>
<th>39</th>
<th>63</th>
<th>87</th>
<th>111</th>
<th>135</th>
<th>159</th>
<th>183</th>
<th>207</th>
<th>231</th>
<th>255</th>
<th>279</th>
<th>(+24)</th>
</tr>
</thead>
<tbody>
<tr>
<td>59</td>
<td>131</td>
<td>203</td>
<td>275</td>
<td>347</td>
<td>419</td>
<td></td>
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<td>(3x+1)/2</td>
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<tr>
<td>89</td>
<td>197</td>
<td>305</td>
<td>413</td>
<td>521</td>
<td>629</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>(3x+1)/2</td>
</tr>
<tr>
<td>22</td>
<td>49</td>
<td>76</td>
<td>103</td>
<td>130</td>
<td>157</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>(x-1)/4</td>
</tr>
</tbody>
</table>

These are the level 4 equation (7+16x) and make another 50% of the remaining provable after running through two cycles of (3x+1)/2 and one cycle of (x-1)/4. The end row is separated by 27. Now 27 may be an interesting coincidence in that starting at that number produces a very long chain. This might prove useful if one wishes to try to determine the length of the chains. I'm not interested in that here.

Let's redraw the remainder we get:

<table>
<thead>
<tr>
<th></th>
<th>15</th>
<th>63</th>
<th>111</th>
<th>159</th>
<th>207</th>
<th>255</th>
<th>303</th>
<th>351</th>
<th>399</th>
<th>447</th>
<th>495</th>
<th>543</th>
<th>(+48)</th>
</tr>
</thead>
<tbody>
<tr>
<td>23</td>
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<td>311</td>
<td>455</td>
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<td>(3x+1)/2</td>
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<tr>
<td>35</td>
<td>251</td>
<td>467</td>
<td>683</td>
<td>899</td>
<td>1115</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td>(3x+1)/2</td>
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<tr>
<td>53</td>
<td>377</td>
<td>701</td>
<td>1025</td>
<td>1349</td>
<td>1673</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td>(3x+1)/2</td>
</tr>
<tr>
<td>13</td>
<td>94</td>
<td>175</td>
<td>256</td>
<td>337</td>
<td>418</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(x-1)/4</td>
</tr>
</tbody>
</table>

These are level 5 equation (15+32x) and proves a further 50% of the remainder. This is after 3 cycles of (3x+1)/2 and one of (x-1)/4. Our list is getting pretty small with these first 5 levels removed. Note that the end row is separated by 81. Are you beginning to see a pattern with this spacing as we go higher in my equations to upper levels. Level 2 (1+4x) has them separated by 3; Level 3 (3+8x) has them separated by 9 = (3 * 3). Level 4 (7+16x) separated by 27 = (3 * 3 * 3). And level 5 needless to say will be 81 as shown above (3 * 3 * 3 * 3 * 3). Cool.

Now we are beginning to step into my realization that is we apply (3x+1)/2 over and over we will reach a point where we hit level 2 after a specific number of iterations...and the final step in that at level 2 we can do the (x-1)/4. Right? That's the cascade I've been pointing out. What I didn't consider is that the resulting number may still be 'even' and further divisible by another (x-1)/4 or simply by 2 or a combination and number of these which results in the final number being smaller than the starting number. So what I am saying is if we end up with a number that is still larger than the starting number and cannot reduce it further with (x-1)/4 or (x/2)...then continue to apply (3x+1)/2 until you can start reducing again. My belief is that no matter the number (multiple of 3) it can be manipulated into provable in short fashion. Let's take the remainder and start a new chart:

<table>
<thead>
<tr>
<th></th>
<th>63</th>
<th>159</th>
<th>255</th>
<th>351</th>
<th>447</th>
<th>543</th>
<th>639</th>
<th>735</th>
<th>831</th>
<th>927</th>
<th>1023</th>
<th>1119</th>
<th>(+96)</th>
</tr>
</thead>
</table>

---

Collatz Conjecture Explored – Complete Proof

Page: 37 of 59
As seen above the same pattern exists for pulling out those entries that are related to the current level we are investigating. In this case they are separated by 192. So let's start a new chart with just those expanded out:

159 351 543 735 927 1119 (+192)
809 1781 2753 3725 4697 6641 7613 8585 9557 10529 11501 4(3x+1)/2
202 445 688 931 1174 1417 1660 1903 2146 2389 2632 2875 (x-1)/4
101 344 587 830 1073 1316 (x/2)
25 111 354 268 597 (x-1)/4
172 177 415 134 658 (x/2)

As can be seen in the above chart every 4th column is not reducable. The other columns through a decernable pattern are reducable well below the starting number. See if you can pick out that pattern yourself...

So at this point we have shown that ¾ of the multiples of 3 are provable. Let's pull out those that were not and start yet another sub-chart:

735 1503 2271 3039 3807 4575 5343 6111 6879 7647 8415 9183 (+768)
931 1903 2875 3847 4819 5791 6763 7735 8707 9679 10651 11623 (prelims)
1397 2855 4313 5771 7229 8687 10145 11603 13061 14519 15977 17435 (3x+1)/2
4283 8657 13031 17505 21979 26453 (3x+1)/2
6425 19547 32669 (3x+1)/2

Continuation of the above chart:

4723 5452 8725 6181 14728 6910 10912 7639 8368 13099 (x-1)/4

Again, it appears that every 4th column do not reduce to provable. Let's pick off the remaining that did not reduce to a provable level into a new chart:

1503 4575 7647 10719 13791 16863 19935 23007 26079 29151 32223 35295 (+3072)
6425 19547 32669 45791 58913 72035 85157 98279 111401 124523 137645 150767 (+13122)
1606 8167 14728 21289 27850 34411 (x-1)/4
803 7364 13925 (x/2)
29321 12251 68687 108053 31934 147419 186785 51617 226151 (3x+1)/2
7330 27013 46696 (x/2)
3665 6753 23348 (x-1)/4
15967 12904 (x-1)/4
18377 103031 221129 339277 (3x+1)/2
4594 55282 84819 (x-1)/4
27641 6910 (x/2)
154547 127229 (3x+1)/2

Collatz Conjecture Explored – Complete Proof
Continuation of the above chart:

