Unification of Classical and Quantum Theory Based on a Combination of Special Relativity and Sampling Theory

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Abstract

The assumption that angular momentum is quantized underpins all of quantum theory. The assumption is at best, questionable and most likely false because it violates the tenets of the scientific method. A new model is proposed for the hydrogen atom which does not rely on this assumption and is instead based on a combination of a sampling process and special relativity. It is postulated that certain orbital velocity terms are themselves affected by relativity. This leads to a planetary model for the atom in which the electron orbits at near light speed and at a constant radius independent of the energy level, thus obviating the need for changes in radius with energy level and the attendant changes in potential energy. The atom is no longer seen as a nebulous cloud, but as a particle in the conventional sense.

The model provides explanations for many incomprehensible phenomena associated with current theories. The dynamics involved are recognizably those of Newton and Einstein. It provides a simple mechanical explanation for the discrete energy levels of the atom, why synchrotron radiation does not occur, the nature of the fine structure constant and zero-point energy and the reason why Planck's constant is a constant. It does so under a single set of physical laws and so it effectively unites classical and quantum mechanics.

Introduction

"To understand hydrogen is to understand all of physics."
Victor Frederick Weisskopf (1908 – 2002)

Hydrogen

First named by Antoine Lavoisier (1743 – 1794), hydrogen is the most abundant of all the atoms in the universe; it is the very stuff that stars are made of. It is estimated that hydrogen accounts for over 90% of all atoms in the universe, comprising 75% of all the mass in the universe. Hydrogen forms the basis of all other types of atoms in the universe. It is structurally the simplest of all the atoms, consisting of a single proton acting as a nucleus and orbited by a single electron. All other atoms have a so called *hydrogenic* form where they are stripped of all but one orbiting electron. Therefore, a comprehensive and detailed understanding of the workings of the hydrogen atom is an essential step to understanding all other, more complicated, atoms.

Background

Atomic Spectra

In the early 1800s the English chemist William Hyde Wollaston (1766-1828) and independently the German physicist Joseph von Fraunhofer (1787-1826) discovered that the spectrum of the sun contained a series of dark lines. Fraunhofer mapped the frequencies of these lines, which are now named after him.

Later it was discovered by Robert Bunsen (1811-1899) and Gustav Kirchhoff (1824-1887) that each chemical element can be associated with a set of these spectral lines. The lines are caused by the absorption of light by the atoms of the element at specific frequencies. The presence and frequencies of these lines are a characteristic of the type of atom, rather like a fingerprint or a barcode. Typical absorption spectra are shown in Figure 1.

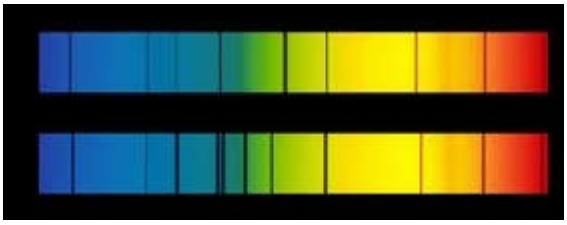

Figure 1 Absorption spectra

The Swiss mathematician and numerologist Joseph Jakob Balmer (1825-1898) was the first to find a mathematical formula for the wavelengths for the absorption spectrum of hydrogen. Balmer's formula was later shown to be a special case of a more general result which was formulated in 1888 by the Swedish physicist Johannes Rydberg (1854-1937).

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \qquad \text{Equation 1}$$

where λ is the wavelength of the spectral line
R_H is called the Rydberg constant for hydrogen
n_1 and n_2 are integers and $n_1 > n_2$

Structure of Atoms

Around 1910 Ernest Rutherford (1871-1937) began to use radiation to probe the structure of the atom. While in Canada Rutherford had noticed that Alpha particles were sometimes scattered as they passed through a thin sheet of mica. Rutherford, now back in Manchester, and assisted by Hans Geiger (1882-1945) together with Ernest Marsden (1889-1970) began to investigate in more detail. Much to their surprise, they found that the particles were sometimes deflected by very large angles, and indeed some were reflected directly towards the source. Rutherford described it as *"... like firing a fifteen-inch shell at a piece of tissue paper and it came back and hit you"*.

Rutherford knew that Alpha particles carried a positive electric charge, and he also knew that this would cause them to be deflected as they passed close to charged particles within the gold atom. Some of these were negatively charged electrons, which were relatively light and would have very little influence on the much heavier alpha particles. However, he also knew that the atom was electrically neutral and therefore had to contain something which was positively charged to balance out the negative charge of the electrons now known to form a part of the atom. He reasoned that the behavior of the alpha particles could be explained if all the positive charges in the atom were concentrated at a single point at the center of the atom. He called this the nucleus. In Rutherford's model the nucleus was positively charged and contained most of the mass of the atom, the negatively charged electrons spread at a distance, resembling a mini solar system.

Rutherford's planetary model of an atom still presented some difficulties. An atom with positive charge concentrated at the center and stationary electrons disposed around it would be unstable. The negatively charged electrons would be attracted inexorably towards the positively charged nucleus. If on the other hand the electrons were in orbit around the nucleus the centrifugal force could balance the electrical force. However, to complete their orbits the electrons would be undergoing a continuous acceleration towards the central nucleus, and when a charged particle is accelerated it radiates energy. This so called "synchrotron radiation" would sap the orbiting electron of its energy causing it to spiral in towards the nucleus.

The Bohr Model

Niels Bohr (1885-1962), working under Rutherford's tutelage in Manchester, sought to find a mathematical model for the atom which would balance the various forces involved and at the same time match the wavelengths predicted by the Rydberg formula. He could then calculate the orbital radius of the electron and its orbital velocity. He knew that Rutherford's planetary model of the atom was unstable. The electrons could not be stationary, or they would collapse into the nucleus, they could not be in motion because they would spiral towards the nucleus as they radiated synchrotron energy. Bohr chose to ignore the problem of stability and instead to concentrate on the dynamics of the atom. Only after he had solved this problem did he intend to look at the problem of stability.

Bohr assumed that the electron orbit was circular and balanced the electrostatic force with the centrifugal force as follows:

$$\frac{Kq^2}{r^2} = \frac{mv^2}{r} \qquad \text{Equation 2}$$

Where K is the electrostatic force constant, q is the electron charge and r is the orbital radius.

He needed a second equation and reasoned that the orbital angular momentum of the electron was related to Planck's constant. Using Newtonian dynamics to balance the forces yields an atom that has only one energy state and so does not match the Rydberg formula. Bohr needed to modify the Newtonian equations. In essence, an integer multiplier was missing to represent the energy level of the atom. Bohr took inspiration from a colleague, John W Nicholson (1881-1955) who had suggested

that the orbital angular momentum of the atom and that it can only take on values that are an integer multiple of Planck's constant.

From the classical equation

$$l = \hbar = mvr \qquad \text{Equation 3}$$

Nicholson developed the modified version

$$l = n\hbar = nmvr \qquad \text{Equation 4}$$

Where $\hbar$ is Planck's constant, m is the mass of the electron, v is the velocity of the electron and n is an integer with values 1, 2, 3, 4, 5 …∞

Bohr now had two equations with two unknowns, r and v. He could solve these to calculate the orbital radius and the velocity of the orbiting electron for each of the different energy levels, n.

$$v_n = \frac{Kq^2}{n\hbar} \qquad \text{Equation 5}$$

And

$$r_n = \frac{n^2\hbar^2}{mKq^2} \qquad \text{Equation 6}$$

For $n = 1$, referred to as the ground state, base state, or lowest energy state:

$$v = 2187803.961 \textbf{ m/s} \qquad \text{Equation 7}$$

This is frequently referred to as the Bohr Velocity and is approximately .07% of the velocity of light.

And

$$r = 5.29149 * 10^{-11} \textbf{ m} \qquad \text{Equation 8}$$

Commonly referred to as the Bohr Radius

The velocity of the orbiting electron varies as the inverse of the energy level. In other words, somewhat paradoxically, the velocity decreases as the energy level increases. While the orbital radius increases as the square of the energy level, indicating that the size of the atom increases with energy level.

At first sight it appears that the energy of the electron in the Bohr model decreases with increasing values of n. However, this is not the case, because in moving from a low energy state the electron loses kinetic energy but gains potential energy. The exact amount is given by the Virial theorem and is equal to double the kinetic energy in each state.

Bohr's model appeared to accurately describe the behavior of the hydrogen atom however, in doing so Bohr had to introduce Nicholson's idea that the angular momentum of the orbiting electron could only take on these very specific values which were integer multiples of Planck's constant. Bohr failed to explain why this should be the case, and another problem defied explanation: To satisfy the requirements of the Bohr model, when the energy of the atom changes from one energy state to another the electron must jump between two energy states, in effect jumping between two orbits of different radii. The electron must make the transition from one orbit to another instantaneously and without ever occupying any position in between the two orbits. Such transitions between orbits represent discontinuities of position and appeared to defy rational explanation. They were given the

name ‘Quantum Leap’ and the term has since come to mean any seemingly improbable change of position.

Louis de Broglie

Louis de Broglie (1892 – 1987) was born into an aristocratic French family in August 1892 in Dieppe. Louis was introduced to physics by his elder brother who had attended the Solvay conference in 1911. In 1905 Einstein showed that the hitherto wavelike nature of light concealed the underlying particle-like behavior. In 1923 Louis de Broglie suggested that maybe this situation could be reversed, and that perhaps what had hitherto been thought of as a particle could be described in terms of a wave. De Broglie discovered that if he assigned a wavelength and a frequency to an electron, he could explain the location of the atoms in the Bohr model of the atom. He found that the orbiting electrons could only occupy orbits which contained a whole number of such wavelengths. De Broglie published his theory in 1924.

De Broglie’s idea hinges on the notion of standing waves. A standing wave occurs in a taught string, anchored at both ends, that is plucked. The fundamental frequency occurs when the whole length of the string vibrates. Other modes of vibration are also possible, for example where the center of the string remains stationary and the two halves of the string each vibrate at what is referred to as a second harmonic frequency. This can also happen at the third, fourth and other higher harmonics as shown in Figure 2. Here then was a possible explanation as to why the electron orbits could only take on whole number multiples of a base value.

