The kilogram, inertial mass, gravitational mass and types of universal free fall

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When gravitational motion of test objects are studied, two distinct concepts of mass are invoked in principle, namely, inertial mass and (passive) gravitational mass. If inertial mass and gravitational mass are quantities, then according to the definition of "quantity" available in the standard literature of metrology, there need to be specific units corresponding to inertial mass and gravitational mass. The possibilities of such definitions and the associated obstacles of reason are discussed. A recent classification of "kilogram" as a unit of inertial mass, that depends on the realization of kilogram through atom count method involving the use of matter wave interferometers, is critically analyzed. The process of reasoning that leads to such a classification is misleading because the equivalence between inertial mass and gravitational mass is an implicit assumption, in the process, both at the macroscopic level (mass measurement of silicon sphere) and the microscopic level (mass measurement of silicon atom with atom wave interferometers) resulting in no a priori distinction between inertial mass and gravitational mass in the concerned literature. Further inquiry reveals a crucial, but hitherto unexplained, difference between the working principles of neutron wave interferometers and atom wave interferometers. In neutron wave interferometers, the universal free fall of neutrons is attributed to the equality of inertial mass and gravitational mass, whereas in atom wave interferometers the distinction between inertial mass and gravitational mass is not made in the first place and universal free fall of atoms is attributed to either mass difference of the different species of atoms or different energy states of the same species of atom. Such observations result in the recognition of different *types* of universal free fall that are studied in the literature which, however, has hitherto not been recognized assertively.

Keywords: The kilogram; metrology; inertial mass and gravitational mass; matter wave interferometers; universal free fall

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Contents

I. INTRODUCTION: INERTIAL MASS, GRAVITATIONAL MASS AND METROLOGY

"Mass" is categorized, at least, as "inertial mass" and "gravitational mass" while studying the motion of test objects under the effect of gravity[1–3, 5–9]. Inertial mass (m_i) appears in the theoretical definition of "force" provided by Newton's laws of motion and gravitational mass (m_g) appears in the Newton's law of gravitation that hypothesizes the gravitational force between two objects both considered as "point mass" in accord with the first axiom of geometry[68]). Thus, the concepts of "inertial mass" and "gravitational mass" are theoretically explicated through classical physics and the *equivalence between inertial mass and gravitational mass* are investigated through experiments involving macroscopic test objects[5, 7–11] which are hypothesized as "point mass" or "material point"[1–3, 68] (owing to which classical mechanics is also called point mechanics[12]).

However, after the advent of quantum mechanics towards the beginning of the twentieth century, the concepts of "inertial mass" and "gravitational mass" have also been extrapolated to the regime of quantum mechanics[15–17][1], where test objects are hypothesized as "wave like systems" as per Schroedinger's postulate[67] (explicitly pointed out from Hilbert's axiomatic point of view in ref.[69]), albeit ignoring any need of an explanation of whether "force" is a quantum mechanical observable or not, so that "inertia" can even be conceived of, in principle, in the language of quantum mechanics[74, 75]. Nevertheless, the equivalence between inertial mass and gravitational mass for the test objects are tested through neutron interferometry[13–25] and atom interferometry[27, 28, 34–36, 38–40, 43–46] (see ref.[47] for an exhaustive list of references), which may be given a benefit of doubt on the basis of Einsteinian realization of physics as a logical system of thought whose essence is justified by its intuitive operational implementation through experiments[92][2].

Now, irrespective of the categorizations as "classical" and "qautnum", as far as the experimental tests of the equivalence between inertial mass and gravitational mass are concerned, certain issues must be addressed regarding the foundations of metrology before we even start speaking about experimental tests of the equivalence between inertial mass and gravitational mass. This is simply because, it is in metrology and quantity calculus that, we actually debate and set out the rules about how to speak and write about experimental measurements i.e. the language of science is set forth for further use[49–61]. A simple but important question from the metrological perspective is that, are there different units corresponding to inertial mass and gravitational mass? Or, in a more concrete fashion, we may pose the following question:

Q: *Is kilogram a unit of inertial mass or gravitational mass?*

It is quite strange that such a simple question has remained unasked for centuries and even the most modern book Schlamminger 2018[66], that is focused on the 2019 redefinition of "kilogram", is devoid of this question. It is only recently that the question **Q** appeared on an academic platform Majhi 2021[71] and a prompt answer followed in Mana et al 2022[72] from the metrology community. To mention, the answer to the question **Q**, as provided in Mana et al 2022[72] is that, "kilogram", as defined in the new 2019 convention[63, 65], is the unit of inertial mass and it can be experimentally realized by counting the atoms of silicon (^{28}Si) sphere[3] through a reinterpretation of the atom count method discussed in Massa et al 2020[89] and Fuji et al 2016[90]. In this work, I intend to discuss why such a conclusion regarding "kilogram" in Mana et al 2022[72] is misleading and how a critical analysis of the underlying process of reasoning unravel some deeper issues concerning the working principles of matter wave interferometers and the notion of universal free fall.

I debrief the structure of this work. In section (II), I discuss the motivation and the significance of the question **Q** as far as metrology and quantity calculus, hence the foundations of physics, are concerned and how such discussion is relevant in association with conceptual explications of the equivalence between inertial mass and gravitational mass. In section (III), I analyze the process of reasoning that led the authors

[1] Also see Kajari et al 2010[26] for a comparatively recent discussion on the distinction between inertial mass and gravitational mass in quantum mechanics.

[2] Indeed, in Staudenmann et al 1980[16], such an Einsteinian tone reverberated in the authors' justification of assuming the correctness of the Hamiltonian, viz. $H = \frac{p^2}{2m_i} - G\frac{Mm_g}{r} - \vec{\omega} \cdot \vec{L}$, involving inertial mass (m_i) and gravitational mass (m_g) of the neutron, to explain the underlying theoretical framework that is necessary for the test of the equivalence between inertial mass and gravitational mass of a neutron through a neutron interferometer: "*we will assume it is also the correct quantum-mechanical Hamiltonian, and then see if the predictions based on it agree with the experiment.*".

[3] See Leistner et al 1986[96] for discussions regarding the choice of sphere over cube.

of Mana et al 2022[72] to conclude that "kilogram" is a unit of inertial mass. I provide some explicit review type analyses of the relevant literature cited in Mana et al 2022[72] to discuss why the equivalence between inertial mass and gravitational mass is an implicit assumption in the process of reasoning that has led Mana et al 2022[72] to such a conclusion regarding "kilogram". In the process, some deeper issues concerning the literature of matter wave interferometry gets unraveled. In section (IV), I discuss the working principles of matter wave interferometers to point out a crucial difference between the working principles of neutron wave interferometers and atom wave interferometers as far as the distinction between inertial mass and gravitational mass, which motivates the recognition of different types of universal free fall. In section (V), I discuss the different constructions of η parameter, only one of which is the Eotvos parameter, which are already there in the literature but not interpreted clearly. These different constructions of η parameter correspond to different types of universal free fall. In section (VI), I conclude with some remarks with an emphasis on the fact that this work is not solely intended for raising any particular objection against Mana et al 2022[72], but it is a general critique of the modern experimental endeavour to perform quantum tests of the equivalence between inertial mass and gravitational mass that germinates very much from a "quantum gravity" mindset[76–78]. Mana et al 2022[72] has been in the focus of discussion here because it is the single most article (to the best of my knowledge) which containing an actual discussion about the units of inertial mass and gravitational mass. The present work adds to a sequence of articles that question the current dogma of "quantum gravity"[68–70], to seek more modesty while such research investigations are concerned[91].