<table>
<thead>
<tr>
<th>38367</th>
<th>41439</th>
<th>44511</th>
<th>47583</th>
<th>50655</th>
<th>53727</th>
<th>56799</th>
<th>59871</th>
<th>62943</th>
<th>66015</th>
<th>69087</th>
<th>(+3072)</th>
</tr>
</thead>
<tbody>
<tr>
<td>163889</td>
<td>177011</td>
<td>190133</td>
<td>203255</td>
<td>216377</td>
<td>229499</td>
<td>242621</td>
<td>255743</td>
<td>268865</td>
<td>281987</td>
<td>295109</td>
<td>(+13122)</td>
</tr>
<tr>
<td>40972</td>
<td>47533</td>
<td>54094</td>
<td>60655</td>
<td>67216</td>
<td>73777</td>
<td>(x-1)/4</td>
<td>(x/2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20486</td>
<td>27047</td>
<td>33608</td>
<td>20486</td>
<td>27047</td>
<td>33608</td>
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<td></td>
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<td>344249</td>
<td>90983</td>
<td>383615</td>
<td>422981</td>
<td>110666</td>
<td>(3x+1)/2</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>66379</td>
<td>86062</td>
<td>43031</td>
<td>23436</td>
<td>(x-1)/4</td>
<td>(x/2)</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>99569</td>
<td>136475</td>
<td>575423</td>
<td>(3x+1)/2</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>24892</td>
<td>(x-1)/4</td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>12446</td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

Numbers really start reducing in these cycles of $(3x+1)/2$. We only have every 16th column left unproven. I am pretty certain all the above numbers are correct. Hmmm, there seems to be spreading out, 4 in the other chart; 16 in this chart. 16 = 4*4. Now if I were to place a bet I would safely assume that if I pulled out the leftovers and expanded into a new chart we would find that we would have leftover columns not reducable after every 64th column. And the following chart would see leftovers after every 256th column; and the next after every 1024th column; etc. Wow! Interesting indeed. So we see ¼ leftover after first chart; 1/16th leftover after the second chart; 1/64th after the third; 1/256th in the fourth and 1/1024th in the 5th. This process continues and shows that with deductive reasoning, an unproven multiple of 3 simply run through another 3 iterations of $(3x+1)/2$, will likely become provable. Right? Maybe not. So I worked outside this report in a spreadsheet to prove that is the case. That was an involved process indeed and placing the results in this report would make it far too long. What I found out is that the very next chart with the leftovers expanded will result in a minimum of $1/64^{th}$ but it could be much less like $1/128^{th}$ ; I decided against trying to find the exact number; the minimum of $1/64^{th}$ fits my theory but I had to run the process through 3 iterations of 3 iterations to reach only $1/64^{th}$ remaining. That's running through 9 iterations of $(3x+1)/2$. I wonder if this has something to do with 2*2 and 3*3. I believe that '3' is important in understanding what is going on but I don't think I need it for the proof. I was able to show that we approach 100% reducable with more iterations of $(3x+1)/2$. Anyways, I'll archive that spreadsheet or find a way to place it as an appendix to this report.

It would appear that each time I do a set of $(3x+1)/2$, I reduce the remaining set by $\frac{3}{4}$ leaving only a quarter. In the next set of $(3x+1)/2$ I reduce the remaining set to $1/16^{th}$. And the next the remaining is reduced to just $1/64^{th}$;... So we have a situation where as we approach an infinite number of $(3x+1)/2$ iterations we reduce the set to very, very, very, very, very tiny. For all intents and purpose we have proven all these multiples of 3? We would have to map out many more numbers in the above chart to show this clearly; that is why I am clearly pointing out this observation. At this state of the charting it appears to be what is going on.

So my idea almost played out in that we could apply further $(x-1)/4$ or $x/2$ to reduce to make provable in $\frac{3}{4}$ of the cases. As I've shown, if we apply that last quarter $(1/4)$ through multiple iterations of $(3x+1)/2$ it then becomes divisible by 4 after subtracting 1. That's another $\frac{3}{4}$ easily proven. That leaves a quarter of a quarter to prove. It appears that if given enough iterations of $(3x+1)/2$ one can reduce any multiple of three to an inductive state! Some of these multiples of 3 are going to consume a very large number of iterations as you can imagine;
almost enough to consider it a runaway growth cycle. But, if you will notice there is again a discernable pattern to all this madness. So even if you do not want to take that last step to having them all provable...you can accept that ¼ of with an additional 15/16th of that final ¼ are easily provable. 75% + 23.4375% = 98.4375% total. For level 6 (31+64x) we easily show that 66% are provable leaving only multiples of 3. Above we have shown that 98.4375% of those multiples of 3 are also easily proven. The remainder are a little questionable. So that works out to 66.666666% + 98.4375% of 33.333333% = 66.666667% + 32.81% = 99.48%. So level 6 has 1.5625% of the natural numbers...with 99.48% of them easily provable... 1.554%. I'll do the next level 7 immediately to show this concept holds in upper levels. If you follow my above reasoning and agree with the mathematics displayed you will notice that we can prove ¾ leaving ¼; of that ¼ we can prove 15/16th of that leaving just 1/16 to prove; one more iteration set and we prove 63/64 leaving 1/64th unproven. Can you see that this is approaching 100% provable after a finite number of steps? Now, a little more statistics (with just shows:

<table>
<thead>
<tr>
<th>Level</th>
<th>Formula</th>
<th>Percentage Provable</th>
<th>Natural Numbers Provable</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0+2x</td>
<td>100%</td>
<td>50%</td>
</tr>
<tr>
<td>2</td>
<td>1+4x</td>
<td>100%</td>
<td>25%</td>
</tr>
<tr>
<td>3</td>
<td>3+8x</td>
<td>100%</td>
<td>12.5%</td>
</tr>
<tr>
<td>4</td>
<td>7+16x</td>
<td>100%</td>
<td>6.25%</td>
</tr>
<tr>
<td>5</td>
<td>15+32x</td>
<td>100%</td>
<td>3.125%</td>
</tr>
<tr>
<td>6</td>
<td>31+64x</td>
<td>99.48%</td>
<td>1.5625%</td>
</tr>
<tr>
<td>7</td>
<td>63+128x</td>
<td>99.48%</td>
<td>0.78125%</td>
</tr>
<tr>
<td>8</td>
<td>127+256x</td>
<td>99.48%</td>
<td>0.3906%</td>
</tr>
<tr>
<td>9</td>
<td>255+512x</td>
<td>99.48%</td>
<td>0.1953%</td>
</tr>
<tr>
<td>10</td>
<td>511+1024x</td>
<td>99.48%</td>
<td>0.0977%</td>
</tr>
</tbody>
</table>

For a grand total of 99.886% easily provable! That's 0.11% not so easily provable but I do believe I was able to show that they are as well. Do you agree that if I put the full observable from above this number approaches 100% provable multiples of 3.

Continuation of above chart to show same patterns...