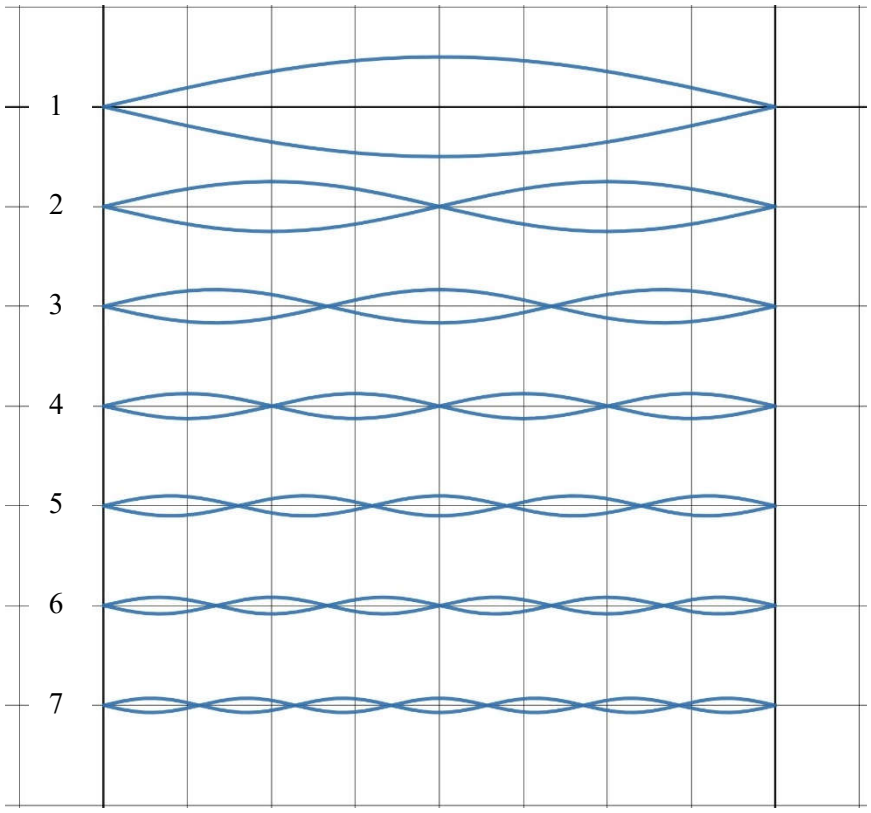

Figure 2 Standing Waves

In the base energy state, the electron is oscillating at the fundamental frequency that is there is one whole cycle of the electron as a wave during one complete orbit. In the second energy state the electron is oscillating at exactly twice the frequency of the orbit of the electron, forming a standing wave where two wavelengths are required to describe one complete orbit. This carries on in the third and higher orbits.

In de Broglie's model the standing waves are disposed around the orbits of the Bohr model. The waves themselves have no physical interpretation but are commonly represented as shown in Figure 3 and Figure 4.

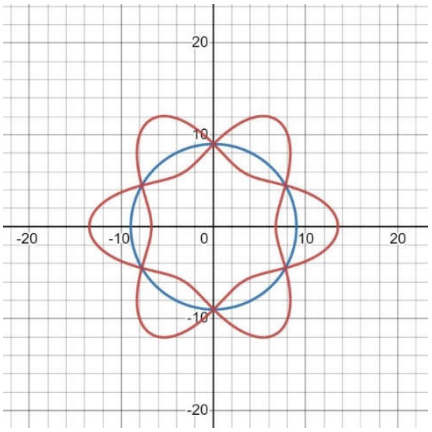

Figure 3 de Broglie's waves for energy level 3

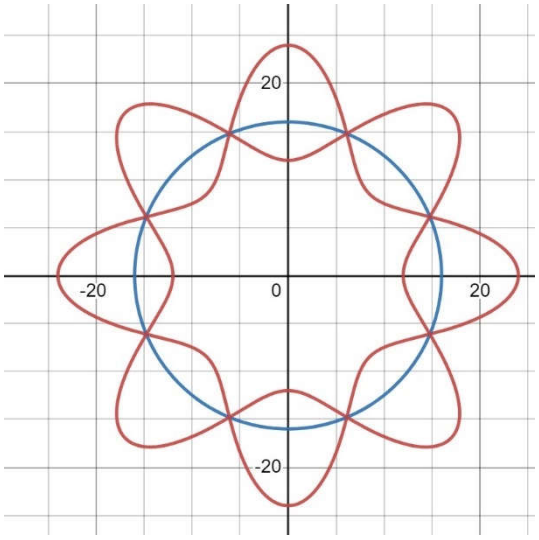

Figure 4 de Broglie's waves for energy level 4

https://www.desmos.com/calculator/holtxm9x7j

The Heisenberg Uncertainty Principle

Werner Heisenberg (1901 – 1976) was a German physicist who studied physics and mathematics in Munich. He studied under Arnold Sommerfeld alongside Wolfgang Pauli (1900-1958).

He chose to represent the frequencies and intensities of the various absorption and emission lines in tabular form. It was while trying to manipulate this table he found that the normal rules of multiplication did not apply. Multiplication of matrices is not commutative, that is [A][B[] ≠ [B][A], the result of multiplying two matrices together is different depending upon the order in which the multiplication is written.

When Heisenberg applied this to the momentum and position of a particle he found the relationship:

$$[X][P] - [P]\{X] = \hbar[I] \qquad \text{Equation 9}$$

Where [I] is known as the identity matrix and is the equivalent of unity in matrix multiplication, so $\hbar$[I] is the equivalent of a constant in matrix multiplication.

Heisenberg realized that the non-commutative nature of this matrix multiplication implied that it was not possible to know both the position and the momentum (velocity) of a particle to an arbitrary degree of accuracy. There was an uncertainty inherent in the system which was related through the constant term to Planck's constant, which appears in the right-hand side of the equation. This subsequently became known as the Heisenberg Uncertainty Principle.

Heisenberg first sought to explain the uncertainty principle by looking at the way in which things are measured. To determine the momentum of a small particle such as an electron it is necessary to measure both its mass and its velocity. Velocity can be determined by measuring position at two points separated by a small interval of time. On the scale of the electron however, the only tools available to measure the position are other subatomic particles such as electrons or photons, which are of comparable size and have comparable energies. Therefore, any attempt to measure the position of the electron will alter its velocity and vice versa, any attempt to measure the velocity will alter its position.

Schrödinger Wave Equation

Independently of Heisenberg the Austrian physicist Erwin Schrödinger (1887 – 1961) took a different approach but arrived at much the same conclusion. Schrödinger had been made aware of de Broglie's ideas that particles could be regarded as waves as a result of correspondence with Einstein. He was asked to give a presentation on de Broglie's waves by Peter Debye (1884-1966). When Schrödinger gave his talk, Debye dismissed it as being childish because it did not contain an equation to describe the said waves. Schrödinger took up the challenge and set about discovering a wave equation to describe de Broglie's matter waves. The resulting equation is a second order partial differential equation involving both distance and time. The Schrödinger wave equation only has meaningful solutions for certain discrete values of the total system energy. In the case of the hydrogen atom these are found to correspond to the discrete energy levels of the atom.

Schrödinger's wave amplitudes have no direct physical interpretation they are the self-same waves as proposed by de Broglie. Max Born (1882 – 1970) went on to show that the square of the amplitude represented the probability distribution of the position of the particle. In the case of the hydrogen atom the peaks in these functions for the various energy levels correspond to the orbital radii predicted by the Bohr model.

Just as with the Bohr model, the equation can only be solved analytically for hydrogen itself and for other so called *hydrogenic* atoms, which are atoms that are ionized to have a single orbiting electron.

Eventually Schrödinger, Carl Ekhart (1902-1973) and Paul Dirac (1902-1984) each independently showed that Heisenberg's matrix mechanics and Schrödinger's wave equations were mathematically equivalent and were in fact just different manifestations of the same phenomenon.

Schrödinger believed that he had eliminated Bohr's awkward quantum leaps. Energy changes in the Schrödinger model were represented by changes in the probability that the electron occupied one particular orbit or another in a way that is analogous to the way in which the frequency of a violin changes from one note to another.

The Copenhagen Interpretation

Bohr thought that Heisenberg's explanation of uncertainty was flawed. Heisenberg's original idea, that uncertainty was a practical problem of measurement, did not sit with Bohr's idea that the laws of physics were different on the atomic scale. Bohr took a holistic view in which the observer and the observed could not be separated and argued that light, for example, exists as both a wave and as a

particle at the same time but that it only manifests itself as one or the other form when it is measured or observed. It is at the point where an observer chooses how to measure the particle that its nature becomes fixed. Instead of being capable of description as either a wave or a particle, objects can be described in terms of a *wave function*. Bohr introduced the principle of complementarity, which states that different experimental setups reveal different aspects of a quantum system. Wave-like and particle-like behaviours are said to be complementary aspects that cannot be observed simultaneously. Quantum systems are said to exist in multiple states simultaneously until measured. For example, an electron can be in a *superposition* of different energy levels. The original wave function contains all the possibilities within it. It manifests itself as one form or another, not because it changes in nature from one form to another, but because the observer is looking for that particular form.

The interpretation emphasizes that quantum mechanics does not provide deterministic outcomes but rather probabilities of various outcomes. This probabilistic nature is inherent and not due to lack of information. When a quantum system is measured, the *wave function* collapses to a single eigenstate. The act of measurement causes the wave function (a mathematical description of the quantum state) to collapse to a definite state. Before measurement, only probabilities of different outcomes can be predicted. The so-called Measurement Problem is that of understanding when and how this collapse takes place and the idea that it does so is a postulate of the Copenhagen interpretation.

The idea was to result in a schism in the world of nuclear physics between those who accepted such a subjective reality and those who did not. The principal protagonists in this debate were Einstein and Bohr, with Bohr an advocate of subjective reality and Einstein an advocate of an alternative objective reality.

While the Copenhagen interpretation is the most widely taught theory of quantum mechanics, it is not without its problems and critics leading to the development of a number of alternative interpretations:

Hidden variables. The EPR Paradox proposed by Einstein, Podolsky, and Rosen argued that quantum mechanics is incomplete and suggested the existence of hidden variables.

Many-Worlds Interpretation: this interpretation suggests that all possible outcomes of a quantum measurement occur, each in a separate, branching universe.

Pilot-Wave Theory: this deterministic interpretation introduces hidden variables and suggests that particles have definite positions guided by a wave function.

Schrödinger's Cat

Schrödinger was deeply skeptical of the Copenhagen interpretation and sought to point out its deficiencies with a thought experiment. In his thought experiment the eponymous cat is locked in a sealed box together with an atomic device that has 50% probability of releasing an alpha particle in a one-hour period. If the alpha particle is emitted it triggers a device which releases a vial of poison which kills the cat. After one hour the box is opened to reveal the fate of the cat. It has been suggested that during the one-hour period that the cat is sealed in the box it is neither alive nor dead but enters a state of superposition where it is both alive and dead at the same time.

The two events, placing the cat in the box and opening it one hour later, are two samples taken one hour apart in a sampling regime that notionally samples the vital signs of the cat at a rate of once per hour. The vital signs of the cat can be thought of as a continuous analog signal. When a signal is sampled in this way the state of the signal between samples, the inter sample value, is unknown and so it is undefined. Quantum theory seeks to suggest that during the one-hour interval the cat is both alive and dead at the same time. However, such fanciful ideas are not necessary. At any instant in time the cat is either alive or dead but there is no way to know which based on the sampled values. The fact that the cat's vital signs are only monitored at these two specific times is sufficient to render them undefined.

Couched in these terms it is evident that Schrödinger's cat is almost a perfect textbook example of the application of Shannon's Sampling Theorem. This states that the sampling rate must be twice the highest frequency of interest in the sampled signal. In the case of the cat this would mean that the death throes of the cat would have to have a duration of at least one hour for the time of death to be ascertained accurately.

In quoting Schrödinger's thought experiment the question that is most often asked is whether the cat is alive and/or dead and, if it died, when precisely did that event occur. However, a far more pertinent question is to ask why it is not possible to monitor the vital signs of the cat at more frequent intervals than once per hour.

In terms of the hydrogen atom the question that needs to be asked is: What is it about the dynamics of the atom that means it is only possible to ascertain the position and velocity of the orbiting electron once per complete orbit?

Sampling

During the 1930's and 40's telecommunications engineers were concerned to increase the capacity of the telephone network. One of the ideas that surfaced was called Time Division Multiplexing (TDM). In this each of a number of incoming telephone lines is sampled by means of a switch, the resulting samples are sent over a trunk line and are decoded by a similar switch at the receiving end before being sent on their way. This allowed the trunk line to carry more telephone traffic without the expense of increasing the number of cables or individual lines. The question facing the engineers at the time was to determine the frequency at which the incoming lines needed to be sampled for the telephone signal to be correctly reconstructed at the receiving end.