II. MOTIVATIONS: DEFINITION OF "QUANTITY", UNIT OF GRAVITATIONAL MASS

The motivation to ask the question **Q** is extremely simple and originates from the basics of metrology and quantity calculus[49, 51, 62–65]. According to the definition of quantity provided by the BIPM[62, 63, 65], quantity is defined in terms of unit and mass is considered as a kind of quantity which is defined in terms of some unit, generally called kilogram. And, if "inertial mass" and "gravitational mass" are types of mass, then there must be different types of unit corresponding to each of inertial mass and gravitational mass, which can be regarded as "unit of inertial mass" and "unit of gravitational mass". Then, the question **Q** follows immediately. Now, if we assert that kilogram is the unit of inertial mass, as argued in Mana et al 2022[72], then the following question arises:

R: *What is the unit of gravitational mass and how is it defined?*

Thus, an answer to the question **Q** gives rise to further basic queries that makes the situation untenable from the metrological perspective.

Let me make the issues more explicit as follows. Definition of **quantity** is given on p.18 of ref.[62] as follows:

"*property of a phenomenon, body, or substance, where the property has a magnitude that can be expressed as a number and a reference*"

which is accompanied by a following clarifying note that

"*A reference can be a measurement unit, a measurement procedure, a reference material, or a combination of such.* "

Then measurement unit or unit of measurement or *unit* is defined on p.26 as follows:

"*real scalar quantity, defined and adopted by convention, with which any other quantity of the same kind can be compared to express the ratio of the two quantities as a number.*"

Since "mass" is a kind of quantity it must be written as some number and a unit. According to the 2019 convention adopted by the BIPM[64, 65], the SI unit of mass , namely kilogram (kg), is defined in terms of the Planck constant h, the velocity of light in vacuum c and the Cesium frequency standard $\Delta\nu_{cs}$ as follows:

$$1 \text{ kg} \approx 1.475 \times 10^{40} \ \frac{h\Delta\nu_{cs}}{c^2}. \tag{1}$$

I have considered the value only up to three places after the decimal as the precision is not the matter of concern in the present discussion. Now, let me call the unit of inertial mass as "inertial kilogram (Ikg)" and the unit of gravitational mass as "gravitational kilogram (Gkg)". Then, according to the definition of quantity, we must be able to write for a given object

$$m_i = n_i \text{ Ikg}, \tag{2}$$
$$m_g = n_g \text{ Gkg}. \tag{3}$$

In ref.[72] it has been argued that kilogram, defined in (1), is a unit of inertial mass i.e.

$$1 \text{ Ikg} \approx 1.475 \times 10^{40} \; \frac{h\Delta\nu_{cs}}{c^2}. \tag{4}$$

Therefore, the question **R** arises i.e. what is the definition of “Gkg”? Unless the answer to this question is provided in metrology, the interpretation of the experimental tests of the equivalence between inertial mass and gravitational mass become susceptible to trivial obstacles of reason that concerns the very language in which we write physics; “gravitational mass” become an empty collection of words having no relevance in the metrological jargon.

One possibility to answer the question **R** is to consider G as a defining constant and define a unit of gravitational mass, say “Gkg”, as follows:

$$1 \text{ Gkg} \approx 2.133 \times 10^{-26} \; \frac{c^3}{G\Delta\nu_{cs}}. \tag{5}$$

Such a possibility is supported by the fact that G is considered to be a fundamental constant in theoretical physics, especially in any theoretical calculation involving Planck length $G\hbar/c^3$.

However, G is not in the list of defining constants and the main reason behind this exclusion is the poorly determined nature of G due to the association of high uncertainty in its measurement[73]. Therefore, a clear necessity arises to resolve a dilemma i.e. whether to include G in the list of defining constants to provide a definition of Gkg (the logical viewpoint), or to exclude G from the list of defining constants on account of its poor measurement accuracy (the operational viewpoint). Even if we are able to resolve this dilemma by some means, a query immediately arises that whether it is possible to independently experimentally measure the inertial mass and the gravitational mass of an object e.g. a chunk of gold; i.e. are there ways to experimentally realize the two independent relations (2) and (3) for the given chunk of gold? To be more precise, can we devise two independent experimental procedures that will yield the numbers n_i and n_g, as observed readings, for the chunk of gold? The significance of such an inquiry is manifested if I focus upon the following statement, regarding an experimental procedure, in Eotvos et al 1922[5]: “*Let us suspend two* 25 g *masses of different substances on both ends of a homogeneous beam of* 40 cm *length.*”, and ask the questions:

- Does the number “25” correspond to “$n_i = 25$” or “$n_g = 25$”?
- Is “g”, which is an abbreviation of “gram”, a unit of inertial mass or gravitational mass (i.e. related to Ikg or Gkg)?

Without the provision of answers to the above questions, it appears that at the operational level during mass measurement of the objects (which yield “25 g”), no distinction has been made between inertial mass and gravitational mass, and their respective units. In that case, the subject matter of Eotvos et al 1922[5] appears to be plagued with an inherent contradiction, if it is interpreted as an experimental test of the equivalence between inertial mass and gravitational mass which, however, is the standard modern understanding of Eotvos' work that became explicit from Roll et al 1964[7] – see Appendix (A) for further details.

To resolve such issues, we must have experimental realizations of Ikg and Gkg independently. Importantly, it has been claimed that Ikg can be experimentally realized by the atom count method, in Mana et al 2022[72]. Certainly, *for the claim to be true in its essence, such an experimental realization must not depend on the assumption of the equivalence between inertial mass and gravitational mass.* However, as I intend to explain here, the atom count method implicitly depends on the assumption of the equivalence between inertial mass and gravitational mass due to the reliance of the method on atom interferometers which work on the basis of such an assumption albeit implicitly.

Digression: Certainly one can avoid such obstacles of reason by pointing out that Einstein designated “mass” as “quality” rather than “quantity” on p.54 of Einstein 2010[3] while explaining the equivalence between inertial mass and gravitational mass: “*The same quality of a body manifests itself according to circumstances as “inertia” or as “weight” (lit. “heaviness”).*”. However, such an objection brings forth the logical debate on "quality"-"quantity" distinction[4] and, more importantly, renders the discussion of Majhi 2021[71] and Mana et al 2022[72] meaningless. In fact, if such an objection holds, then "quality" needs a definition in metrology alongside "quantity", which is not there till date[62, 63, 65]. Therefore, for the present discussion I have disregarded Einstein's designation of "mass" as "quality" and consider it as "quantity" in accord with the standard practice in metrology.