<table>
<thead>
<tr>
<th>63</th>
<th>255</th>
<th>447</th>
<th>639</th>
<th>831</th>
<th>1023</th>
<th>1215</th>
<th>1407</th>
<th>1599</th>
<th>1791</th>
<th>1983</th>
<th>2175</th>
<th>(+192 )</th>
</tr>
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<tr>
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<td>1823</td>
<td>2399</td>
<td>2975</td>
<td>(3x+1)/2</td>
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<td></td>
<td></td>
</tr>
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<td></td>
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</tr>
<tr>
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<td>10043</td>
<td>(3x+1)/2</td>
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</tr>
<tr>
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<td>3401</td>
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<td>9233</td>
<td>12149</td>
<td>15065</td>
<td>(3x+1)/2</td>
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<td></td>
<td></td>
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<td></td>
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<td></td>
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</tr>
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<td>(x/2)</td>
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<td>(x-1)/4</td>
<td></td>
<td></td>
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</tr>
<tr>
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<td>72</td>
<td>(x-1)/4</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Continuation of above chart to show same patterns...

<table>
<thead>
<tr>
<th>2367</th>
<th>2559</th>
<th>2751</th>
<th>2943</th>
<th>3135</th>
<th>3327</th>
<th>3519</th>
<th>3711</th>
<th>3903</th>
<th>4095</th>
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<th>(+192 )</th>
</tr>
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<tbody>
<tr>
<td>3551</td>
<td>4127</td>
<td>4703</td>
<td>5279</td>
<td>5855</td>
<td>6431</td>
<td>(3x+1)/2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
This is getting somewhat involved. I hope you can appreciate that if you end up with unprovables simply pass them through \(\frac{x-1}{4}\) and \(\frac{x}{2}\) as many times as needed to reduce to odd and if the number is still larger than the start number apply \(\frac{3x+1}{2}\) however many times to get it reducable once again using \(\frac{x-1}{4}\) and \(\frac{x}{2}\). As seen in the above detailed work with level 6 the exact same trends hold in this level. I didn't go into as great detail; just enough to show this was the case. It is. So 75\% are easily provable with 93.75\% of the remaining quarter also easily provable, and so on...with 98.4375\% of that remaining 1/16th also provable...

I spoke about this aspect in an upper section where I believed that if you apply \(\frac{3x+1}{2}\) three times in a row you make it possible to extract \(\frac{x}{2}\) and/or \(\frac{x-1}{4}\) a number of times. At that time I wasn't clear how it worked in Collatz...but now it is becoming very clear. You can see it is a little involved but the basic premis is there.

After having done all the work above I made a discovery that really simplifies proving all multiples of 3, whether they be even or odd. I think you're going to enjoy this piece since it is so obvious after having done all the other research. I'm going to start by putting together several charts I mastered last night:

<table>
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<th>Multiple</th>
<th>(\frac{3x}{2})</th>
<th>(\frac{x-1}{4})</th>
<th>(\frac{x}{2})</th>
</tr>
</thead>
<tbody>
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<td>5</td>
<td>(\frac{1}{2})</td>
<td>(\frac{1}{4})</td>
</tr>
<tr>
<td>6</td>
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<td>(\frac{1}{4})</td>
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<td>(\frac{1}{4})</td>
</tr>
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<tr>
<td>33</td>
<td>42</td>
<td>(\frac{1}{2})</td>
<td>(\frac{1}{4})</td>
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<td>(\frac{1}{2})</td>
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</tr>
<tr>
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<td>(\frac{1}{2})</td>
<td>(\frac{1}{4})</td>
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<td>68</td>
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<td>(\frac{1}{4})</td>
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<td>77</td>
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<td>(\frac{1}{4})</td>
</tr>
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<td>51</td>
<td>80</td>
<td>(\frac{1}{2})</td>
<td>(\frac{1}{4})</td>
</tr>
<tr>
<td>54</td>
<td>83</td>
<td>(\frac{1}{2})</td>
<td>(\frac{1}{4})</td>
</tr>
</tbody>
</table>
57 → \((3 \times 57) + 1\)/2 = 86; 86/2 = 43
60 → 60/2 = 30
63
66 → 66/2 = 33
69 → \((3 \times 69) + 1\)/2 = 104; 104/2 = 52
72 → 72/2 = 36
75 → \((3 \times 75) + 1\)/2 = 113; \((3 \times 113) + 1\)/2 = 170; 170/2 = 85; (85-1)/3 = 28
78 → 78/2 = 39
81 → \((3 \times 81) + 1\)/2 = 122; 122/2 = 61
84 → 84/2 = 42
87
90 → 90/2 = 45
93 → \((3 \times 93) + 1\)/2 = 140; 140/2 = 70
96 → 96/2 = 48
99 → \((3 \times 99) + 1\)/2 = 149; \((3 \times 149) + 1\)/2 = 224; 224/2 = 112; (112-1)/3 = 37
102 → 102/2 = 51
105 → \((3 \times 105) + 1\)/2 = 158; 158/2 = 79
108 → 108/2 = 54
111
114 → 114/2 = 57
117 → \((3 \times 117) + 1\)/2 = 176; 176/2 = 88

Starting another chart with the left overs from above chart:

15 → \((3 \times 15) + 1\)/2 = 23; \((3 \times 23) + 1\)/2 = 35; \((3 \times 35) + 1\)/2 = 53; \((3 \times 53) + 1\)/2 = 80; 80/2 = 40; (40-1)/3 = 13
39 → \((3 \times 39) + 1\)/2 = 59; 89; 134; 134/2 = 67; (67-1)/3 = 22 (2nd column chart 1)
63 →
87 → \((3 \times 87) + 1\)/2 = 131; 197; 296; 296/2 = 148; (148-1)/3 = 49
111 → \((3 \times 111) + 1\)/2 = 167; 251; 377; 566; 566/2 = 283; (283-1)/3 = 94
135 → \((3 \times 135) + 1\)/2 = 203; 305; 458; 458/2 = 229; (229-1)/3 = 76
159 →
183 → \((3 \times 183) + 1\)/2 = 275; 413; 620; 620/2 = 310; (310-1)/3 = 103
207 → \((3 \times 207) + 1\)/2 = 311; 467; 701; 1052; 1052/2 = 526; (526-1)/3 = 175
231 → \((3 \times 231) + 1\)/2 = 347; 521; 782; 782/2 = 391; (391-1)/3 = 130
255 →
279 → \((3 \times 279) + 1\)/2 = 419; 629; 944; 944/2 = 472; (472-1)/3 = 157
303 → \((3 \times 303) + 1\)/2 = 455; 683; 1025; 1538; 1538/2 = 769; (769-1)/3 = 256
327 → \((3 \times 327) + 1\)/2 = 491; 737; 1106; 1106/2 = 553; (553-1)/3 = 184
351 →
375 → \((3 \times 375) + 1\)/2 = 563; 845; 1268; 1268/2 = 634; (634-1)/3 = 211
399 → \((3 \times 399) + 1\)/2 = 599; 899; 1349; 2024; 2024/2 = 1012; (1012-1)/3 = 337
423 → \((3 \times 423) + 1\)/2 = 635; 953; 1430; 1430/2 = 715; (715-1)/3 = 238
447 →
471 → \((3 \times 471) + 1\)/2 = 707; 1061; 1592; 1592/2 = 796; (796-1)/3 = 265
495 → \((3 \times 495) + 1\)/2 = 743; 1115; 1673; 2510; 2510/2 = 1255; (1255-1)/3 = 418
519 → \((3 \times 519) + 1\)/2 = 779; 1169; 1754; 1754/2 = 877; (877-1)/3 = 292
543 →

As you can see from the above chart reflected in the two charts below; the next sequence in each will immediately reduce through existing columns. And all the remaining upper sequences do the exact same thing.
so I will not bore you with more charting. The two following charts puts it in a compact easy to understand package.