The solution to this problem was arrived at independently by a number of investigators but is now largely credited to two engineers. The so-called Nyquist-Shannon sampling theorem is named after Harry Ny Nyquist[i] (1889-1976) and Claude Shannon[ii] (1916 – 2001) who were both working at Bell Labs at the time. The theorem states that to reproduce a signal with no loss of information then the sampling frequency must be at least twice the highest frequency of interest in the signal itself. The theorem forms the basis of modern information theory, and its range of applications extends well beyond transmission of analog telephone calls, it underpins much of the digital revolution that has taken place in recent years.

What concerned Shannon and Nyquist was to sample a signal and then to be able to reproduce that signal at some remote location without any distortion, but a corollary to their work is to ask what happens if the frequency of interest extends beyond this Shannon limit? In this condition, sometimes called under sampling, there are frequency components in the sampled signal that extend beyond the Shannon limit and maybe even beyond the sampling frequency itself.

A simple example can be used to illustrate the phenomenon. Suppose there is a cannon on top of a hill, some distance away is an observer equipped with a stopwatch, Figure 5. The job of the observer is to calculate the distance from his current location to the cannon. Sound travels in air at roughly 340 m/s. So, it is simply a matter of the observer looking for the flash as the cannon fires and timing the interval until he hears the bang. Multiplying the result by 340 will give the distance to the cannon in meters, let's call this distance D.

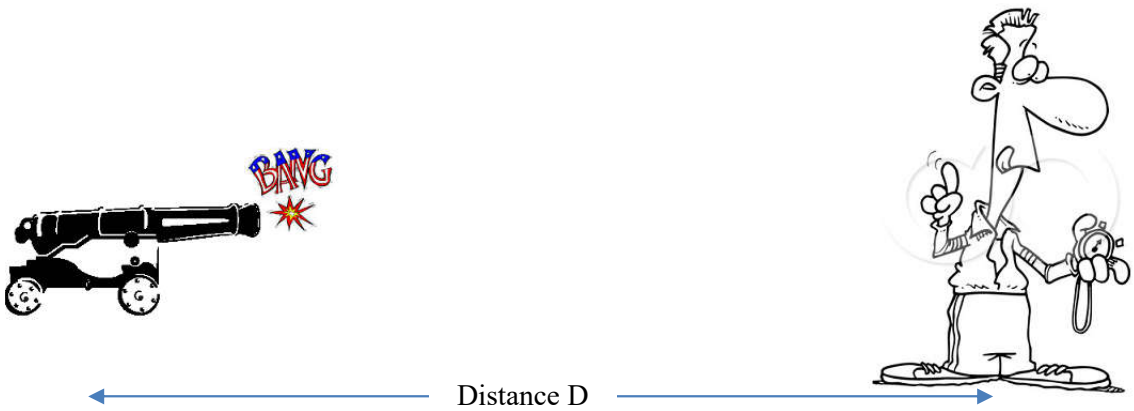

Figure 5 Measuring Distance

This is fine if the cannon just fires a single shot, but suppose the cannon is rigged to fire at regular intervals, say T seconds apart. For the sake of argument and to simplify things, consider the case where T equals one. If the observer knows he is less than 340 m from the cannon there is no problem. He just makes the measurement as before and calculates the distance D. If on the other hand, there is no restriction placed on his distance to the cannon then there is a problem. There is no way that the observer knows which bang is associated with which flash, so he might be located at any one of a number of different distances from the cannon. Not just any old distance will do, however. The observer must be at a distance D or D + 340 or D + 680 and so on, in general D + 340n. The distance calculated as a result of measuring the time interval between bang and flash is ambiguous. In fact, there are an infinite number of discrete distances which could be the result of any particular measured value.

Restricting the observer to within 340 m of the cannon is a way of imposing Shannon's sampling limit and removing this restriction opens the possibility of ambiguity in determining the position of the observer due to a phenomenon called aliasing.

Turning the problem around, if instead of measuring the distance to the cannon the position of the observer is fixed. Once again to make things simpler, the distance can be chosen to be 340m. This time however it is possible to adjust the rate of fire of the cannon until the observer hears the bang and sees the flash as occurring simultaneously. If the rate of fire is one shot per second then the time taken for the slower bang to reach the observer exactly matches the interval between shots and so the two events, the bang and the flash, are seen as being synchronous.

If the rate of fire is increased then at first, for a small increment, the bang and the flash are no longer in sync. They come back into sync however when the rate of fire is exactly two shots per second, and again when the rate is three shots per second. If the cannon had a fast enough rate of fire, this sequence would extend to infinity for a rate of fire which is an integer number of shots per second. Notice that now the bang no longer relates to the previous flash, but to a previous flash. It is interesting to note also that if the rate of fire is reduced from once per second, then the observer will never hear and see the bang and the flash in sync with one another and so once per second represents the minimum rate of fire which will lead to a synchronous bang and flash. Here is a system that has as its solution a base frequency and an infinite set of harmonic frequencies.

This phenomenon is called aliasing because each such occurrence where the bang and the flash coincide is an alias for the first or base solution. In the former case the variable is the distance between the observer and the cannon, while in the latter case it is the interval between the bang and the flash that is ambiguous. The two examples with the cannon illustrate two different types of aliasing. In the first the sample interval is fixed, and it is the distance that is subject to aliasing, while Distance D 10 in the second it is the distance that is fixed and the sample interval that is subject to

aliasing. However, in both cases all the possible values are valid at any one time, but only one is true. The distance or the time can be said to be determinate, but not determinable.

Aliasing and Harmonic Series

In this latter example, where it is the period that is being altered and by implication the frequency of the events, the aliases form a harmonic sequence. In any sampled signal where all the samples have the same value, then there is no way to know whether the signal is at the sample frequency or at an integer multiple of the sample frequency. This can be seen in Figure 6 which shows how a fundamental frequency and its harmonics all return the same sampled signal, note that the sample values are all the same and depend only on the phase relationship between the signal being sampled and the sampling function. The important point here is that all the waveforms coincide at the sampled instants even though the waveforms have different frequencies.

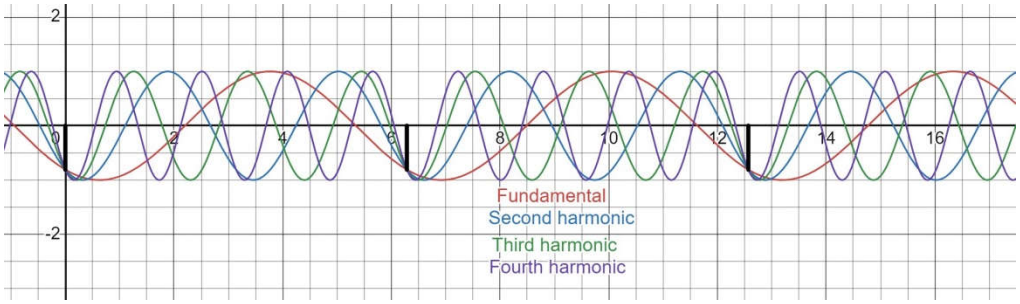

Figure 6 Sampling and Harmonics

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Wherever there is a harmonic series in nature there must always be a corresponding sampling process. This becomes evident from consideration of the Fourier representation of a harmonic series. Such a Fourier representation comprises a series of spikes equally spaced along the frequency axis. For a real function these are disposed equally on both the positive and negative frequency axes. These spikes are referred to as Dirac or Delta functions and such a collection of equally spaced and equally weighted Dirac functions is referred to as a Dirac comb.

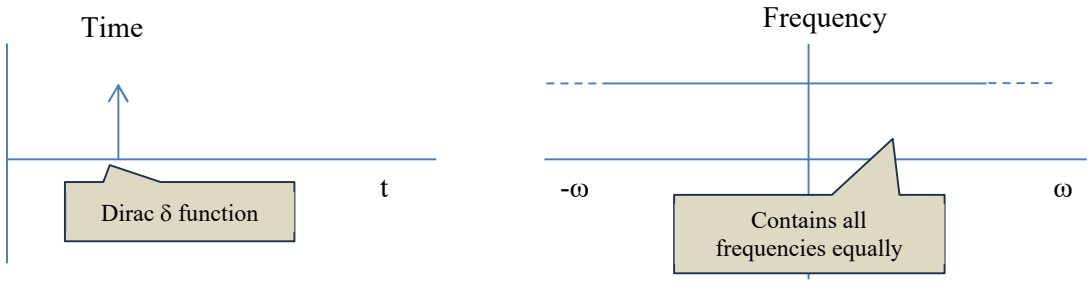

Figure 7 Fourier transform of a single Dirac function

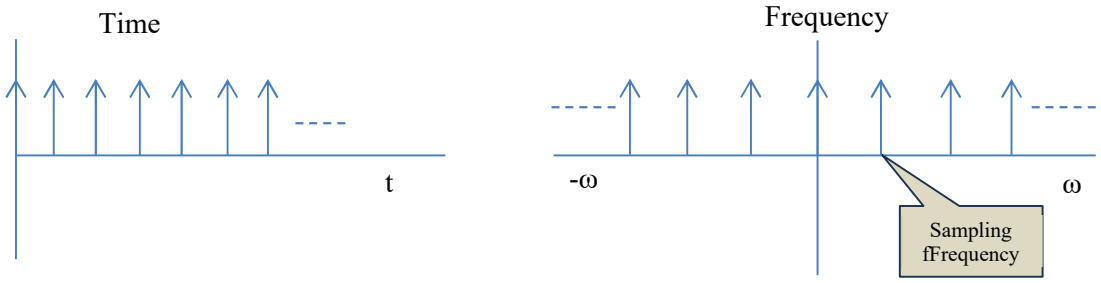

Figure 8 Fourier transform of a Dirac comb

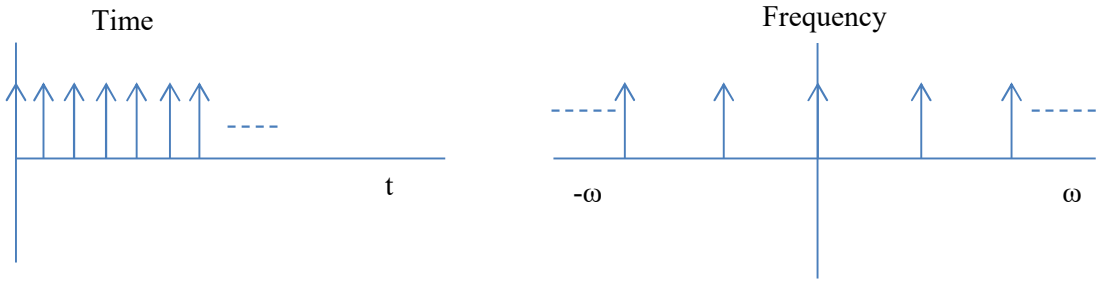

Figure 9 Fourier transform of a higher frequency Dirac comb

The Fourier transform of a Dirac comb in the frequency domain is a function in the time domain that is itself another Dirac comb[iii]. Such a Dirac comb in the time domain can be regarded as a sampling function, since if it is multiplied by any other signal, it effectively takes a sample at regular intervals in time. The sampling frequency corresponds to the lowest frequency in the harmonic series, which in the case of the atom is the orbital frequency of the base energy state of the atom, Figure 7, Figure 8 and Figure 9. The orbital frequencies of the higher energy states are signals of interest and so can be regarded as aliases of this base band signal.[1]

Problems with Quantum Theory

The fundamental problem with the Bohr model is that it results in the physically impossible quantum leap. The origin of this phenomenon can be traced back through the force balance equations and rests with the assumption that angular momentum is quantized. The quantum leap represents a discontinuity of position with respect to time. Not only is such a discontinuity a physical impossibility but it renders the concept of velocity invalid since velocity requires that position with respect to time is differentiable, single valued[iv]. and continuous.