III. ANALYZING THE STEPS TO REALIZE "KILOGRAM" AS A UNIT OF INERTIAL MASS FROM MANA ET AL 2022[72]

The atom count method has been considered as a procedure to realize kilogram as a unit of mass since a long time e.g. see the review article Massa et al 2020[89] and the references therein. However, it is in Mana et al 2022[72], the authors have claimed that the atom count method is actually a realization of kilogram as a unit of *inertial* mass. In order for this claim to be valid, no step of reasoning in the principles of such an experimental procedure can depend on the assumption of the equivalence between inertial mass and gravitational mass of the atoms under consideration. However, actually it does. This is because the experimental procedure involve –

1. the study of free fall of atoms under gravity where the associated mass of the atoms should be interpreted as gravitational mass in principle,
2. the measurement of the mass of the surface layer on the silicon crystal sphere through a weighing process where the mass of the surface layer should be interpreted as gravitational mass in principle,

but it becomes evident from a literature survey that during the above steps of observational study, the usual practice is not to distinguish between inertial mass and gravitational mass i.e. the equivalence between inertial mass and gravitational mass is a hidden assumption, both for the working principle of the atom interferometer experiments and the measurement of the mass of the surface layer of the silicon crystal. Here, I shall focus on the particular section of the literature that has been cited in Mana et al 2022[72] which should be convincing enough for the present course of reasoning. The explication of the realization of kilogram as a unit of inertial mass, as has been provided in Mana et al 2022[72], is comprised of two steps, which I discuss one after the other.

A. The First Step

In section (2) of Mana et al 2022[72], named *Atom Count*, the arguments that lead to the experimental realization of kilogram as a unit of inertial mass come in two steps, of which, "*The first realisation step is recoiling ^{133}Cs or ^{87}Rb atoms atoms by photons in an atom interferometer to measure the ratios between their inertial masses and the Planck constant [..., ...]*", where the citations are that of Clade et al 2016[79] and Yu et al 2019[83]. I may note that "inertial mass" are the words used by the authors of Mana et al 2022[72] in contrast to the use of only the word "mass" by the authors of Clade et al 2016[79] and Yu et al 2019[83]. Here, I shall revisit Clade et al 2016[79] and Yu et al 2019[83] one by one to examine the steps of reasoning in these two references where the equivalence between inertial mass and gravitational mass of the atoms has been presumed, in principle.

1. Comments on Clade et al 2016[79] and other relevant references

In Clade et al 2016[79], there are two instances where the effect of gravity on the experimental set up has been addressed, through the following statements:

- "*As shown in figure 4, the synthesized frequency results from the mixing of a fixed frequency (6.834 GHz), a frequency ramp to compensate the fall of atoms in the gravity field (about 25 MHz s^{-1}) and the probe frequency.*"
- "*For each beam direction we accelerate the atoms alternatively upward and downward to get rid of the change in velocity due to the free fall of atoms in the gravity field*"

I may note that to explain the free fall of atoms in the gravity field, one needs to associate the concept of gravitational mass with the atom, in principle. Since the authors do not provide any clarification regarding this fact, I may conclude that the equivalence between inertial mass and gravitational mass has been assumed in Clade et al 2016[79]. To delve for further details, I note that Clade et al 2016[79] actually cites Bouchendira et al 2011[80] and Bouchendira et al 2013[81] for further details of the experimental protocol. Therefore, it is worth revisiting these two articles to look for any clarification regarding the distinction between inertial mass and gravitational mass of the atoms.

In Bouchendira et al 2011[80], the authors "*present a new measurement of the ratio h/m_{Rb} between the Planck constant and the mass of ^{87}Rb atom*" (and obtain a new value of the fine structure constant), albeit without providing any clarification of whether m_{Rb} is the inertial mass or the gravitational mass of the ^{87}Rb atom – the authors just write "mass", without any distinction as "inertial" or "gravitational". This clarification becomes necessary when gravity is involved in the process, which is indeed the case as the authors address the effect of gravity on the experimental set up through the following statement.

- "*To cancel the velocity variation gT due to the gravity g ($T = 19$ ms is the delay between the two pairs of $\pi/2$ pulses), the atoms are accelerated alternatively upward and downward and the difference between the results eliminate gT.*"

Certainly, here the mass of the atom that relates the gravitational force to gravitational acceleration g is the gravitational mass of the atom. Since the authors do not provide any clarification regarding this issue in Bouchendira et al 2011[80], then I may conclude that the equivalence between inertial mass and gravitational mass has been assumed in the process, in principle, so that it suffices to write "m_{Rb} is the mass of ^{87}Rb atom" without any further distinction.

In Bouchendira et al 2013[81] also, which is sort of a review of Bouchendira et al 2011[80], the authors do not distinguish between inertial mass and gravitational mass of the atoms, but address the effect of gravity on the experimental set up through the following statements:

- "*As shown in figure 3, the synthesized frequency results from a mixing of a fixed frequency (6.84 GHz), a frequency ramp to compensate the fall of atoms in the gravity field (25 kHz/s) and the probe frequency.*"
- "*Two spectra allow to get rid of the change in velocity due to the free fall of atoms in the gravity field. They are obtained by accelerating the atoms alternatively upward and downward. The difference between the results eliminate gT, where T is the spacing time between the two pairs of Raman $\pi/2$ pulses.*"

It is quiet obvious that to explain the fall of the atoms due to gravity, one must implicitly designate the mass of the atom as gravitational mass. Since the authors do not provide such clarification in Bouchendira et al 2013[81], then I may conclude that the equivalence between inertial mass and gravitational mass has been implicitly assumed, in principle.