From the above chart you can see the multiples of 3 form two distinct charts below. I've simplified that in the quick table right below:

<table>
<thead>
<tr>
<th>Column</th>
<th>Sequence</th>
<th>Starting</th>
<th>Sequence</th>
<th>Starting</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-1</td>
<td>3*1=3</td>
<td>2nd</td>
<td>0</td>
<td>1st</td>
</tr>
<tr>
<td>1-1</td>
<td>3*4=12</td>
<td></td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>2-2</td>
<td>3*8=24</td>
<td>9</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>1-2</td>
<td>3*16=48</td>
<td>1st</td>
<td>39</td>
<td>40</td>
</tr>
<tr>
<td>2-3</td>
<td>3*32=96</td>
<td>2nd</td>
<td>15</td>
<td>2nd</td>
</tr>
<tr>
<td>1-3</td>
<td>3*64=192</td>
<td>1st</td>
<td>159</td>
<td>160</td>
</tr>
<tr>
<td>2-4</td>
<td>3*128=384</td>
<td>2nd</td>
<td>63</td>
<td>2nd</td>
</tr>
<tr>
<td>1-4</td>
<td>3*256=768</td>
<td>1st</td>
<td>639</td>
<td>640</td>
</tr>
<tr>
<td>2-5</td>
<td>3*512=1536</td>
<td>2nd</td>
<td>255</td>
<td>2nd</td>
</tr>
<tr>
<td>1-5</td>
<td>3*1024=3072</td>
<td>1st</td>
<td>2559</td>
<td>2nd</td>
</tr>
</tbody>
</table>

It may not be obvious from the first sequence(s) in each chart but they have items/members that are automatically provable. In the case of the second chart the first two sequences fit that bill. I've highlighted one in red, one in green and the other in blue (above). You can also see that the even multiples of 3 are being accounted for in the first column of the second chart – that's because they are also easily proven by simply dividing by 2. Right? Just in case that doesn't work for you you'll find that all the even multiples of 3 are found in the second chart in the A column. The double lettered columns in each chart are simply the sequences (first half) with the last half being the result of multiple (3x+1)/2 and x/2 until final (x-1)/3 possible. For example take 3; ((3*3)+1)/2=5; ((3*5)+1)/2=8; 8/2=4; (4-1)/3=1. Using this feature you'll notice that there is a cascade of all multiples of 3 through to the first columns which are provable so they are all provable too. Right?

A quick explanation of the following charts in case you didn't immediately see it. The single letter columns are arranged so that each column up (to the right) is simply 3x+1

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H/I</th>
<th>J/K</th>
<th>L/M</th>
<th>N/O</th>
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<td>729</td>
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<td>48/27</td>
<td>192/243</td>
<td>768/2187</td>
</tr>
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<td>445</td>
<td>1336</td>
<td>4009</td>
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<td>87/49</td>
<td>351/445</td>
<td>1407/4009</td>
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</tr>
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<td>25</td>
<td>76</td>
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<td>688</td>
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<td>183/103</td>
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<td>999/562</td>
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<td></td>
</tr>
</tbody>
</table>

Collatz Conjecture Explored – Complete Proof
Note that duality plays an important function for even numbers that show up in the above charts and make this all possible. I should also mention that we can likely create another two sets of equations something like my original ones that may be useful in expanding upon this proof. Those new equations would look very similar to mine. I, without realizing it in previous work had stumbled across this without realizing it's full potential. We may even be able to make similar charts for the other two subsets... multiples of 3 minus 1; and multiples of 3 minus two and use those in a proof. I'll leave that up to the reader to explore. Actually, I've reconsider and I will come up with those charts below; because last evening after publishing a new version of this updated report I found there is in fact a chart much like the above two that really simplifies the entire proof.

Unfortunately we had to go through all this other work to come to that realization.

Let's do a visual to fix this idea in place so that you can easily agree with the concept as all encompassing for these multiples of 3.

<table>
<thead>
<tr>
<th>607 – 1214 – 2428 …</th>
</tr>
</thead>
<tbody>
<tr>
<td>202 ← 809 – 1618 …</td>
</tr>
</tbody>
</table>
I believe you see it clearly now. This is the case and concept for all multiples of 3. 159 quickly reduces to a much smaller number than the starting 159.

So, we were left with a subset of multiples of 3 we could not easily prove with other explored methods previously explored and the route I originally took became far too cumbersome to use for this proof. I left it there as a precursor to why I went this route. The above discussion exclusively dedicated to multiples of 3 shows that ALL are provable through simple induction because of the cascades through the two charts. The column headers have numbers immediately below them which indicate the separation of the sequence elements following in those columns. There are very nice patterns there. So having said that the remainder of outstanding multiples of 3 are previously proven if we consider the charts above and hence the proof is COMPLETE.