The physical impossibility of the quantum leap renders the Bohr model invalid, but equally important it renders the assumption that angular momentum is quantized invalid as well. If the assumption is invalid and therefore any model which relies on this assumption is in direct violation of the scientific method.

It is frequently argued that the Bohr model has been superseded by other more sophisticated models such as that of le Broglie, Schrödinger and Heisenberg, but this is not the case. All these models are highly dependent on the Bohr model and incorporate a version of the quantum leap as evidenced by the fact that a change in energy level incurs a change in potential energy. On closer examination it is also evident that the Bohr model forms an integral part of these later models. Figure 3 and Figure 4

[1] A proof of aliasing is provided in Appendix 1

show how de Broglie's waves are disposed around the orbits of the Bohr model, so rather than superseding the Bohr model these later models rely on it.

The received wisdom is that quantum theory is correct but not complete and that it is necessary to find some new postulate in addition to the quantization of angular momentum which will cause everything to fall into place. It is argued that this new assumption must lie in the domain of quantum theory itself and not in our understanding of classical mechanics. However, the fact that the assumption that angular momentum is quantized is invalid means that there must be an alternative postulate in the domain of classical mechanics, otherwise all that is left is Newtonian dynamics which is insufficient to account for the discrete energy levels. It is therefore necessary to rethink quantum theory from the ground up in such a way that it retains the existing results but does not require changes in potential energy between stable states.

The Rydberg Formula and Series

The Rydberg formula describes the relationships between the various energy levels of the hydrogen atom. It is important to understand that Rydberg's formula is based on the results of experiments and observation. It does not seek to explain the spectral lines, rather it seeks to describe them, and it is complete, that is it describes objectively all of the spectral lines for hydrogen. The formula deals only with the differences in energy between the various energy states. It has nothing to say about the absolute value of energy carried by the orbiting electron. The Rydberg formula forms a sort of gold standard against which any successful model for the hydrogen atom may be tested.

The Rydberg formula was given in Equation 1. In this form it uses the somewhat obscure wave number ($1/\lambda$). It can be expressed more usefully in terms of the energy emitted or absorbed when a transition takes place. This is achieved by multiplying both sides of Equation 1 first by *c*, the velocity of light and then by *h*, Planck's constant[2]. Gathering terms and substituting the analytical value for R_H gives:

$$E_{n_1,n_2} = \frac{1}{2}mc^2\alpha^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \qquad \text{Equation 10}$$

Where *m* is the rest mass of the electron and α is the Sommerfeld Fine Structure Constant (Alpha)

The Rydberg formula gives the amount of energy released when the electron orbiting the hydrogen nucleus makes a transition from the n_2^{th} energy state to the n_1^{th} energy state, or conversely the amount of energy absorbed if the transition is in the other direction. Letting $n_2 = \infty$ gives the energy associated with a transition to or from the maximum possible energy state of the electron and its energy in the n^{th} energy state, that is the *energy potential*[3] of the atom in the n^{th} energy state.

Doing so leads to the Rydberg Series

$$E_\infty - E_n = \frac{1}{2}mc^2\frac{\alpha^2}{n^2} \qquad \text{Equation 11}$$

The Rydberg Series is particularly useful because it makes it easy to calculate the energy associated with any transition simply by taking the difference between two values in the series.

It is reasoned here that the electron orbiting the atomic nucleus must do so at the constant radius, for every energy state. Anything other than this would imply the existence of the physically impossible 'quantum leap'. This in turn means that there can be no change in potential energy when the electron

[2] Note that this is the long form of Planck's constant
[3] Note that energy potential is not the same as potential energy.

transitions from one energy state to another energy state. All changes in energy must therefore be kinetic in nature.

E_n is the energy potential of the n^{th} energy state and represents the difference between the energy of the electron in the n^{th} energy state and that in the most energetic energy state possible, the ∞ energy state or *energy ceiling* of the atom. Since nothing can ever travel faster than the speed of light, the energy ceiling is limited by the speed of light to be

$$E_\infty = \frac{1}{2}mc^2 \qquad \text{Equation 12}$$

And the energy of the electron in the n^{th} energy state is

$$E_n = \frac{1}{2}mv_n^2 \qquad \text{Equation 13}$$

Where v_n is the orbital velocity in the n^{th} energy state

It is the difference between the energy ceiling and the energy in the n^{th} energy state that is expressed in the Rydberg series, so by definition

$$\frac{1}{2}mc^2 - \frac{1}{2}m\,v_n^2 = \frac{1}{2}mc^2\frac{\alpha^2}{n^2} \qquad \text{Equation 14}$$

Equation 14 can be simplified to give

$$c^2 - v_n^2 = c^2\frac{\alpha^2}{n^2} \qquad \text{Equation 15}$$

And further simplified to give

$$\sqrt{\frac{c^2}{c^2 - v_n^2}} = \frac{n}{\alpha} \qquad \text{Equation 16}$$

The term on the left-hand side of Equation 16 will be recognized from as the Lorentz factor Gamma (γ) and hence

$$\gamma_n = \frac{n}{\alpha} \qquad \text{Equation 17}$$

From Equation 17 it is evident that the variable of quantization is Gamma, the Lorentz factor, and that the electron is orbiting at near light speed. While Equation 17 tells what is quantized and how it is quantized it does not tell why it is quantized. The Lorentz factor, Gamma, is not inherently quantized. There are many examples where Gamma is known to take on values which are not related to the fine structure constant in any way. This means that there must be something about the dynamics of the hydrogen atom which causes Gamma to only take on these discrete values.

Special Relativity and Objects in Circular Orbit

It was Albert Einstein to whom we owe our present understanding of how relativity affects distance, time, frequency and mass. He did so initially for objects travelling at constant speed in what is now called Special Relativity. Later, he was to deal with objects that are accelerating or decelerating in what has come to be known as General Relativity. The electron is considered to be travelling at constant speed while it occupies a particular energy level and so it is only necessary to be concerned with Special Relativity

Central to Einstein's theory of special relativity is the idea of a reference frame. A reference frame is simply a co-ordinate system in which the objects move at the same constant speed and so maintain their position relative to one another. Einstein postulated that the laws of physics were the same in any reference frame and that the speed of light for any observers in any reference frame was the same. In adopting this second postulate Einstein had to give up on the notion that time was universal. Time, according to Special Relativity, runs at different rates in different reference frames, depending on how fast they are moving with respect to one another. For a stationary observer, time in a moving reference frame runs slower than it does in his or her own reference frame. The extent to which this occurs is a factor, Gamma, the Lorentz, factor which is given by the equation.

$$\gamma = \frac{c}{\sqrt{c^2 - v^2}} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad \text{Equation 18}$$

A graph of Gamma versus speed shows that it is close to unity for objects moving slowly but rises with increasing rapidity as the speed approaches the speed of light, see Figure 10.

Time for the moving object moves at a slower rate than that of stationary observer and so

$$\tau = \frac{t}{\gamma} \qquad \text{Equation 19}$$

Where τ is the time elapsed in the domain of the moving object and *t* is the time elapsed in the domain of the stationary observer.

A direct consequence of this slowing down of time is to increase the perception of frequency. Frequency is defined as the number of events per unit of time. Because time is running at different rates the perception of frequency will be different for stationary and moving observers. Slowing down of time causes an increase in frequency for the moving observer compared to that of the stationary observer. For the moving observer the frequency is multiplied by the Lorentz factor (Gamma).
Distance is also affected by relativity. Distances measured by a moving observer are shorter than those measured by a stationary observer. The extent to which this foreshortening occurs is the same Lorentz factor Gamma as affects the passage of time.

$$D = \frac{dt}{\gamma} \qquad \text{Equation 20}$$

Where *D* is the distance measured in the domain of the moving object and *d* is the distance measured in the domain of the stationary observer between the same two points.

Finally, the mass of a moving object appears greater to a stationary observer by the factor Gamma than it does to a moving observer, leading to the concept of a massive object having a rest mass, that is the mass as it is seen by an observer in the same reference frame as the object.

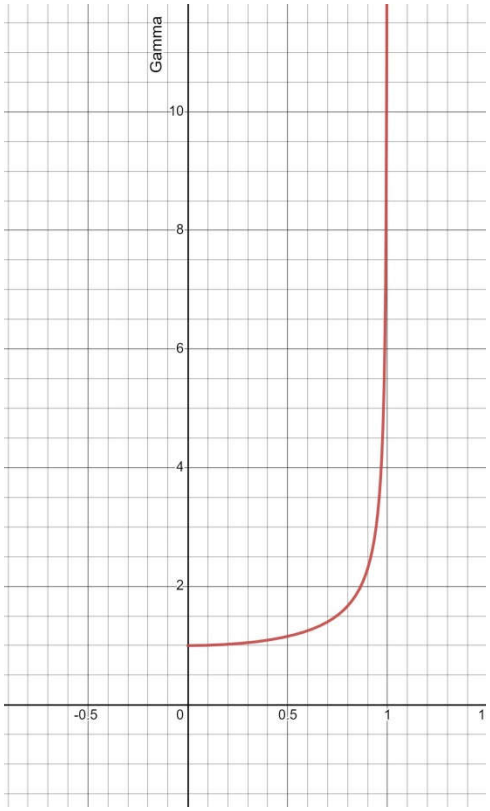

Figure 10 Gamma vs v/c

For an object travelling at 99.5% of the speed of light the value of Gamma is approximately 10. An observer travelling on such a moving object would therefore experience time at one tenth the rate of a stationary observer. The distance travelled by such an observer appears shorter by a factor of 10 and to a stationary observer the mass of an object moving at 99.5% of the speed of light appears to have increased by a factor of 10.

Speed is said to be invariant with respect to relativity. That is to say that speed is perceived by both stationary and moving observers to be the same. This comes about because speed is the result of dividing distance by time, and both are equally affected in a way which cancels out.

$$v = \frac{D}{\tau} = \frac{d/\gamma}{t/\gamma} = \frac{d}{t}$$ Equation 21

The effect of relativity on distance can be difficult to visualize because it forces involved non-Euclidian geometry. This can be thought of as if the measurements are made with a tape measure made of elastic, the faster one travels; the more the tape measure is stretched, but only when making measurements in the direction of travel. So for the astronaut travelling at 99.5% of the speed of light, any measurements made in the direction of travel are done so with a tape measure which has been stretched by a factor of 10 and so distances will appear to be less by the factor 10.