2. *Comments on Yu et al 2019[83] and other relevant references*

In Yu et al 2019[83], the authors review Parker et al 2018[84], but with some added clarifications. In Yu et al 2019[83], the following statements appear regarding the effects of gravity on the experimental set up:

- "*The phase difference between the upper and the lower interferometer[...], measured through ellipse fitting techniques, can be written as[...]:*

$$\Phi_d = \Delta\phi_1 - \Delta\phi_2 = -2nT\omega_m + 16n(n+N)\omega_r T + \Phi_\gamma + \Phi_{splitting} + \Phi_0 + \Phi_{sys} \tag{6}$$

  .... Φ_γ *is the phase caused by the acceleration gradient, such as the gravity gradient;...*"
- "*Another main systematic source is the gravity gradient. The fundamental tools of our experiment are the simultaneous conjugate interferometers, which cancel the signal from gravity and vibrations.... Since the two interferometers have a vertical off-set relative to each other, however, the cancellation of gravity is not perfect. A residual that is proportional the the gravity gradient $\gamma = \partial g/\partial z$, where g is the acceleration of free fall and z the vertical coordinate, remains.*"

- "*Based on the phase caused by gravity gradient[...], if we set it to $\delta z = 2(n+N)v_r T/(3n)$, where $v_r = \hbar k/m_{At}$ is the recoil velocity, the sensitivity to the gravity gradient will vanish.*"

Here, I make the following observations. Since the free fall of the atoms under gravity is concerned, "m_{At}" should be interpreted as the gravitational mass of the concerned atoms, in principle. However, the authors do not clarify whether "m_{At}" is the inertial mass or the gravitational mass of the concerned atoms which freely fall in a gravitational field. Therefore, I may conclude that the authors assume the equivalence between the inertial mass and the gravitational mass of the atoms, in principle. In fact, the equivalence between the inertial mass and the gravitational mass of the concerned atom, in principle, is a hidden assumption that underlies the theoretical explanations of cancellation of gravity effects in the phase shifts of atom waves in modern atom interferometers discussed in Parker et al 2018[84], Yu's thesis 2018[85], Peter's thesis 1998[29], D'Amico et al 2017[39], Roura 2017[86] and Zhong et al 2020[87], because none of these references make a distinction between the inertial mass and the gravitational mass while discussing the free fall of the atoms under gravity. Shortly, I shall point out in a separate section how the assumption of the equivalence between inertial mass and gravitational mass is a part of the theoretical principle of the atom interferometers.

B. The Second Step

Now, after the first step of realization of kilogram as a unit of "inertial" mass in Mana et al 2022[72]: "*In the second step, the kilogram is realised by the atom counting. To determine the count in practice, a ^{28}Si monocrystal is shaped as a quasi-perfect ball; the number N_{Si} of atoms in it is obtained from the measurement of the ball volume V and the lattice parameter a_0 according to $8V/a_0^3$ where $a_0^3/8$ is the atom volume, and 8 is the number of atoms in the cubic unit cell.*

Making reference, for instance, to the m_e/h quotient, the measurement equation is

$$\frac{m_i(^{28}\mathrm{Si\ ball})}{h} = \frac{8V}{a_0^3}\frac{M(^{28}\mathrm{Si})}{M(e)}\frac{m_e}{h}, \tag{7}$$

where $m_i(^{28}\mathrm{Si\ ball})$ is the ball's inertial mass and $M(X)$ indicates the X's molar mass[...]. "

The citation "[...]", in the above excerpt, is that of Massa et al 2020[89]. Now, it is important to note that what has been called "inertial mass" and designated as "m_i" in the above equation from Mana et al 2022[72], actually has been called "mass" and designated as "m" in the respective equation in Massa et al 2020[89]. If a distinction between inertial mass and gravitational mass is made, then a detailed study of Massa et al 2020[89], and the concerned references therein, reveals that "m" should actually be interpreted as gravitational mass of the silicon crystal, as opposed to inertial mass Mana et al 2022[72], because of the underlying procedure for its measurement. However, the distinction between inertial mass and gravitational mass has not been made in Massa et al 2020[89], and in the concerned references, which justifies the use of only one concept of "mass" and consequently indicates that the equivalence between inertial mass and gravitational mass is a hidden assumption in Massa et al 2020[89] and the concerned references.

1. *Comments on Massa et al 2020[89] and other relevant references*

Let me be more explicit and quote the relevant excerpt from the section named *Realising the Kilogram by Counting Si Atoms* in Massa et al 2020[89]:

"*Therefore, the mass of a pure, enriched, and perfect* ^{28}Si *single-crystal can be determined by counting its atoms [...]. .... After measuring the lattice parameter and crystal volume V, the count is given by $N_{Si} = 8V/a^3$. Hence, for instance, the quotient of the crystal mass, m, and the Planck constant is*

$$\frac{m}{h} = \frac{8V}{a^3}\frac{M(^{28}\mathrm{Si})}{M(e)}\frac{m_e}{h}, \tag{8}$$

where $M(-)$ indicates the molar mass."

The citation "[...]", in the above excerpt, is that of Fuji et al 2016[90] where the realization of the newly defined kilogram through the x-ray crystal density(XRCD) method has been discussed. The principle of the XRCD method depends on the experimental determination of h/m_{Si} using silicon atom recoil measurement with atom interferometry. In such a procedure the free fall of the silicon atoms under gravity is considered which requires, in principle, "m_{Si}" to be interpreted as the gravitational mass of the silicon atom. Since such a distinction does not appear in Fuji et al 2016[90], therefore, the equivalence between inertial mass and gravitational mass for the silicon atoms is an underlying hidden assumption so that it suffices to interpret "m_{Si}" as the mass of the silicon atom. The relevant excerpt from section(2), named *Principle of the XRCD method*, of Fuji et al 2016[90] goes as follows:

"*Another relation between N_A and h can be derived from the ratio $h/m(X)$ for an atom X, which can be determined by atom recoil measurements using atom interferometry [...].*"

The citation "[...]", in the above excerpt, is that of Clade et al 2016[79], where, as I have pointed out previously, the equivalence between inertial mass and gravitational mass is a hidden assumption, in principle, because of the use of only one concept of "mass" to explain the free fall of the atoms under gravity. Now, the mass of the silicon crystal (m_{crystal}) is related to the mean mass of a silicon atom (m_{Si}) through a relationship as $m_{\text{Si}} = m_{\text{crystal}}/N$, where N is the number of atoms in the crystal. If m_{Si} is interpreted as gravitational mass of the atom, then m_{crystal} must also be interpreted as the gravitational mass of the silicon crystal. However, the distinction between inertial mass and gravitational mass has not been made in Fuji et al 2016[90]. Therefore, the equivalence between inertial mass and gravitational mass has been implicitly assumed in Fuji et al 2016[90]. This implicit assumption is manifest in a different explicit way through the experimental procedure by which m_{crystal} is directly estimated so as to take into account the mass of impurities that contribute as a surface layer on the crystal.