Here is my look and result of the other two subsets – multiples of 3 minus 1 and multiples of 3 – 2.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C/D</th>
<th>E/F</th>
<th>G/H</th>
<th>I/J</th>
<th>K/L</th>
<th>M/N</th>
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<td>11/13</td>
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</table>

Collatz Conjecture Explored – Complete Proof
The above chart is a combination of the 3s-1 and 3s-2 sequences. There are only 2 single letter columns compared to the other two charts that have an infinite number. The two columns however are generated in the same fashion with A being (B-1)/3. Just like what happened in the other two charts. The first double letter column is the sequence formed by 3s-2 (all of them in one giant sequence). The other double letter columns cover off the 3s-1 in an infinite set again. I arrived at these in much the same way as the multiple of 3s columns.

\[
\begin{align*}
D &= (C-1)/3 \\
F &= E/2 \\
H &= (((3*G)+1)/2)/2 \\
J &= ((3*I)+1)/2 \rightarrow (((3*prior)+1)/2)/2 \\
L &= ((3*K)+1)/2 \rightarrow ((3*prior)+1)/2 \rightarrow (((3*prior)+1)/2)/2 \\
N &= ((3*M)+1)/2 \rightarrow ((3*prior)+1)/2 \rightarrow ((3*prior)+1)/2 \rightarrow (((3*prior)+1)/2)/2 \\
&\ldots
\end{align*}
\]

Obviously this covers off all the elements in the union of 3s-1 and 3s-2. Study the chart and you easily see the pattern. Again there is a cascade going on. This like the above two charts for the 3s makes it easy to show by induction due to the cascades. As well, duality of even numbers plays into this. I'll do one as an example with small numbers so you can see what is happening:

\[
\begin{align*}
121 &\rightarrow 242 \rightarrow 484 \ldots \\
| &\quad | \\
40 &\leftarrow 161 \rightarrow 322 \ldots \\
| &\quad | \\
13 &\quad 107 \rightarrow 214 \ldots \\
| &\quad | \\
4 &\quad 71 \rightarrow 142 \ldots \\
| &\quad | \\
1 &\quad 47
\end{align*}
\]

So, with these final three charts and a deep understanding of where they came from one can easily through induction that PROVE that the Collatz Conjecture is completely true. So it can now be referred to as the Colatz Theorem!

Could this be the elusive proof for the remainder of the multiples of 3 I could not prove with the other above methods? The end result is once again induction where the end number is less than the starting number and thus in 1 to k. But as you saw, carrying the idea I used to study the multiples of 3 to the other two subsets of 3s-1 and 3s-2, I was able to come up with a third chart that made the proof much easier to understand. I'm sure you agree now that you've seen it in action.

I am not going to go any deeper with the above levels because the numbers are going to get scary large quickly. I just wanted to get the concept across. With each additional level we halve the number of elements remaining and achieve an amazing 100% provable. Actually we were 'approaching' 100% provable. I did not believe I could get any closer to proving this conjecture, but as you saw above, once I reconsidered the multiples of 3 in it's own subset the proof became obvious and 100% achievable.
It is not worth investigating here but I do wonder if what I last did to prove that small subset of multiples of 3 can also be used for any number as a more complicated way to a proof. I have a feeling it can. It may be worth investigating at some future date.

This all got me to thinking why my original equations did not display the same features of the 3 charts above? I did some more analysis and now that I clearly understand the mechanism, it is possible. I was pleased with myself for going back to check. Here is the resulting chart I formulated using my infinite set of equations:

<table>
<thead>
<tr>
<th>A</th>
<th>B/C</th>
<th>D/E</th>
<th>F/G</th>
<th>H/I</th>
<th>J/K</th>
<th>L/M</th>
<th>N/O</th>
<th>P/Q</th>
</tr>
</thead>
<tbody>
<tr>
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<td>4/3</td>
<td>8/3</td>
<td>16/9</td>
<td>32/27</td>
<td>64/81</td>
<td>128/243</td>
<td>256/729</td>
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<td>11/4</td>
<td>23/13</td>
<td>47/40</td>
<td>95/121</td>
<td>191/364</td>
<td>383/1093</td>
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</tr>
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<td>7/4</td>
<td>15/13</td>
<td>31/40</td>
<td>63/121</td>
<td>127/364</td>
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</tr>
<tr>
<td>4</td>
<td>7/4</td>
<td>15/13</td>
<td>31/40</td>
<td>63/121</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\{0+2x\} & \quad C=B/2 \\
\{1+4x\} & \quad E=((3*D)+1)/2 \to \text{prior}/2 \\
\{3+8x\} & \quad G=\left((3*F+1)/2 \to ((3*\text{prior}+1)/2 \to \text{prior}/2 \to (\text{prior}-1)/3ight) \\
\{7+16x\} & \quad I=\left((3*H+1)/2 \to ((3*\text{prior}+1)/2 \to ((3*\text{prior}+1)/2 \to \text{prior}/2 \to (\text{prior}-1)/3ight) \\
\{15+32x\} & \quad K=\left((3*J+1)/2 \to 3 \text{ iterations} ((3*\text{prior}+1)/2 \to \text{prior}/2 \to (\text{prior}-1)/3ight) \\
\{31+64x\} & \quad M=\left((3*L+1)/2 \to 4 \text{ iterations} ((3*\text{prior}+1)/2 \to \text{prior}/2 \to (\text{prior}-1)/3ight) \\
\{63+128x\} & \quad O=\left((3*N+1)/2 \to 5 \text{ iterations} ((3*\text{prior}+1)/2 \to \text{prior}/2 \to (\text{prior}-1)/3ight) \\
\{127+256x\} & \quad Q=\left((3*P+1)/2 \to 6 \text{ iterations} ((3*\text{prior}+1)/2 \to \text{prior}/2 \to (\text{prior}-1)/3ight) \\
\{255+512x\} & \quad S=\left((3*R+1)/2 \to 7 \text{ iterations} ((3*\text{prior}+1)/2 \to \text{prior}/2 \to (\text{prior}-1)/3ight) \\
\end{align*}
\]
Note the separation numbers under the letter headers; there is a connection through each... '*2' and '*3'.

Also take note that like the other charts the second number in the dual letter headers when \((x-1)/3\) gives the result in the prior column on the exact same row...and this feature cascades all the way back to the second and third columns (D/E & F/G) which have identical end parts...with those two columns again \((x-1)/3\) to give the second entry in the very first column B/C. The reason D/E and F/G have the identical second numbers has to do with the fact that I didn't do that part for D/E in the equation above. It was not required because D/E is already provable after doing the two steps; the third step \((x-1)/3\) is not required. It is also a fact that all those entities in D/E are subject to \((x-1)/4\), right? So they are provable through two distinct paths...either cascade through using \((x-1)/3\) or do the cascade using \((x-1)/4\). This is the only column that can do that.

The chart explains the process in a tiny package for sure. You can see the cascade that occurs. I added the column A to show that each number is consumed in the Collatz tree structure. Because of duality of even numbers they appear many times in the chart but that is unimportant because these duality entries are invisible in the tree...they occur under the sheets.

Here is an example to show the workings:

5467 – 10934 – 21868 …
   | 1822  7289 – 14578 …
   | 609   4859 – 9718 …
   | 202   3239 – 6478 …
   | 67    2159 – 4318 …
   | 22    1439 – 2878 …
   | 7     959 – 1918 …
   | 2     639

How about an example that does not start with multiple of 3:

3280 – 6560 – 13120 …
   | 1093  4373 – 8746 …
   | 364   2915 – 5830 …
   | 121   1943 – 3886 …
   | 40    1295 – 2590 …
   | 13    863 – 1726 …
   | 4     575 – 1150 …
   | 1     383
Do they look familiar? It is hard to believe that the entire proof boils down to this one chart.