Relativity and Objects in Circular Orbit

The orbital radius is the same for all energy levels. This is because it is measured at right angles to the direction of travel and so is and does not alter and so is not affected. Since the orbital velocity is very close to the speed of light, the orbital angular momentum is therefore the same in all energy states. It is reasoned that is equal to Planck's constant and so

$$\hbar = mRc$$ Equation 22

From which

$$R = \frac{\hbar}{mc}$$ Equation 23

Relativity means that the orbital path length is foreshortened by the Lorentz factor, Gamma, but this is simply another way of saying that it is scaled by the factor 1/Gamma. Taking the reciprocal of Gamma from Equation 18 it is evident that this is the equation of a circle, more specifically a quadrant of a unit circle since *v* is constrained to lie between 0 and *c*.

$$\frac{1}{\gamma} = \sqrt{1 - \frac{v^2}{c^2}}$$ Equation 24

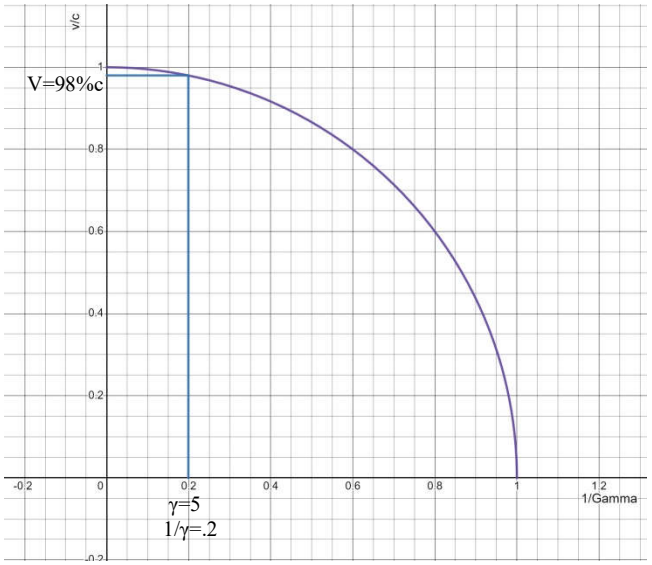

Figure 11 Reciprocal of Gamma

By superimposing this quadrant on the circular orbital path, the effects of relativity on such a path can be explored, see Figure 11 and Figure 12.

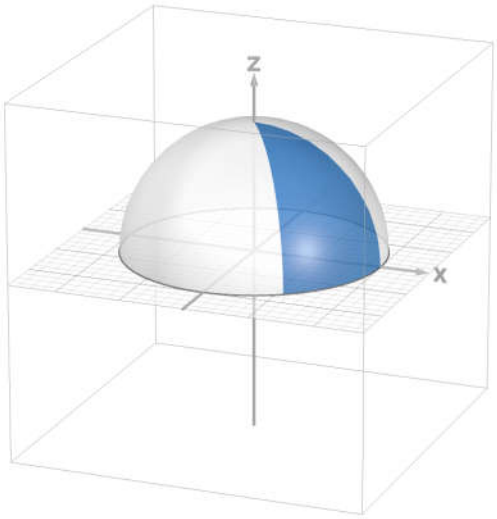

Figure 12 Spherical representation of relativity

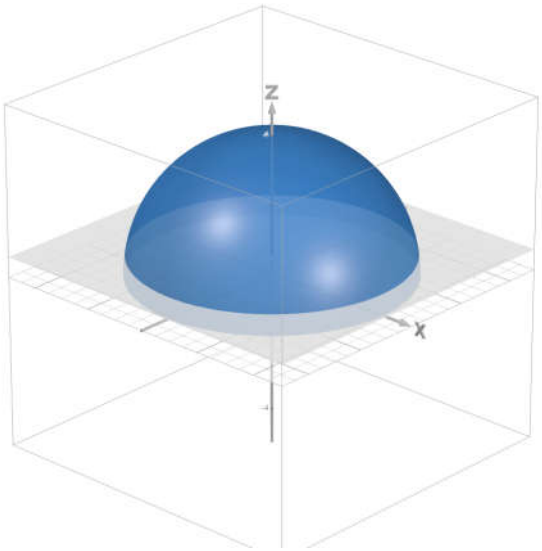

Figure 13 Effect of relativity at 15% c

Objects orbiting at non-relativistic speeds see the path length around the orbit as being equal in length to the equator, while objects orbiting at higher speeds follow a path length described by a line of latitude on the hemisphere. An object orbiting at the theoretical maximum speed of light would then be pirouetting at the pole. The orbital path can be considered as being represented by the line of latitude formed by a slicing plane which cuts through the hemisphere parallel to the equatorial plane. In In Figure 13 this is at approximately 15% of the speed of light c and so the orbital path length is just a little less than the equatorial path length, around 99%.

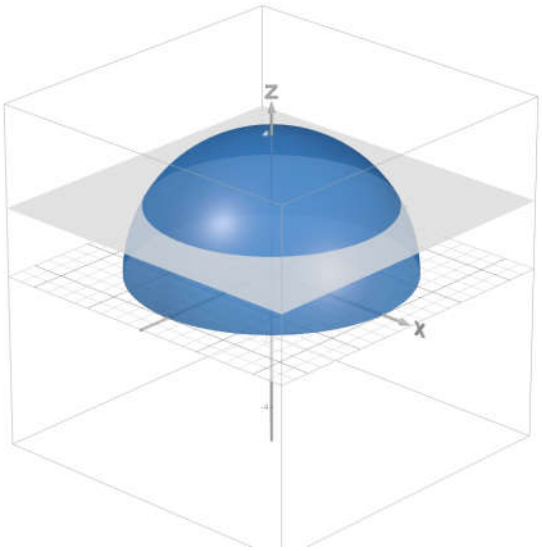

Figure 14 Effect of relativity at approx. 50% c

Figure 14 shows the situation where the orbital velocity is 50%c and so the orbital path length is approximately 70% that at the equator.

In Figure 15 the orbital velocity is around 98% of the speed of light and the corresponding orbital path length is approximately 20% of that for non-relativistic motion.

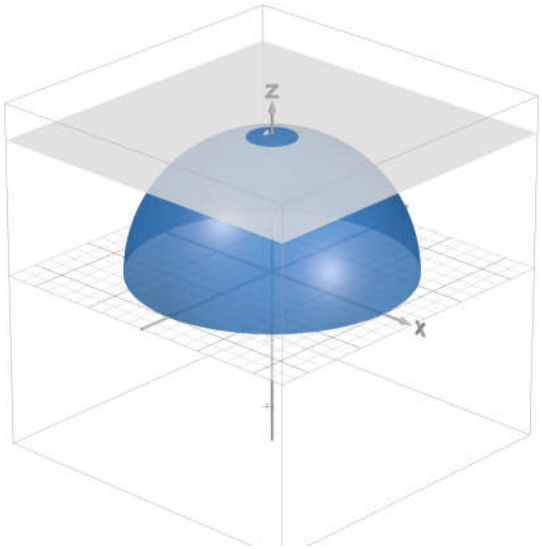

Figure 15 Effect of Relativity at Approximately 99% of c

This hemispheric model of the motion of an orbiting object is useful because it shows how orbital radius is unaffected by relativity, being the distance to the center of the sphere. The orbital geometry is non-Euclidian and in reality, it all takes place in just one plane. The introduction of this third dimension is just a device used to visualize what is going on. The orbiting object sees the distance it travels around one orbit as being reduced by a factor Gamma, but nevertheless sees the orbital radius as being unaffected by relativity since this is at right angles to the direction of travel.

The elastic tape measure analogy can be extended to include such a circular orbital path for all the possible lines of latitude on the hemisphere. As before this allows the effect relativity to visualized without distorting the geometry of space.

Instead of regarding distance as being compressed, the distance scale that is regarded as being stretched then the hemisphere of Figure 12 is transformed into a cylinder. This cylindrical representation can then be unwrapped to create a flat two-dimensional mapping in much the same way that a map projection can be used to draw the surface of a spherical earth onto a flat piece of paper. This is exactly analogous to the elastic tape measure described above, Figure 16.

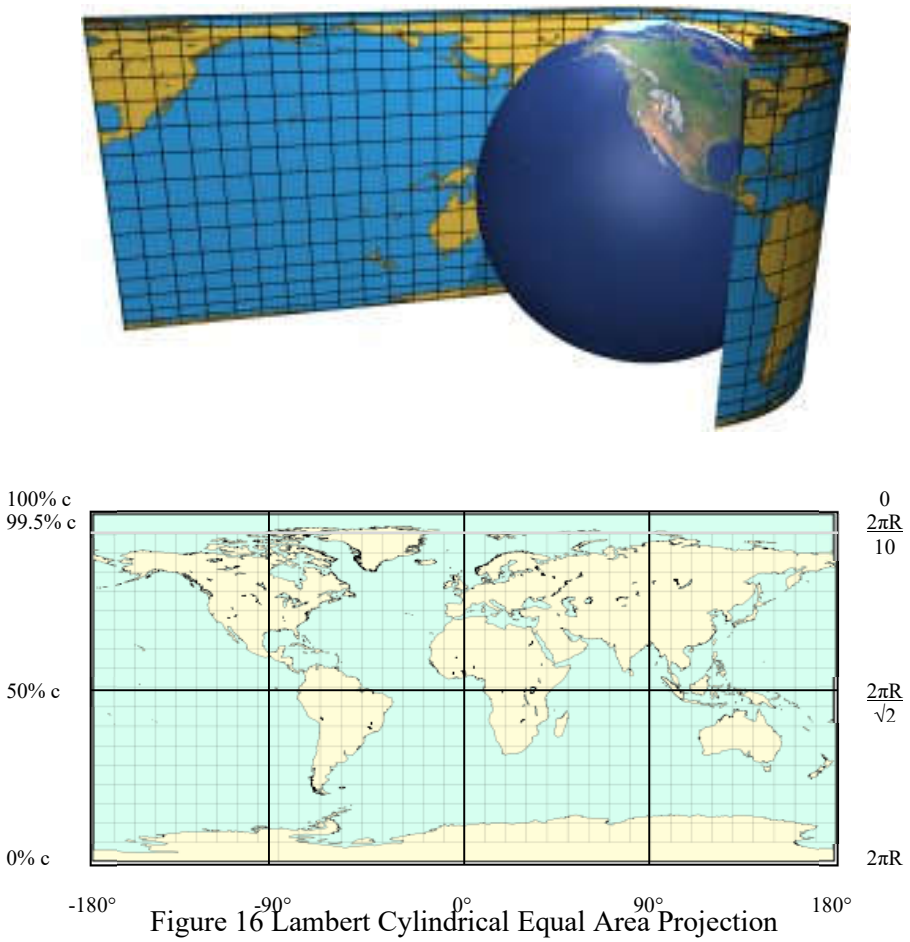

Figure 16 Lambert Cylindrical Equal Area Projection

The distance around the equatorial orbit is referred to here as the Actual Distance travelled by the astronaut. The distance as it is perceived at any other latitude, compressed by the factor Gamma, is referred to as the Relativistic Distance.