In practice, the mass of silicon crystal sphere (m_{sphere}) consists of the mass of the pure silicon core (m_{core}) and the mass of a surface layer of impurities (m_{SL}), so that we write $m_{\text{sphere}} = m_{\text{core}} + m_{\text{SL}}$. It is m_{core} that plays the role of m_{crystal} while relating to m_{Si}. The equivalence between inertial mass and gravitational mass has been assumed at a macroscopic level in the XRCD method in Fuji et al 2016[90] while estimating m_{SL}. To explain the argument, let me quote the relevant excerpt from section(2), named *Principle of the XRCD method*, of Fuji et al 2016[90] as follows:

"*After determination of the sphere mass, m_{sphere}, and the mass of the surface layers, m_{SL}, the core mass is obtained as*

$$m_{\text{core}} = m_{\text{sphere}} - m_{\text{SL}}, \tag{9}$$

and the mean mass of a silicon atom is obtained as

$$m(\text{Si}) = m_{\text{core}}/N = m_{\text{core}}a^3/(8V_{\text{core}})." \tag{10}$$

In the above equation, "$m(\text{Si})$" is the mass of the silicon atom which I designated as "m_{Si}". Here, m_{SL} contains the mass of the physically absorbed water(PWL) by the silicon sphere and it is measured through a weighing process. The relevant excerpt from section(7), named *Surface evaluation of silicon spheres*, of Fuji et al 2016[90] goes as follows:

"*The PWL mass can be measured by gravimetry. NMIJ determined the PWL mass of the two Si spheres by weighing them in nitrogen gas at a pressure of 1200 Pa, and in water vapour at a pressure of 1200 Pa [...].*"

The citation "[...]" is that of Azuma et al 2015[93]. Here, the PWL mass must be interpreted as gravitational mass in principle, if one wants to make a distinction between inertial mass and gravitational mass of an object or a substance. Since it is the gravitational mass of the PWL that contributes to m_{SL}, then m_{SL} must be interpreted as gravitational mass as well (otherwise, there needs to be discussions regarding whether inertial mass and gravitational mass can be added, an answer to which will give rise to a further questions regarding whether, instead of mass dimension, do we need to consider inertial mass dimension and gravitational mass dimension, etc.). On similar grounds, if m_{SL} is gravitational mass then, it being subtracted from m_{sphere}, m_{sphere} must also be interpreted as gravitational mass. Hence, m_{core} must be interpreted as gravitational mass. Consequently, $m(\text{Si})$ must also be interpreted as gravitational mass of the silicon atom (as opposed to inertial mass in Mana et al 2022[72]). However, no such specific declarations have been made in Fuji et al 2016[90]. Therefore, certainly the equivalence between inertial mass and gravitational mass has been assumed, in principle, so that only the word "mass" can be used accordingly in Fuji et al 2016[90] and in Azuma et al 2015[93] as well (in this context it is also worth consulting Andreas et al 2011[94] and Bartl et al 2017[95]).

IV. ANALYZING THE THEORETICAL PRINCIPLES BEHIND THE MATTER WAVE INTERFEROMETERS

In this section I shall now point out exactly how the equivalence between inertial mass and gravitational is an assumption in the theoretical principles which form the basis of atom wave interferometers. The relevant literature cited in Mana et al 2022[72] that concerns atom wave interferometry is Yu et al 2019[83]. However, the phase calculation of the atom waves using the Hamiltonian, the Lagrangian and the path integral formulation can be traced back from Yu et al 2019[85] to Yu 2018[85] to Peters [29] to the detailed exposition in the tutorial article Storey et al 1992[97] which was preceded by the relevant experimental article Kasevich et al 1992[98] that also contained explanation of the principles. Interestingly Kasevich et al 1992[98] refers to the Collela et al 1975[14] to point out that gravitational effects have been experimentally demonstrated through neutron wave interferometers and that the atom wave interferometers hold an advantage over the neutron wave interferometers due to the greater mass and slower velocities of the atoms. However, none of these make a distinction between inertial mass and gravitational mass of the particles under study. This distinction was made, in the regime of quantum mechanics, for the first time in literature in Staudenmann et al 1980[16]. The theoretical principles, based on which the experimental tests of the equivalence between inertial mass and gravitational mass of a neutron were clearly and elaborately explained for the first time in the literature in Staudenmann et al 1980[16] and now it is available in an excellent textbook concerning neutron wave interferometers Rauch et al 2015[25].

A. Inertial mass/gravitational mass distinction in neutron wave interferometers

Now, the distinction between inertial mass and gravitational mass is generally explained by writing the classical laws "$F = m_i a, F \propto m_g M/r^2$", which do not apply directly to the quantum mechanical regime where we deal with the Hamiltonian operators, path integral formulations based on a Lagrangian, etc. So, the relevant question is the following:

How is the distinction between inertial mass and the gravitational mass made in terms of equations so that one can construct a theoretical working principle?

The answer was provided by Staudenmann et al 1980[16] through a proposition that could only be justified by its use. The description of the principles started with a proposed Hamiltonian, that manifested an explicit distinction between inertial mass and gravitational mass, in Staudenmann et al 1980[16]:

"*Classically, the Hamiltonian governing the neutron's motion in the gravitational field of the rotating Earth is [citing Landau and Lifshitz[100]]*

$$H = \frac{p^2}{2m_i} - G\frac{Mm_g}{r} - \vec{\omega} \cdot \vec{L}. \tag{11}$$

Here the angular momentum of the neutron's motion about the center of the Earth ($\vec{r} = \vec{0}$),

$$\vec{L} = \vec{r} \times \vec{p}; \tag{12}$$

$\vec{p}$ is the canonical momentum of the neutron, ω is the angular rotation velocity of the earth, M the mass of the Earth, m_i the inertial mass of the neutron, and m_g the gravitational mass of the neutron. From an epistemological point of view it is not possible to be confident that this Hamiltonian correctly describes quantum-mechanical phenomena, especially those involving interference. However, for lack of evidence to the contrary, we will assume it is also the correct quantum-mechanical Hamiltonian, and then see if the predictions based on it agree with the experiment. The principle of equivalence would require that the inertial mass m_i and the gravitational mass m_g in Eq.(11) are equal.

Since the distances involved within the nuetron interferometer are very small compared to the radius R of the Earth, we can write (11) as

$$H = \frac{p^2}{2m_i} + m_g\vec{g}_0 \cdot \vec{r} - \vec{\omega} \cdot \vec{L} + V_0, \tag{13}$$

where V_0 is the gravitational potential energy at some reference height above the Earth (say, the center of the interferometer), and $\vec{g}_0$ is the acceleration due to gravity. "

The reason behind calling the Hamiltonian in eq.(11) "a proposition" is that, the cited book of Landau and Lifshitz[100] does not distinguish between inertial mass and gravitational mass. In other words, the authors of Staudenmann et al 1980[16] began with an assumption that inertial mass and gravitational mass are not a priori equivalent and implemented this assumption through the proposition of the Hamiltonian in eq.(11). This assumption of inequivalence between inertial mass and gravitational mass is mandatory because it is the equivalence between inertial mass and gravitational mass that is under examination. Without an assumption of the inequivalence, the work becomes tautologous i.e. if there is no theoretical principle with an a priori distinction between inertial mass and gravitational mass to begin with, then there is no question of an experimental verification of the equivalence between inertial mass and gravitational mass.