It may be worth mentioning here, that it is now obvious from my 'final' chart that there is only the one trivial loop \( \{1 - 4 - 2\} \). Why? Because all the natural counting numbers are accounted for and they all reduce to that one trivial loop. Right? Column D/E has the only occurrence of a number retruning back to itself...specifically 1/1. A little further in this discussion I'm going to show the three possible loops in the \( \{3n-1; n/2\} \) sequence with positive natural counting numbers.

It would be interesting to see what my chart(s) would look like when we pass the negative natural numbers through the Collatz function. I suspect there would be three indiviudal charts – one for each of the three loops. This is another of those fun after the fact investigations that are not required for this proof. Instead of passing the negative natural counting numbers through the Colatz function, I will instead pass the positive natural counting numbers through the \( \{3n-1; n/2\} \) function since it will give the same three loops.

<table>
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<th>B/C</th>
<th>D/E</th>
<th>D/F</th>
<th>D/G</th>
<th>D/H</th>
<th>D/I</th>
<th>D/J</th>
<th>D/K</th>
<th>L/M</th>
<th>N/O</th>
<th>P/Q</th>
<th>R/S</th>
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<tr>
<td>40</td>
<td>80/40</td>
<td>157/-</td>
<td>157/21</td>
<td></td>
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<tr>
<td>41</td>
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<td>161/-</td>
<td>161/23</td>
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<tr>
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<td>84/42</td>
<td>165/-</td>
<td>165/22</td>
<td></td>
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<tr>
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<tr>
<td>44</td>
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<td>173/23</td>
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<tr>
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<td>181/24</td>
<td></td>
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<tr>
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<tr>
<td>49</td>
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<td>193/14</td>
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<td>197/26</td>
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</table>

\[
\begin{align*}
\{0+2x\} & \quad C=B/2 \\
\{1+4x\} & \quad E=((3*D)-1)/2 \\
\{1+4x\} & \quad F=((3*D)-1)/2 \rightarrow ((3*prior)-1)/4 \rightarrow 1 \text{ iterations } \{\text{prior}+1\}/3 \\
\{1+4x\} & \quad G=((3*D)-1)/2 \rightarrow 1 \text{ iteration } \{(3*prior)-1\}/2 \rightarrow ((3*prior)-1)/4 \rightarrow 1 \{(prior+1)/3\} \\
\{1+4x\} & \quad H=((3*D)-1)/2 \rightarrow 2 \{(3*prior)-1\}/2 \rightarrow ((3*prior)-1)/4 \rightarrow 2 \{(prior+1)/3\} \\
\{1+4x\} & \quad I=((3*D)-1)/2 \rightarrow 3 \{(3*prior)-1\}/2 \rightarrow ((3*prior)-1)/4 \rightarrow 3 \{(prior+1)/3\} \\
\{1+4x\} & \quad J=((3*D)-1)/2 \rightarrow 4 \{(3*prior)-1\}/2 \rightarrow ((3*prior)-1)/4 \rightarrow 4 \{(prior+1)/3\} \\
\{1+4x\} & \quad K=((3*D)-1)/2 \rightarrow 5 \{(3*prior)-1\}/2 \rightarrow ((3*prior)-1)/4 \rightarrow 5 \{(prior+1)/3\} \\
\{3+8x\} & \quad M=((3*L)-1)/4 \\
\{7+16x\} & \quad O=((3*N)-1)/4 \\
\{15+32x\} & \quad Q=((3*P)-1)/4 \\
\{31+64x\} & \quad S=((3*R)-1)/4 \\
\{63+128x\} & \quad U=((3*T)-1)/4 \\
\{127+256x\} & \\
\end{align*}
\]

So we do not get three separate charts but instead one super chart with different features. The double letter columns are truncated shortly after the first 10 elements since the patterns are continuous beyond there. I've given the first 5 of those infinite possible columns; again because the patterns hold true for all those further out. The pattern is easy to see. What is interesting is the D/? columns; that one column breaks down into an infinite number of sub-columns that have half the number of elements as prior sub-column. I've shown the first 7 of them. D/? continues on well past D/K. Notice the only element in D/E is 1/1 initial loop. In column D/F the first element is 5/2 but that is misleading because it only indicates overall chart patterns to show all numbers are reducable and that there are no runaways. If we were to run it through the \{3n-1; n/2\} sequence we would find that it is actually 5/5. Ohhh! Another loop... Doing the same at 17/5 will give 17/17 yet another loop... the third and final loop. We know there are only the three loops because of earlier discussions on their origin. See the snippet below:

\[
\begin{align*}
A & \quad B/C \quad D/E \quad D/F \quad D/G \quad D/H \quad D/I \\
2/1 & \quad 4/- \quad 8/1 \quad 16/9 \quad 32/9 \quad 64/9 \\
1 & \quad 2/1 \quad 1/1 \\
2 & \quad 4/2 \quad 5/- \quad 5/5 \\
3 & \quad 6/3 \quad 9/- \quad 9/5 \\
4 & \quad 8/4 \quad 13/- \quad 13/3 \\
5 & \quad 10/5 \quad 17/- \quad 17/17 \\
\end{align*}
\]
<table>
<thead>
<tr>
<th></th>
<th>12/6</th>
<th>21/-</th>
<th>21/4</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>14/7</td>
<td>25/-</td>
<td>25/14</td>
</tr>
<tr>
<td>8</td>
<td>16/8</td>
<td>29/-</td>
<td>29/5</td>
</tr>
<tr>
<td>9</td>
<td>18/9</td>
<td>33/-</td>
<td>33/5</td>
</tr>
<tr>
<td>10</td>
<td>20/10</td>
<td>37/-</td>
<td>37/6</td>
</tr>
</tbody>
</table>

Only 3 possible loops and they all originate in the D/? sub-columns. You will also note that 5/5 occurs 2* D/E (or 2*4=8)... and 17/17 occurs 4* D/F (or 4*8=32).