Waves and Duality

Waves

It is important to be clear about exactly what is a wave. A wave cannot exist in isolation, there must always be some 'thing' that is waving. In the context of the orbiting electron one such wave is the displacement of the electron about some mean position, either the *x* or the *y* component of its position around its circular orbit. For an object orbiting at constant speed in a circular orbit the wave form is a pure sinusoid. The difference between the *x* and the *y* components is a phase shift of 90°. Because of this close relationship between *x* and *y* components common practice to refer to either just the *x* component or just the y component unless circumstances dictate otherwise. The general equation of such a wave has the form

$$y = R\sin(\omega t + \varphi)$$ Equation 25

Where R is the orbital radius, ω the angular frequency and φ a phase shift (often taken to be zero)

The distance between successive peaks, the wavelength (λ) is equal to the orbital circumference ($2\pi R$). Since the orbital velocity is constant the equation can be written in terms of distance instead of time in which case

$$y = R\sin(\frac{\omega x}{v} + \varphi)$$ Equation 26

And is shown graphically in Figure 17

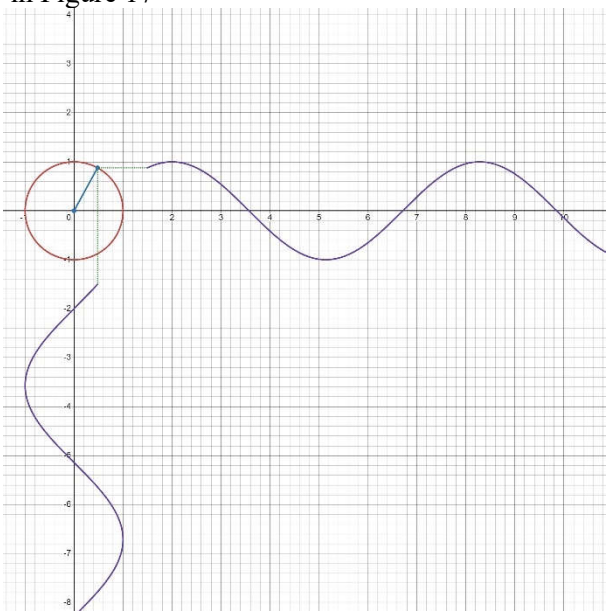

Figure 17 x and y components

https://www.desmos.com/calculator/yle1n5bwfc

Velocity is the first derivative with respect to time of the position and so it too has sinusoidal waves as its *x* and *y* components.

Duality

Duality is an inevitable consequence of special relativity, for every measurement there are two results, one for the stationary observer and one for the moving observer. At low relative speeds these are almost identical but diverge rapidly as the relative speed approaches that of light.

As the orbital velocity approaches the speed of light, the effects of relativity are to reduce the orbital period causing the frequency to increase and to foreshorten the orbital wavelength. The path that the electron follows does not alter, rather it is the perception of the length of this path that is less as far as the electron is concerned. Consequently, there are two sets of waves that describe the orbital motion of the electron in the hydrogen atom, one that is seen by a stationary observer and one that is seen by the moving electron.

As far as the stationary observer is concerned the electron is orbiting at near light speed in all its energy states. The orbital speed and hence the frequency can be calculated from Equation 17 and Equation 18 and is

$$\frac{v}{c} = \sqrt{1 - \frac{a^2}{n^2}}$$ Equation 27

$$\omega = \frac{c}{R}\sqrt{1 - \frac{a^2}{n^2}}$$ Equation 28

For the stationary observer the orbital frequency remains substantially constant, its dynamic range being less than .003%. The resulting orbital velocities and frequencies for the first 7 energy states are given in Table 1. The electron is orbiting at near light speed and its kinetic energy is close to $\frac{1}{2}mc^2$. The differences between the energy levels are relatively small compared to the total energy but their differences match the values of the Rydberg formula.

The situation for the moving electron is somewhat different. The sampling frequency of the stationary observer in the domain of *t* translates into a sampling frequency in the domain of τ, that of the moving electron, by dividing by α and so $\Omega_s = \omega_s/\alpha$ the sampling frequency is, by definition, the same as the orbital frequency in the base energy state. The base energy state is associated with a value of Gamma of $1/\alpha$, but the higher energy states are associated with values of Gamma which are *n* times this and so form a harmonic series as shown in Figure 18.

n	$v_{n\,/c}$	γ_n	ω_n	Energy eV	Energy Potential eV
1	0.9999733776803	548.145820680354	7.7632391423E+20	255485.925	13.607
2	0.9999933437358	274.075646892332	7.7633941477E+20	255496.130	3.402
3	0.9999970416220	411.110429716811	7.7634228560E+20	255498.020	1.512
4	0.9999983359049	548.145820680228	7.7634329041E+20	255498.682	0.850
5	0.9999989349769	685.181454892904	7.7634375550E+20	255498.988	0.544
6	0.9999992603998	822.217210731314	7.7634400814E+20	255499.154	0.378
7	0.9999994566199	959.253036067305	7.7634416047E+20	255499.255	0.278
∞	1.0000000000000	0.000000000	7.763451838E+20	255499.532	0.000

Table 1 Orbital velocity, frequency and energy

Taking, as an example, just the second harmonic: the wavelength is shortened by a factor two relative to that of the base energy state, alternatively this can be viewed as a dilation of time by a factor two, Figure 19.

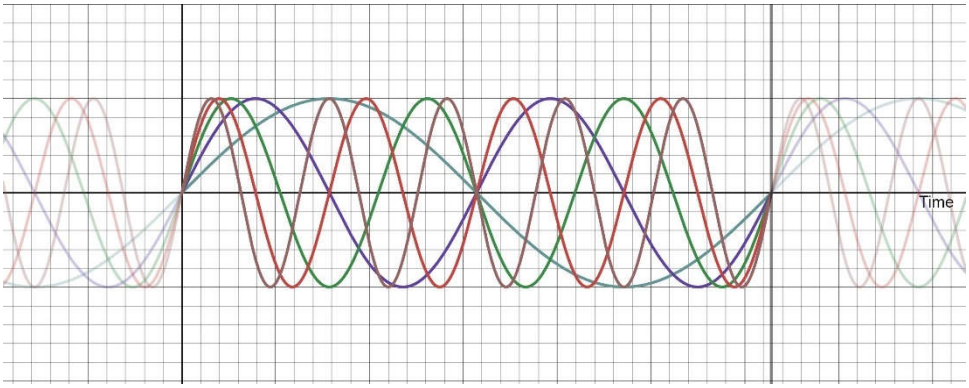

Figure 18 Harmonic series

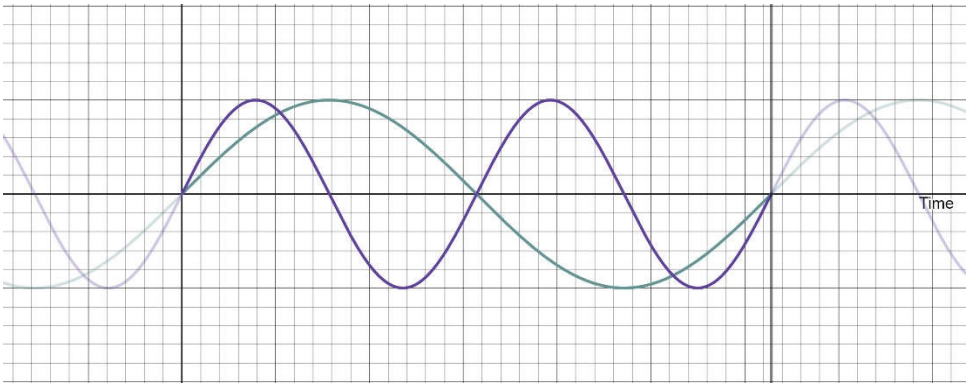

Figure 19 Fundamental and second harmonic

Relativistic Velocity

Relativity combined with sampling provides an explanation for the presence of the harmonic series associated with the discrete energy levels of the hydrogen atom. But this begs the question as to exactly what is sampled and how this affects the dynamics of the atom. Given that the assumption that angular momentum is quantized cannot be correct, it follows that somehow this must lead to a change in our understanding of the laws of classical dynamics.

The sampling process is a function of time with a sampling interval of once per orbit. This results in the position of the electron appearing to remain the same from sample to sample and the exact number of orbits between samples being ambiguous. The distance traveled by the electron in the reference frame of the stationary observer is more or less constant, but in the reference frame of the moving electron varies as the inverse of the energy level due to the foreshortening effect of relativity.

The conventional wisdom holds that both the stationary observer and the moving observer agree on their relative velocity, that velocity is invariant with respect to relativity. Such invariance is axiomatic to the derivation of Special Relativity.

To measure the speed of an object moving at close to the speed of light in real time it is necessary for a stationary observer to use two clocks, at least conceptually. One clock must be set up at the point of departure and another at the point of arrival. The two clocks must then be synchronized before the measurement can begin[4]. The time that the moving object leaves the point of departure is noted on the departure clock and the time of its arrival is noted on the arrival clock. At least one of these measurements must then be transmitted to the other location before the difference can be taken and the speed calculated. Any attempt to measure such a velocity in real time is thwarted by the fact that

[4] Since the two clocks are stationary with respect to one another they will run at the same rate and therefore it is possible to synchronize them.

the clock would have to move with the moving object and so would itself be slowed down by relativity.

There is however one circumstance where this is not the case, where it is possible to measure velocity using just a single clock; that is when the moving object is in orbit. Under such circumstances the object returns to its point of origin once per orbit and so it is possible, conceptually at least, to measure its orbital velocity in real time using a single clock provided the measurement is made over one or more complete orbits. Any attempt to measure the speed between two separate points on the circumference is thwarted by the same two clock problem outlined above. The restriction that orbital period can only be measured or experienced over a whole number of complete orbits amounts to a sampling process and such a sampling processes lead to aliasing.

Using this as a starting point and combining it with the effects of relativity on orbital motion, it is possible to define a hybrid velocity term which straddles the two reference frames: that of the stationary observer and that of the moving electron. Such a velocity is calculated as the distance measured by the moving observer, and foreshortened by relativity, divided by the time as measured or experienced by the stationary observer. Hence this 'Relativistic Velocity' term is also reduced by a factor of Gamma compared to the Actual Velocity as measured within the two respective reference frames and is invariant.

Within the reference frame of the stationary user the normal conditions apply, and the orbital velocity is seen as being close to the speed of light. The same is true for the electron, within its frame of reference distance is foreshortened and time is slowed down. Hence Relativistic Velocity only applies when considering phenomena which act between the two reference frames. Therefore it is postulated that this type of Relativistic Velocity is what applies when calculating momentum, angular momentum, centripetal and centrifugal force and acceleration.

When orbital velocity is measured over a complete orbit, the distance value which contributes to the measurement of velocity is measured in the reference frame of the moving object and is subject to aliasing. The orbital period however is measured in the reference frame of the stationary observer and so is not subject to aliasing. The orbiting electron has no inbuilt counter and cannot count the orbits, it is only capable of relating the distance travelled to the time taken once per orbit and since the distance travelled is subject to aliasing then so is the effective orbital velocity.

Balance of Forces

The Newtonian force balance equation is

$$\frac{Kq^2}{R^2} = \frac{mv^2}{R} \qquad \text{Equation 29}$$

However, it is necessary to consider the effects of relativity and it is postulated that the orbital velocity is close to the speed of light and so is affected by relativity. Under such circumstances the mass of the electron would be increased by a factor Gamma. The velocity term is taken to be the Relativistic Velocity and so is the Actual Velocity reduced by the same factor Gamma and so

$$\frac{Kq^2}{R^2} = \frac{m\gamma c^2}{R\gamma^2} \qquad \text{Equation 30}$$

This simplifies to

$$\frac{Kq^2}{R^2} = \frac{mc^2}{R\gamma} \qquad \text{Equation 31}$$

John W Nicholson argued that the orbital angular momentum of the electron was equal to Planck's constant. He based this idea on the fact that Planck's constant has units which are the same as those for angular momentum. He further assumed that the orbital angular momentum of the electron could take on values which were an integer multiple of this. It is this latter assumption which leads directly to the idea of the quantum leap. Here it is argued that the quantum leap is a physical impossibility and so the idea of quantizing angular momentum is rejected in favour of Gamma being the variable of quantization. However, the idea that the orbital angular momentum is equal to Planck's constant remains valid.