B. Equivalence between inertial mass and gravitational mass in atom wave interferometers

In light of the above discussion concerning neutron wave interferometers, now let me focus on Yu 2018[85] (also Yu et al 2019[83]) which is the relevant citation of Mana et al 2022[72]. The free particle Hamiltonian operator, in the presence of gravity, has been written in Yu 2018[85] as follows:

$$\hat{H} = \frac{\hat{p}^2}{2m} + mg\hat{z}, \tag{14}$$

where m is the mass of the particle under consideration. The Lagrangian for a particle falling under a gravitational field with gravity gradient, has been written in Yu 2018[85] as follows:

$$L = \frac{1}{2}m\dot{z}^2 - mgz - \frac{1}{2}mz^2\frac{dg}{dz}. \tag{15}$$

In case one wants to test the equivalence between inertial mass(m_i) and gravitational mass(m_g), through such atom wave interferometers, then the above mentioned Hamiltonian operator should have to be written as

$$\hat{H} = \frac{\hat{p}^2}{2m_i} + m_g g\hat{z}, \tag{16}$$

following the proposal of Staudenmann et al 1980[16] (where the concerned particle was neutron), and the above mentioned Lagrangian should have to be written as

$$L = \frac{1}{2}m_i\dot{z}^2 - m_g gz - \frac{1}{2}m_g z^2\frac{dg}{dz}. \tag{17}$$

Since such a distinction has not been made in Yu 2018[85] and Yu et al 2019[83], therefore, the equivalence between inertial mass and gravitational mass is an implicit assumption in the working principles of the atom wave interferometers involved in both of these references i.e. there is only one concept of mass $m = m_i = m_g$. Indeed, one can verify this fact by tracing back through the literature to Fray et al 2004[40] which I shall focus on in a forthcoming section.

As Yu 2018[85], Yu et al 2019[83] are concerned with the measurement of h/m of the concerned atoms, here "m" is just the mass of the atom – it is neither inertial mass nor gravitational mass of the atom. This clarifies how exactly the assumption of the equivalence between inertial mass and gravitational mass underlies the first step of the realization of kilogram in Mana et al 2022[72]. While this provides a concrete obstacle of reason that one must face while asserting "kilogram is a unit of inertial mass", some deeper issues concerning the tests of "universal free fall" are unraveled too, which however justifies the significance of the discussions of Mana et al 2022[72]. To be very specific, from a study of the concerned literature, it appears that there needs to be the recognition of different types of universal free fall that I discuss in the next section.

V. RECOGNIZING THE TYPES OF UNIVERSAL FREE FALL

The tests of *universal free fall* (UFF) of test objects under gravity of the earth are generally performed by estimating the value of the following parameter:

$$\eta(A,B) \;:=\; \frac{\text{difference in acceleration of two test objects } A \text{ and } B \text{ due to gravity of the earth } (\Delta g)}{\text{mean acceleration of the two test objects } A \text{ and } B \text{ due to gravity of the earth } (g_{\text{mean}})}$$

i.e.

$$\eta(A,B) \;:=\; \frac{\Delta g}{g_{\text{mean}}} \quad : \quad \Delta g := (g_A \sim g_B),\; g_{\text{mean}} := (g_A + g_B)/2, \tag{18}$$

where g_A and g_B are the accelerations of the test objects A and B, respectively, due to earth's gravity. Further, assuming that $\Delta g \lll g_i \forall i \in [A, B]$, one may also consider $g_A \simeq g \simeq g_B$ and consider the following:

$$\eta(A,B) = \frac{\Delta g}{g} \quad : \quad g \text{ is either } g_A \text{ or } g_B. \tag{19}$$

Now, it is quite obvious that measurement of $\eta(A, B)$ does not, by itself, justifies the outcome. That is, it only shows that whether there is a difference in acceleration of the objects A and B, but it does not justify why there is a difference. For such a justification, a theoretical interpretation is needed. As I shall explain here, there are different types of justifications available in the literature indicating different types of UFF, which has not yet been recognized till date (as far as my knowledge is concerned).

A. UFF-type 1: Distinction between inertial mass and gravitational mass - the Eotvos type (Classical)

Considering two different concepts of mass, i.e. inertial mass (m_i) and gravitational mass (m_g), associated with each of the objects A and B, and considering that the ratios $(m_g/m_i)_A$ and $(m_g/m_i)_B$ are dependent on the material nature of the respective objects, one can write (e.g. see Roll et al 1964[7])

$$\eta(A,B) = 2.\frac{\left(\frac{m_g}{m_i}\right)_A - \left(\frac{m_g}{m_i}\right)_B}{\left(\frac{m_g}{m_i}\right)_A + \left(\frac{m_g}{m_i}\right)_B}. \tag{20}$$

A null outcome of the η-measurement, which signifies UFF, is then interpreted as the equivalence between inertial mass and gravitational mass i.e. the ratio (m_g/m_i) is independent of the material nature of the concerned test object Eotvos et al 1922[5]. I may call this *UFF-type 1.* This is the scenario with Eotvos type experiments with torsion balance and macroscopic test objects of different material nature.

B. UFF-type 2: Different magnitudes of mass of different atoms (Quantum)

In case of UFF tests of different species of atoms with atom interferometers, the theory that is used to interpret η-measurement, is devoid of any distinction between inertial mass and gravitational mass – only one concept of mass is used for the atoms. Following the notable works that discuss UFF tests for atoms (see refs. [43–47]), one may trace back to Fray et al 2004[40] to find the underlying theory and the different interpretations of η-measurement[4]. To understand that no distinction is made between inertial mass and

[4] The theoretical works like that of Lammerzahl 1996[42] and Viola et al 1997[41] where "*Investigations of the equivalence principle on an atomic basis have been proposed ..., motivated by the quest to provide new tests of theories which merge quantum mechanics and relativity.*" inspired the experimental effort of Fray et al 2004[40]. It is worth noting that none of the above two references provide a metrological definition of inertial mass and gravitational mass, which is the prime requisite while motivating experiments through theoretical analysis. Rather, these two references manifest a concrete example of

gravitational mass, it is just enough to look at the Hamiltonian that has been used to begin with in Fray et al 2004[40] (also see Chebotayev et al 1985[48]):

"*The Hamiltonian of the system is* <u>*assumed*</u> *to be*

$$H = \frac{\vec{p}^{\,2}}{2m} + (\hbar\omega_{eg} - i\hbar\Gamma/2)|e\rangle\langle e| - \vec{r}\cdot\vec{E} \tag{21}$$

where $\hbar\omega_{eg}$ denotes the atomic energy spacing between levels $|g_D\rangle$ and $|e\rangle$."

The reader may look into the concerned references for the detailed explanation of all the terms that constitute the Hamiltonian (21). However, there should be no doubt that only one concept of mass, designated by m, has been associated with the atom while assuming the Hamiltonian (21). That is, if one has in mind 'distinction like m_i and m_g', then the assumption of the Hamiltonian (21) is based on the implicit presumption of 'NO distinction like m_i and m_g' such that there is only 'one concept m, devoid of any subscript'. Thus, the interpretation of η-measurement in such a scenario can not be UFF-type 1. In Fray et al 2004[40] there are two different definitions of η and the respective interpretations have been provided based on the Hamiltonian (21).