After doing my own analysis of the first 336 odd numbers from 1 to 671, I found the following distribution: 34.2% belong to 1/1 loop; 36.6% belong to 5/5 loop; and the final 34.2% belong to 17/17 loop. The more odd numbers I include the closer each one gets to 33.333%. In other words they approach 33.333% as the number of odds approach infinity. That does make sense in a convoluted way. The loops are stripped out or break away very early in the tree building process so all three of them growing in the same way would be roughly the same size as we approach infinite number of elements in each. That's like taking a sappling that has three branches and breaking off two to become their own sapplings; but remember one loop breaks off after a branch has limbed...

```
 a   b         c
 |    |    |      |
 |    |    |     /|
 |    |    |   / /
 |    |    | /  \
 |    |    |   \
 |    |    |
|    |    |
|    |    |
|    |    |
|    |    |
|    |    |
|    |    |
```

We can strip off 'a' very early on to mimic 5/5 loop giving:

```
 a
 |    |
 |    |
 |    |
 |    |
|    |
|    |
```

And strip off 17/17 loop on another limb like so:

```
 c
 |    |
 |    |
|    |
|    |
```

This leaves:
Note that the two mutant chopped off branches can't grow any longer. The new 'a', 'b' and 'c' will continue to grow independently into their own respective trees. This is the process so that you can imagine how 33.33% becomes the magic division of members among the 3 trees (3 separate loops). Since this process began very early; as the number of members approach infinity the total number of elements in each of the 3 trees approach 33.33% of the total.

From the above discussion in earlier sections it is obvious that there is an identical charts for the \{ 3x-1; x/2 \} sequence using negative natural counting numbers...but it has negative numbers and a chart for positive counting numbers. Right?

I am convinced that similar equations and charts can be created to prove other similar sequences. Since that is not the point of this paper, I will pass that over and leave up to the reader to try it out for themselves.

I believe this is the 'best' part of what is required for a proof! Now to put it in a more 'proofy' format. Or can this be considered the proof?

Now, I wonder if this can tell us anything about the length of the path? Or the maximum number reached in the chain? Let's see what happens with the infamous 27:

<table>
<thead>
<tr>
<th>31 – 62 – 124 …</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 – 41 – 82 …</td>
</tr>
<tr>
<td>3 – 27</td>
</tr>
</tbody>
</table>

I don't see any hints on how to gauge the path length...do you? I wonder if we were to extend out 10 by repeatedly applying 3x+1... then let's do the same for 3...

3 – 10 – 31 - … continues on the same path as above; which makes perfect sense.

What if we extends them out with 4x+1?

10 – 41 – 165 – 661 – 2645 – 10581

The max value for 27 is 9232 and it has 111 steps in it's path. If we take 9232 and repeatedly divide by 2... 9232/2= 4616/2= 2308/2 = 1154/2 = 577. Now we notice that 577 can cascade through (x-1)/3... (577-1)/3 = 192 and now can once again divide by 2 over and over... 192/2= 96/2 = 48/2 = 24/2 = 12/2 = 6/2 = 3. We also
notice that instead of \((x-1)/3\) on 577...(x-1)/4 also works giving 144 which is divisible by 2 over and over...
144/2= 72/2 =36/2 = 18/2 = 9. These don't seem to help.

**Section 13 – Simplifying Induction for ALL Odd number Nodes (ALL Odds)**

In all of my discussions above I failed to mention something I noticed early on in my investigations. I did not consider it important or useful in a proof. What I noticed is that the Collatz structure as I have drawn it has some odds less one showing up directly under that odd. For example consider...

\[
\begin{align*}
2 & \rightarrow 4 \rightarrow 8 \rightarrow 16 \rightarrow 32 \rightarrow 64 \rightarrow 128 \rightarrow 256... \\
& \quad | \quad | \quad | \quad | \quad | \quad | \quad | \quad | \\
& \quad | \quad | \quad | \quad | \quad | \quad | \\
& \quad \quad 85 \rightarrow 170... \\
& \quad | \quad | \quad | \quad | \quad | \quad | \\
& \quad \quad 21 \rightarrow 42 \rightarrow 84... \\
& \quad | \quad | \quad | \quad | \\
& \quad \quad 5 \rightarrow 10 \rightarrow 20 \rightarrow 40 \rightarrow 80... \\
& \quad | \quad | \quad | \quad | \\
& \quad \quad 1 \rightarrow 2 \rightarrow 4 \rightarrow 8...
\end{align*}
\]

Remember from above discussions 4x+1 and how it plays into the Collatz structure? Needless to say all odd numbers that play into the 4x+1 'game' are easily induced as true. Right? The resulting number is one less the starting odd! Two examples are highlighted above: 85 & 84; 21 & 20. But you will notice the very first number 5 does not fit that pattern... or does it? Let me redraw with some added intel from duality of even numbers; also discussed above:

\[
\begin{align*}
2 & \rightarrow 4 \rightarrow 8 \rightarrow 16 \rightarrow 32 \rightarrow 64 \rightarrow 128 \rightarrow 256... \\
& \quad | \quad | \quad | \quad | \quad | \quad | \quad | \quad | \\
& \quad | \quad | \quad | \quad | \quad | \quad | \\
& \quad \quad 85 \rightarrow 170... \\
& \quad | \quad | \quad | \quad | \quad | \quad | \\
& \quad \quad 21 \rightarrow 42 \rightarrow 84... \\
& \quad | \quad | \quad | \quad | \\
& \quad \quad 5 \rightarrow 10 \rightarrow 20 \rightarrow 40 \rightarrow 80... \\
& \quad | \quad | \quad | \quad | \\
& \quad \quad 1 \rightarrow 2 \rightarrow 4 \rightarrow 8...
\end{align*}
\]

So, all the odd numbers that are part of a 4x+1 run can be proven in this way. Cool, eh? Now, you also likely realize that all odd number nodes display this feature. So all ODD numbers can be proven in this fashion?

Let's check out some not so obvious examples:

\[
\begin{align*}
5 & \rightarrow 10 \rightarrow 20 \rightarrow 40 \rightarrow 80... \\
& \quad | \quad | \quad | \quad | \\
& \quad \quad 13
\end{align*}
\]
So 3 is the first number in its 4x+1 chain and is also provable by using duality feature:

\[
\begin{align*}
4 & \rightarrow 8 \rightarrow 16 \rightarrow 32 \\
5 & \rightarrow 10 \rightarrow 20 \rightarrow 40 \rightarrow 80 \\
1 & \rightarrow 2 \\
3 & \rightarrow 6 \rightarrow 12 \rightarrow 24 \\
1 & \rightarrow 2
\end{align*}
\]

Likely you have noticed that 4 can be extended in this fashion to prove not only 5 but also 3. You will find this can be done for all such cases...these are the infamous chains described above.