Again, the mass term is affected by relativity and the velocity term by the inverse, and so

$$\hbar = m\gamma r\frac{c}{\gamma} \qquad \text{Equation 32}$$

From which

$$\hbar = mrc \qquad \text{Equation 33}$$

And since *ħ, m* and *c* are all constants, it follows that the orbital radius is constant and is numerically $3.86159*10^{-13}$ **m.** This value is not unknown, it is sometimes referred to as the Reduced Compton Wavelength; the Compton Wavelength is then the orbital circumference.

$$R = \frac{\hbar}{mc} \qquad \text{Equation 34}$$

Substituting for R in Equation 31 and simplifying gives

$$\frac{Kq^2}{\hbar c} = \frac{1}{\gamma} \qquad \text{Equation 35}$$

Alpha has been calculated analytically based on the value of other well-known constants and is given by

$$\alpha = \frac{Kq^2}{\hbar c} \qquad \text{Equation 36}$$

Where *K* is the Coulomb force constant, and *q* is the charge on the electron and the proton.

So

$$\alpha = \frac{1}{\gamma} \qquad \text{Equation 37}$$

From this it is possible to calculate the Actual Velocity and the Relativistic Velocity in the base energy state. The actual orbital velocity is 99.9973372% of *c*. The Actual Velocity for the infinite energy state is *c* itself and so the dynamic range of the Actual Velocity is extremely narrow.

The Relativistic Velocity is the Actual Velocity multiplied by Alpha, the Fine Structure Constant, and is therefore $2\pi R\alpha/T$. Numerically it is 2187309.8 **m/s**, a value that is known as the Bohr velocity as it is the same velocity as the electron in the base energy state of the Bohr model.

But from Equation 17 $\gamma_n = \frac{n}{\alpha}$ so it is necessary to consider why, if $\alpha = \frac{1}{\gamma}$ is a solution $\alpha = \frac{n}{\gamma}$ should also be a solution?

Why n/Alpha is a solution if 1/Alpha is a solution

The atom is stable if the Relativistic velocity is equal to the Bohr velocity. As the Actual Velocity approaches the speed of light this first occurs when Gamma is equal to 1/Alpha. However, the orbital path length is subject to aliasing and therefore has multiple values, all of which are equally valid.

Consequently, the Relativistic Velocity also has multiple values, all of which are equally valid as shown in Table 2. The atom is stable when the Actual Velocity is such that one of these values is true.

n / Gamma	1	2	3	4	5	6	7
Relativistic velocities							
137.036	2187309.8	4374619.5	6561929.3	8749239.1	10936548.8	13123858.6	15311168.4
274.720	1093654.9	2187309.8	3280964.7	4374619.5	5468274.4	6561929.3	7655584.2
411.108	729103.3	1458206.5	2187309.8	2916413.0	3645516.3	4374619.5	5103722.8
548.144	546827.4	1093654.9	1640482.3	2187309.8	2734137.2	3280964.7	3827792.1
685.180	437462.0	874923.9	1312385.9	1749847.8	2187309.8	2624771.7	3062233.7
822.216	364551.6	729103.3	1093654.9	1458206.5	1822758.1	2187309.8	2551861.4
959.252	312472.8	624945.6	937418.5	1249891.3	1562364.1	1874836.9	2187309.8

Table 2 Relativistic Velocities for the Stable States of the Atom

For the atom to be stable the electron must occupy the same position around the orbit at time $t = nT$ or $\tau = n\mathbf{T}$ where $\mathbf{T} = T/\gamma$. The wavelength seen by the orbiting electron decreases as Gamma increases and it first enters stable orbit when Gamma is equal to 137.036.

It can be seen how this influences the stability of the orbiting electron by recognizing from Equation 19 that $\tau = t/\gamma$ and plotting the orbital waveform as γ varies. Figure 20 shows the situation where Gamma is less than 137. In this situation the atom is not stable because, while the electron is overhead at time 9 = 0, it is not overhead at time $t = T/\gamma$.

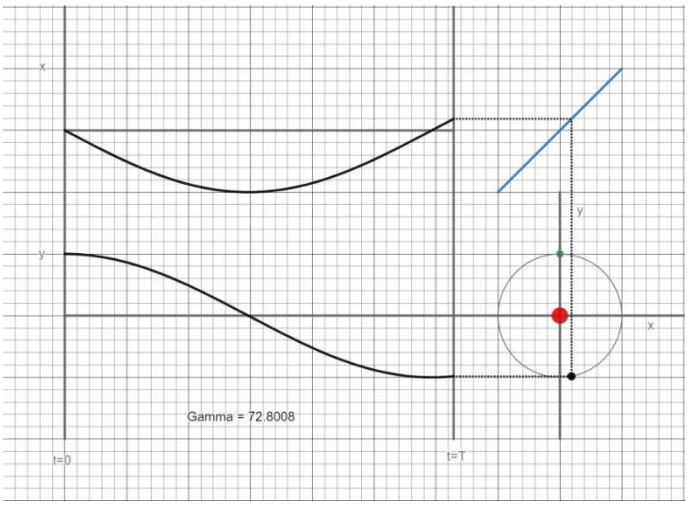

Figure 20 Gamma<137

As the value of Gamma is increased eventually the situation is reached shown in Figure 21. Here Gamma has a value of 137.036 and so the electron is found overhead at each sample instant and therefore the electron is orbiting in a stable state.

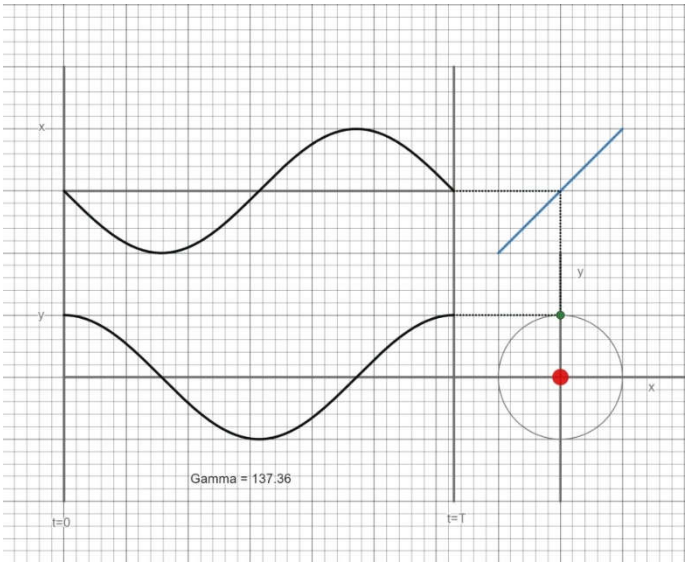

Figure 21 The Base Energy State

A further increase Gamma results in the situation shown in Figure 22 where once again the electron is to be found overhead at time $t = 0$ but not at time $t = T$ resulting in an unstable atom.

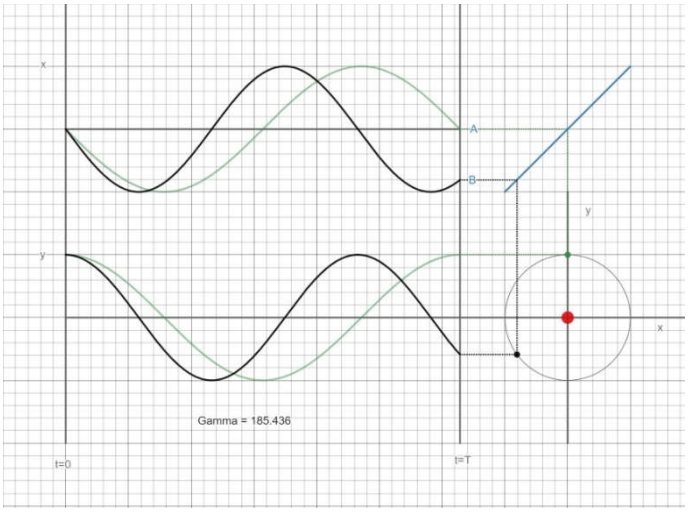

Figure 22 An Unstable State

The situation persists as the value of Gamma increases until Gamma reaches a value of 274.072, shown in Figure 23, at which point the observer located on the nucleus will see the electron directly overhead at both $t = 0$ and $t = T$. The wavelength seen by the electron has been foreshortened by relativity to the extent that two cycles now fit in the space of the wavelength of the base energy state (the sample interval).

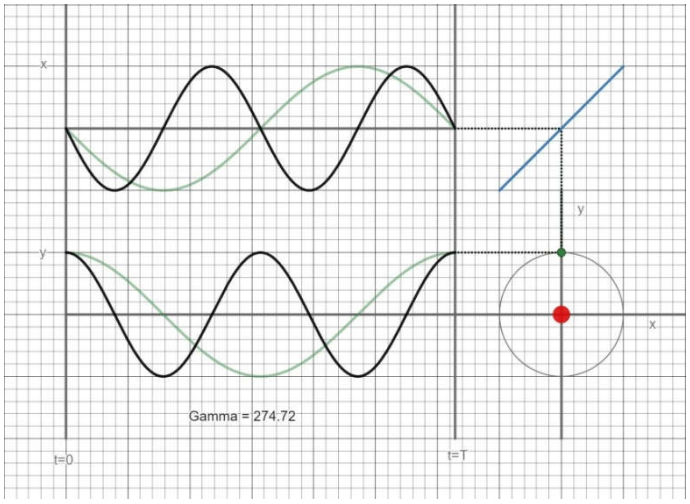

Figure 23 The Second Energy State

As Gamma increases beyond 274.072, the pattern will repeat such that the atom will not be stable until Gamma reaches $3 * 137.036$. The situation for this third energy state is shown in Figure 24, where $\gamma = 411.108$.

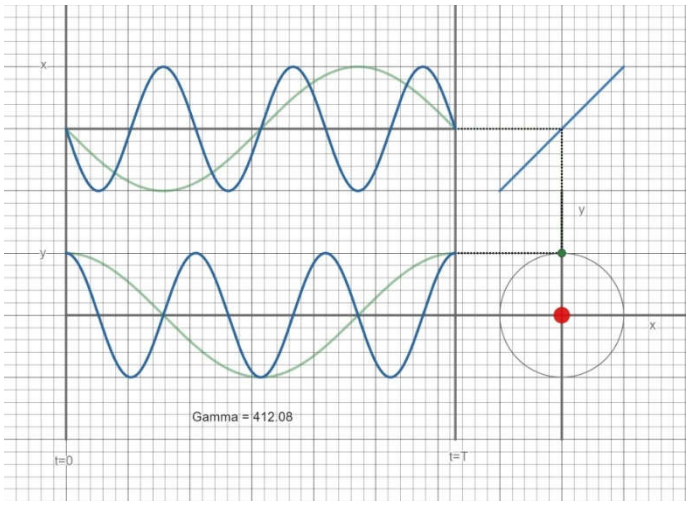

Figure 24 The Third Energy State

https://www.desmos.com/calculator/lgx8g7sffy

The stable states of the atom occur when one of the aliased values for the orbital path length matched against the sample interval results in a Relativistic Velocity equal to the Bohr velocity.

The Infinite Well

The infinite well is a concept used in quantum theory when solving the Schrödinger wave equation to study the behavior of the electron around the atom. Since it is dealing with the de Broglie's waves it has no direct physical interpretation. Here the concepts associated with the infinite well can be developed around the ideas outlined above, but with the essential difference that the electron is seen to be a real particle, the waves are intimately associated with its physical location and so it does have a physical interpretation.