One definition is the following:

$$\eta(^{85}Rb, ^{87}Rb) = \frac{g_{^{85}Rb} - g_{^{87}Rb}}{g_{^{85}Rb}}, \tag{22}$$

where $g_{^{85}Rb}$ and $g_{^{87}Rb}$ are the acceleration of ^{85}Rb atoms and ^{87}Rb atoms, due to gravity of the earth, respectively. It may be noted that the mean acceleration, i.e. $(g_{^{85}Rb} + g_{^{87}Rb})/2$, has not been used in the denominator i.e. this is actually the definition (19) with test objects A and B equivalent to ^{85}Rb and ^{87}Rb atoms, respectively, and g is replaced by g_A or $g_{^{85}Rb}$. It is the mass of the atom m that is related to the de Broglie wavelength, the wave vector and, hence, the momentum of the atom. Therefore, it is the magnitude of m, which differs for ^{85}Rb and ^{87}Rb atoms, that affects the phase of the atom wave that is responsible for the interference. In this case, a null outcome of η-measurement can not be interpreted as UFF-type 1. It can only be attributed to the mass difference of the two isotopes, as far as the underlying theory is concerned. I may call this *UFF-type 2*.

C. UFF-type 3: Different energy states of the same atom (Quantum)

The other definition in Fray et al 2004[40] is the following:

$$\eta = \frac{g_{F=3} - g_{F=2}}{g_{F=3}}, \tag{23}$$

where $g_{F=3}$ and $g_{F=2}$ are the acceleration of ^{85}Rb atoms in different energy states designated by $F = 3$ and $F = 2$. Here, too, the mean acceleration, i.e. $(g_{F=3} + g_{F=2})/2$, has not been used in the denominator i.e. this is actually the definition (19) with test objects A and B equivalent to $^{85}Rb[F = 3]$ and $^{85}Rb[g_{F=2}]$ atoms, respectively, and g is replaced by g_A or $g_{F=3}$. Here, by "$^{85}Rb[F = 3]$" and "$^{85}Rb[g_{F=2}]$", I have designated ^{85}Rb atoms in the energy states $F = 3$ and $F = 2$ respectively. In this case, there is a comparison between the acceleration of a particular species of atom, viz. (^{85}Rb), having a particular mass m, but at different internal energy states, viz. designated by $F = 3$ and $F = 2$. So, the difference in acceleration can neither be attributed to inertial mass-gravitational mass distinction, nor be attributed to a difference in the magnitude of mass, but it has to be attributed to the difference between internal energy states of an atom. Thus, the null outcome of η-measurement can neither be interpreted as UFF-type 1, nor be interpreted as UFF-type 2. I may call this *UFF-type 3*.

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the absence of a general consensus regarding what "equivalence" between inertial mass and gravitational mass means – is it "proportionality" or "equality"? Lammerzahl 1996[42] considers "equality" as the authors write "*In the framework of Newtonian mechanics the weak EP amounts to the statement that inertial mass and gravitational mass are equal.*"; whereas, Viola et al 1997[41] considers "proportionality".

D. UFF-type 4: Classical vs Quantum – the case of neutrons

It is worth focusing on the UFF of neutrons, studied through neutron wave interferometric tests, because of the underlying theoretical interpretation is completely different the above discussed types of UFF. To analyze the difference, I shall quote a portion from Koester 1976[15]:

"*From the experimentally..confirmed principle of the universality of free fall it follows that the gravitational accelerations of the free neutron* (g_f) *and of bulk matter, the local value* (g_0)*, are equal. Thus, neutron experiments which result in a value for the equivalence factor* $\gamma = (m_i/m)(g_f/g_0)$ *provide a test for the equality of inertial* (m_i) *and passive gravitational mass* (m) *for the neutron. A verification of this equality with* $\gamma = 1$ *would confirm that the universality of free fall implies the Einstein equivalence principle since* $g_f/g_0 = m/m_i$*.* "

The differences are the following:

1. Instead of "proportionality", it is "equality" between inertial mass and gravitational mass that is considered as a signature of UFF.

2. There is no construction of any η-parameter.

3. Instead of comparison between accelerations within the regime of a single theory (i.e. either classical or quantum), there is a comparison between accelerations between two different theories i.e. g_f belongs to quantum theory and g_0 belongs to classical theory.

The first point provides a notion of "equivalence" (i.e. "equality") between inertial mass and gravitational mass that is only applicable for the literature concerning neutron wave interferometers and does not apply to any other part of the literature where "proportionality" between inertial mass and gravitational mass considered in accord with Eotvos et al 1922[5]. This issue regarding "proportionality" and "equality" has also been recently discussed in Yarman et al 2019[101] and the relevant references therein. The third point signifies a fourth type of UFF where an acceleration measurement based on quantum theory is compared with an acceleration measurement based on classical theory i.e. "universality" spans the regime of theories, namely, classical and quantum. I may call this *UFF-type 4*.

VI. CONCLUSION

Considering inertial mass and gravitational mass as distinct quantities[3, 6, 7], the units of inertial mass and gravitational mass need to be specified in accord with the definition of "quantity" provided in the metrology literature[62, 63, 65]. An obvious query arises within the tenets of currently accepted jargon of physics, that is, whether "kilogram" is a unit of inertial mass or gravitational mass. Strangely such a basic question has remained unasked until very recently it appeared on an academic platform[71]. An attempt to answer this question, promptly followed, in favour of "unit of inertial mass" in Mana et al 2022[72]. A careful and critical analysis of the underlying process of reasoning, which led to such a conclusion, in Mana et al 2022[72] and the concerned literature cited therein, reveal that the equivalence between inertial mass and gravitational mass has been implicitly assumed, both at the microscopic and the microscopic level, in the process of such a conclusion. Consequently, the distinction between inertial mass and gravitational mass, and their corresponding units, as argued in Mana et al 2022[72], becomes unreasonable if not unfounded. However, the significance of Mana et al 2022[72] lies in the fact that it is the sole attempt till date (as far as my knowledge is concerned) to discuss the units of inertial mass and gravitational mass, which however makes it the unique piece of literature that comes under scrutiny. Even more significant is that, such a scrutiny indeed brings forth certain deeper metrological issues concerning the working principles of matter wave interferometric tests of universal free fall (UFF) of quantum objects. of the equivalence between inertial mass and gravitaitonal mass. While UFF is associated with the equivalence between inertial mass and gravitational mass in neutron interferometers, in atom interferometers UFF is associated with the acceleration being independent of the magnitude of mass (without any distinction between inertial mass and gravitational mass). A thorough review of the literature, with specific focus on the construction of the η parameter whose null measurement signifies UFF, leads to the recognition of "types" of UFF. Since these observations have not been made, or have been overlooked, in the literature till date, the situation calls for a renewed focus on the foundations of gravitational physics which concerns the interface of experiment and theory in the modern era of the quantum. To be specific, in contrast to the theoretical sophistication practiced by theoretical physicists under the name of "quantum gravity" that motivates much of the modern experiments involving quantum tests of gravitational principles, now the concerned experimental physicists may advice for some modesty to the "quantum gravity" enthusiasts by posing some elementary questions from the metrological perspective for the benefit of science.