Let's look at a longer chain to see if there is a similar pattern:

\[
\begin{align*}
121 & \rightarrow 242 \rightarrow 484 \rightarrow 968 \\
161 & \rightarrow 322 \\
40 & \rightarrow 80 \rightarrow 160 \rightarrow 320 \\
53 & \rightarrow 106 \rightarrow 212 \rightarrow 424 \\
35 & \rightarrow 70 \rightarrow 140 \rightarrow 280 \\
23 & \rightarrow 46 \rightarrow 92 \rightarrow 184 \\
15 & \rightarrow 30 \rightarrow 60 \rightarrow 120 \rightarrow 240 \\
1 & \rightarrow 2
\end{align*}
\]

There is an obvious pattern here; actually two of them...4x+1 and 2x+1. Bet you didn't see that at first? For any odd that is not part of a cascading chain we can easily find 4x+1 relationship. However, when you are looking at the odds in the chain, the first one follows 4x+1 but each odd in the rest of the cascade is 2x+1. In the above example 161 is the start of a cascade so we can find 40 which is 40*4+1=161. The next member is 107; but it is 53*2+1=107; 35*2+1=71; and so forth... or 2x+1.

You can readily see that 31 is the last in this chain so it can follow the 4x+1 to cover the 41, Right? (31-1)/3=10 (which is duality again)...10*4+1=41?

If you think about it a little deeper you'll realize that all odd numbers are part of a chain ranging in size from 1 step on up. The very first step will follow 4x+1 for all chains; the remainder of chains will use 2x+1.
Odd numbers that are multiples of 3 are what I call dead end rows that can't spawn further limbs...so proving that first member is all that is required and that can be done using either $4x+1$ or $2x+1$, Right?

This approach covers all odd numbers. So they are very simple to prove through induction because they are automatically one less and in the assumed 1 to K range of already assumed proven. Having established where specific loops arise and why; we have eliminated the possibility that any loop other than 1 – 4 – 2 is impossible no matter how far out one looks. We have shown that all x from 1 to infinity are included in the Collatz structure once; by use of my infinite set of equations. Proving an even number was extremely simple because dividing them by 2 automatically proves this case. Adding this quick approach to odd numbers completes the proof. Right?

I'm going to throw the following in here so that you can visualize the link and what led me to this approach. It starts by looking at my $1+4x$ (second equation of the infinite set) 1, 5, 9, 13, … Take any number from that set and continually apply $(x-1)/2^y$; that is subtract 1 then divide by some power of 2 (the biggest one you can find). Doing this step over and over will lead to '1' for all members of that set. This is the only equation where you can divide all members by a power of 2 after subtracting one (except 1 because it is already '1').
Look for the pattern...it is not so obvious at first. Suffice to say, all members of this equation will eventually go to 1 by applying \((x-1)/2^y\) continuously.

My other equations leading out toward infinity are interesting in that after applying one iteration of \((x-1)/2^y\) they will be the exact members of the previous equation.
The above chart shows my equations $3+8x; 7+16x$ and $15+32x$. The first column starting with 3 are the members of $3+8x$. The column starting with 7 is the $7+16x$ equation members. And finally, the column starting with 15 are the $15+32x$ members. This chart could be extended out to show each and every equation approaching infinity do the same thing. If you subtract 1 and divide by 2 for each member you end up with all the members in the previous equation. Right? You do not have to search for larger powers of 2. 2 is the largest in all these upper level equations. It makes perfect sense since that is the way the equations can cascade through one another.

You are likely asking why I did not include the evens equation; specifically $0+2x$. Let's see what happens with the very first equation ( $0+2x$ ):

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<th>(3-1)/2 = 1</th>
<th>7</th>
<th>(7-1)/2 = 3</th>
<th>15</th>
<th>(15-1)/2 = 7</th>
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<tbody>
<tr>
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<td>(11-1)/2 = 5</td>
<td>23</td>
<td>(23-1)/2 = 11</td>
<td>47</td>
<td>(47-1)/2 = 23</td>
</tr>
<tr>
<td>19</td>
<td>(19-1)/2 = 9</td>
<td>39</td>
<td>(39-1)/2 = 19</td>
<td>79</td>
<td>39</td>
</tr>
<tr>
<td>27</td>
<td>(27-1)/2 = 13</td>
<td>55</td>
<td>27</td>
<td>111</td>
<td>55</td>
</tr>
<tr>
<td>35</td>
<td>(35-1)/2 = 17</td>
<td>71</td>
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The above chart shows my equations $3+8x; 7+16x$ and $15+32x$. The first column starting with 3 are the members of $3+8x$. The column starting with 7 is the $7+16x$ equation members. And finally, the column starting with 15 are the $15+32x$ members. This chart could be extended out to show each and every equation approaching infinity do the same thing. If you subtract 1 and divide by 2 for each member you end up with all the members in the previous equation. Right? You do not have to search for larger powers of 2. 2 is the largest in all these upper level equations. It makes perfect sense since that is the way the equations can cascade through one another.

You are likely asking why I did not include the evens equation; specifically $0+2x$. Let's see what happens with the very first equation ( $0+2x$ ):
Because the even numbers depicted by \(0+2x\) equation do not need to have one subtracted they are automatically divisible by some power of 2...so we simply divide by the largest power of 2 possible and something very interesting happens. This new column contains the exact same members as the second column of my very first chart for \(1+4x\). The colouring gives it away. There is a repeating pattern there as well.

What has by now become obvious is that all the other equations end up in the \(1+4x\) equation after very little work. That's a very important equation. You can finally see why. All roads lead to 'it' and all it's members lead to '1'!

I'm not going to do the leg work to show this approach where there are 3 possible loops; suffice to say that there will be fundamental differences. If someone is interested they can expand upon my research. I do not require that aspect/investigation for the Collatz proof.

### Section 14 - Conclusion

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With at least two methods found for original proofs; I can safely conclude that the Collatz Conjecture is in fact the Collatz Theorem. Both of these methods make use of induction.

Using these methods I was able to prove not only the Collatz series \((3n+1; n/2)\) but the Anti-Collatz \((3n-1; n/2)\) with negative whole numbers. Both these series are in effect mirror images of one another (different directions – Positive versus negative) with the magnitude remaining constant.

Personally, I doubt if these approaches can be used to prove similar series since they are keyed to '3' and '2' and their interrelationship with one another. They are for Collatz-like series. Examples that are not Collatz-like include \((5n+1;n/2); (5n+1;n/4); (9n+1;n/8);\) and so on. It might be interesting to investigate \((9n+1;n/8)\) since it involves multiples of 3 and 2...but I'll leave that to the reader.

I see no way to tease out the length of a path on its way to 1 or how large that number will grow. It in all likelyhood has something to do with '2'; specifically powers of two. I've been unable to find a mathematical method to show this connection.

I am not a mathematician so my technical terminology leaves a lot to be desired. But I hope I have successfully made my case. I wrote this paper in a style that shows the readers my thinking process. It lead to an unusually long paper but well worth it. Maybe some day it can be used as a case study on how to approach a complex problem.

I believe it may be possible for others to simplify or even improve upon my concepts, but please do give me the due credit for my research.

It has been a joy working on this 'unsolvable' problem. It's not so unsolvable, after all!