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Appendix A: A Critical Analysis of Eotvos et al 1922[5]

Here, I shall provide a critical analysis of certain relevant portions of Eotvos et al 1922[5] in light of the issues which I have raised from the metrological perspective. I shall quote the relevant portions from Eotvos et al 1922[5], but with suitably modified notations so as to make it familiar from the modern perspective.

"*The Newtonian law can be expressed as follows: Each smallest part of a body attracts every other with a force whose direction coincides with the connecting line of both parts, and where magnitude is proportional to the product of their masses and inversely proportional to their mutual distance. If M and m are two masses with r being their separation, then the magnitude of their mutual attraction is*

$$F = G\frac{Mm}{r^2}. \tag{A1}$$

According to the principles of the Galilean-Newtonian mechanics, the acceleration of the piece of mass m towards M is then

$$g = G\frac{M}{r^2}. \tag{A2}$$

Hence proportionality of inertia and gravity is equivalent to G being a constant (Newton's constant of gravitation). "

"*Above all we must here remember the arguments which Newton himself put forward in his Principles for the proportionality of inertia and gravity of different bodies. These are twofold: ...... Both kinds of measuring yield, according Newton, the result that although a difference of only* 1/1000 *in the gravitational force of attraction of different bodies of equal mass and equal position should be detected by such observations, the gravitational attraction seems to be independent of the material nature of the bodies.*"

If we consider that Eotvos et al 1922[5] tested the equivalence between inertial mass and gravitational mass, which is the standard understanding (e.g. see Roll et al 1964 [7]), then the distinction between inertial mass and gravitational mass has to be an a priori consideration, in principle. Therefore, it is mandatory to specify whether "different bodies of equal mass" mean that the different bodies are of "equal inertial mass" or "equal gravitational mass", along with a clarification of how such "equality of inertial mass of two different bodies" or "equality of gravitational mass of two different bodies" is determined experimentally. Without such declarations and clarifications it appears that while determining the equality of "mass" of the different bodies, no distinction has been made between inertial mass and gravitational mass of the different bodies, which is equivalent to assuming the equivalence between inertial mass and gravitational mass of the different bodies. So, the process of reasoning contains two conflicting truths – (i) distinction between inertial mass and gravitational mass is considered, a priori, in principle (ii) no distinction between inertial mass and gravitational mass is considered while asserting "equality of mass of two different bodies" during setting up of the experiment. This exposes an inherent contradiction in the procedure, and its interpretation, of Newton's experiment as explained by the authors of Eotvos et al 1922[5] and, therefore, of Eotvos' experiment explained in Eotvos et al 1922[5], if the statement "proportionality of inertia and gravity" is interpreted as "equivalence between inertial mass and gravitational mass". This doubt regarding such modern interpretation, which is rooted to Einstein's explanation on p.53 of ref.[3] that was used to define the η parameter in terms of inertial mass and gravitational mass in Roll et al 1964[7], because Eotvos interprets "proportionality of inertia and gravity of different bodies" in terms of "force", and not "mass", which becomes apparent from the emphasis on the force-aspect (not mass-aspect) of gravity in the phrase "the gravitational attraction seems to be independent of the material nature of the bodies". While I plan to elaborate on this issue in near future[102], I shall continue here to analysis a few more relevant portions of Eotvos et al 1922[5] to establish the obstacles of reason in the path of interpreting "proportionality of inertia and gravity" as "equivalence between inertial mass and gravitational mass".

"*In the spirit of this investigation, if we allow for the possibility that the attraction of bodies of equal mass but different nature is different, the quantities F and G .... are to be regarded as dependent on this nature of the bodies.*"

When the authors write "bodies of equal mass but different nature", it is not declared whether the bodies are of equal inertial mass or of equal gravitational mass. It appears from the statement that even if the distinction between inertial mass and gravitational mass is not made, resulting in only one concept of mass, then also the force of attraction can depend on the nature of the bodies. Which means, the material nature of the bodies is NOT characterized by the distinction between inertial mass and gravitational mass of a body. Or, in other words, the distinction between inertial mass and gravitational mass is not theoretically adequate, or wholly unnecessary, to account for the dependence of a body's gravitational acceleration on its material nature. The authors indeed write about F (force) being dependent on the nature of the bodies.

"*If the pendant of the torsion balance consists of masses of different materials m_1, m_2, m_3, etc., then, according to our considerations, the axis of rotation represented by the torsion wire should deviate towards the poles from the direction of the gravity of water by an easily determined angle E.*"

The subscript of "m" stands for different materials. There is no subscript corresponding to the distinction between inertial mass and gravitational mass. In case a distinction between inertial mass and gravitational mass were made, then we should have to write – inertial masses of different materials m_{i1}, m_{i2}, m_{i3}, etc. and gravitational masses of different materials m_{g1}, m_{g2}, m_{g3}, etc. Without such declarations, it must be presumed that no such distinction has been made.

"*Let us suspend two 25 gram masses of different substances on both ends of a homogeneous beam of 40 cm length.*"

The following questions arise:

- Is "25 gram" inertial mass and gravitational mass?
- How is "25 gram" measured? Is that measuring principle devoid of a presumption of the equivalence between inertial mass and gravitational mass?
- Is "gram" a unit of inertial mass or gravitational mass?

If answers are not provided to the above questions then it can not be understood whether the analysis is based on a presumed distinction of inertial mass and gravitational mass. And, if there is no a priori distinction between inertial mass and gravitational mass, then it can not be claimed that the experiment is designed to test the equivalence between the two i.e. the standard interpretation of Eotvos' experiment, which gave rise to new theories of gravitation Roll et al 1964[7], becomes untenable.

If the provision of such answers is not a requirement, then distinction between inertial mass and gravitational mass has nothing to do with "different substances" or material nature of a body. In that case, the present understanding of the situation needs a revision. A possibility is to make no distinction like "inertial" mass and "gravitational" mass. Instead, "proportionality of inertia and gravity" can be quantitatively written as "proportionality of inertial force ($F_i = ma$) and gravitational force ($F_g = GMm/r^2$)" i.e. $ma \propto GMm/r^2$, along with a proposition that the proportionality factor is dependent on the material nature of the test object of mass m. In that way all the obstacles of reason, from the metrological perspective, concerning inertial mass and gravitational mass does not arise, yet it remains consistent with Eotvos' experiment (except Eotvos' verbal mention of "gravitational mass" at one place in Eotvos et al 1922[5] which has no whatsoever effect on the calculations). I plan to provide an elaborate explanation of the scenario in near future[102].

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