We can best develop the ideas around the 'infinite well' and the relationship between Gamma and the stable states of the atom with a series of three-dimensional graphics. The axes of these are the displacement of the atom in either the *x* or *y* directions, time in the domain of τ, and γ, the Lorentz factor as it applies to the moving electron.

In Figure 25 the Z axis represents the amplitude of the wave; the X axis represents the time in the domain of τ and the Y axis represent the value of Gamma. The resulting surface shows how the wave of the *y* component of the electron's position varies as Gamma varies. The blue trace represents the shape of the waveform when Gamma is equal to 137.036 in the base energy state.

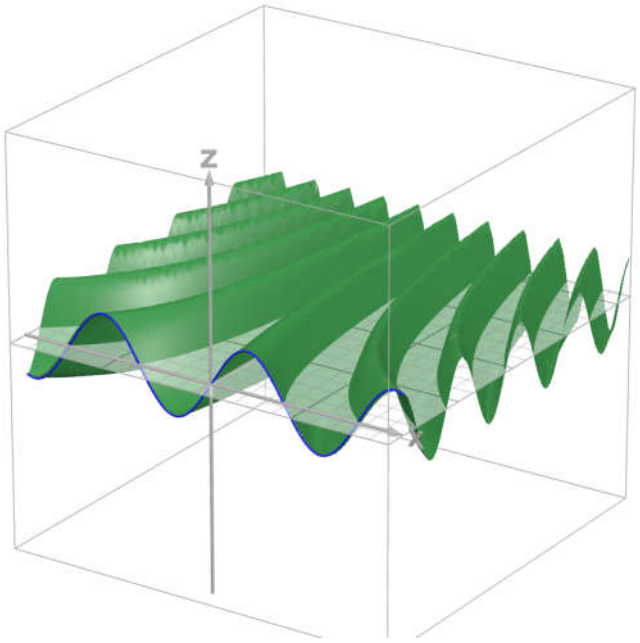

Figure 25 Amplitude as a function of Gamma and time in the domain τ

Since the system is periodic it is sufficient to consider just a single cycle corresponding to one complete orbit in the base state of the atom and since the orbital velocity is constant the X axis can be considered as representing either time or distance. In this case it is the foreshortened distance, or the dilated time as measured in the reference frame of the moving electron.

This means that the surface can be constrained to lie between zero and 2π in the X axis to show just a single orbit of the base energy state; Figure 26.

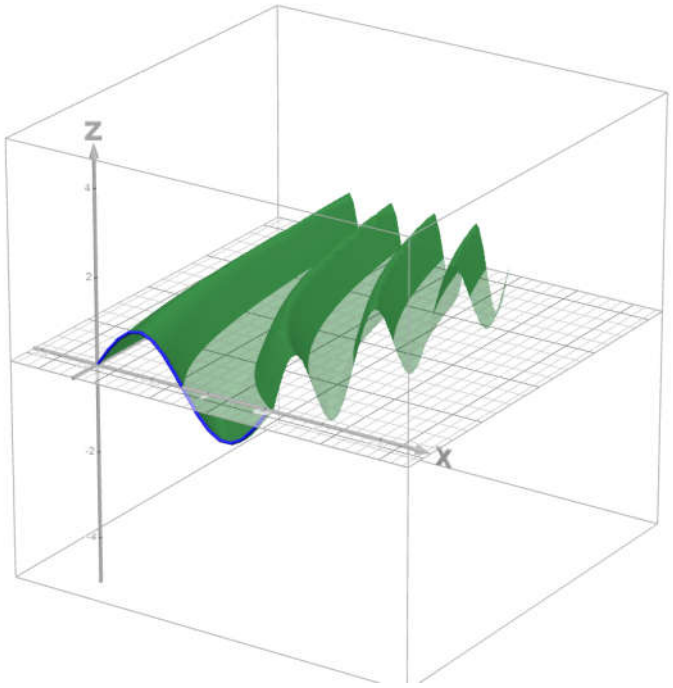

Figure 26 The Infinite Well with the fundamental frequency

The blue sine wave represents the y component of the position for single orbit of the electron as seen by the electron itself, that is with its orbital path length foreshortened by the Lorentz factor with a having a value of 137.036. It is also possible to draw a similar surface representing the x component. The zero crossing of the sine wave at 0 and 2π can be considered as the point at which the electron is directly overhead an observer placed at the atomic nucleus.

The curves of Figure 21 to Figure 24 show how the frequency of the waveform varies as Gamma is increased. When Gamma equals 274.072 exactly 2 wavelengths fit in the Infinite Well representing the second stable state of the atom as shown in Figure 27. This is consistent with the second alias of the orbital path length foreshortened by relativity.

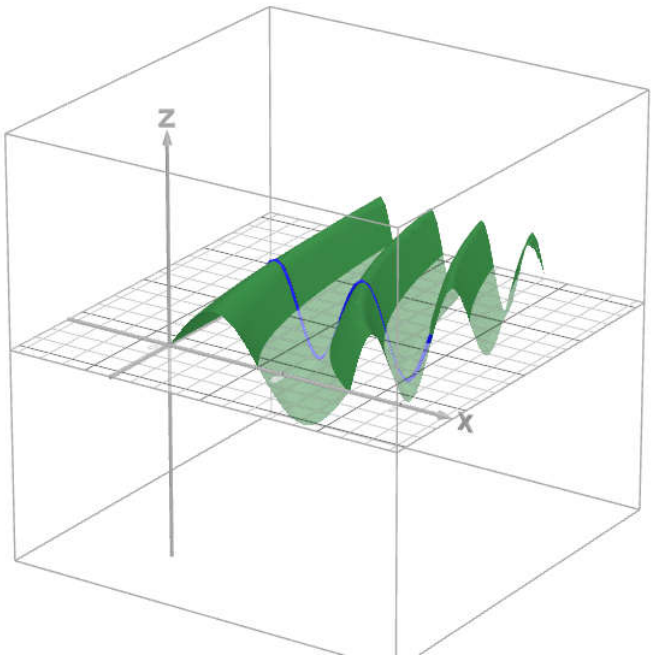

Figure 27 The second harmonic in the Infinite Well

This pattern repeats as Gamma increases further. Figure 28 shows the waveforms for the higher stable states. They occur at intervals of 137.036 in the Gamma axis and each waveform contains a whole number of complete cycles consistent with the electron being overhead at the sample instants. It is these cross sections which form Figure 21 Figure 23 and Figure 24.

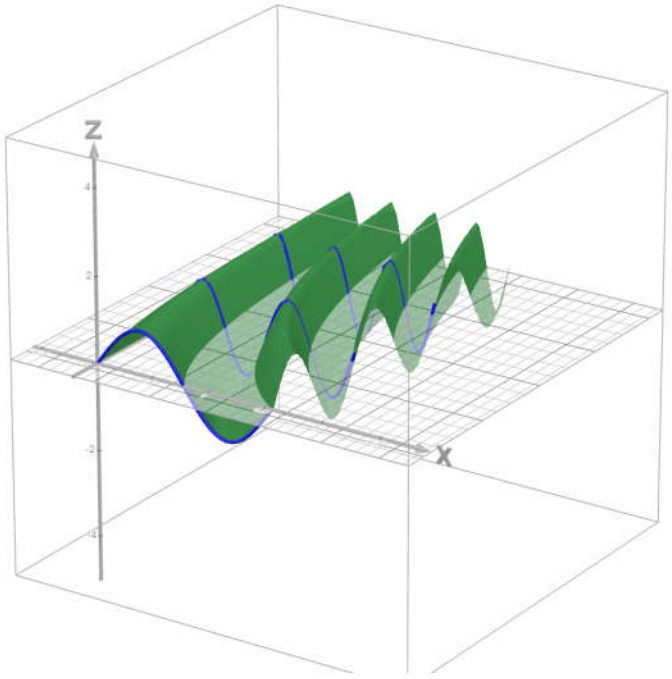

Figure 28 Higher harmonics in the Infinite Well

Figure 29 shows where the surface intersects the boundary of the Infinite Well forming a cosine curve with period 137.036 and whose zero crossings coincide with the point where the electron is directly overhead.

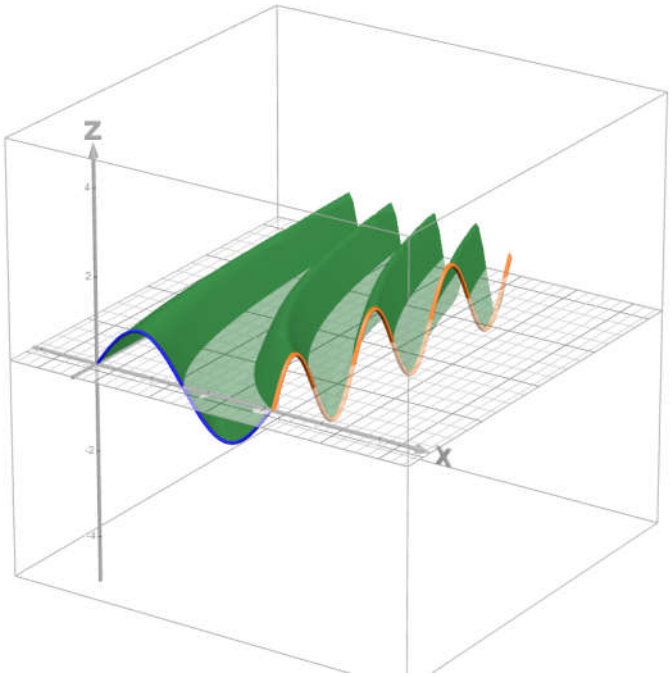

Figure 29 Boundary of the Infinite Well

Overall, this can be thought of rather like the detents of a mechanical gearbox. A very small change in the orbital velocity causes a large change in the value of Gamma. If that change is a multiple of 137.036 then the atom will shift from one energy level to another.

The functions x(τ,γ) and y(τ,γ) together describe the position of the electron in terms of x and y components. These can be combined into a 3D view showing the various energy states in the Infinite Well Figure 30. Here the Well has been extended in the Gamma axis to include the position of the electron in the domain of the stationary observer where Gamma equals one (shown in blue) and to emphasize that the radius is constant in all the different energy states. In this perspective the actual distance around the orbit is preserved.

In Figure 30 the position of the electron is show at some arbitrary time $t = 0$ in each energy state.

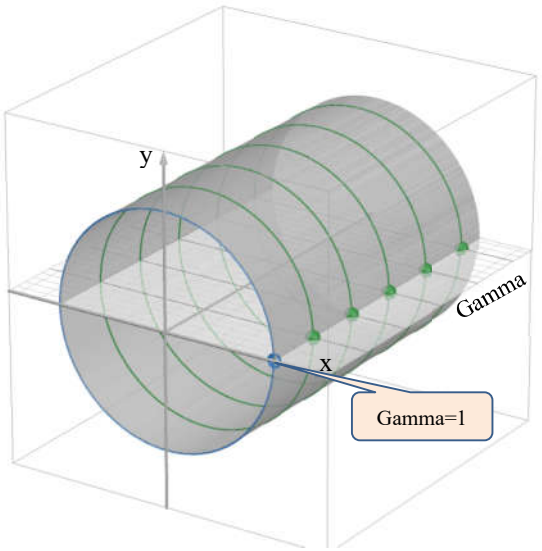

Figure 30 Positions of the Stable States at Time t=0.

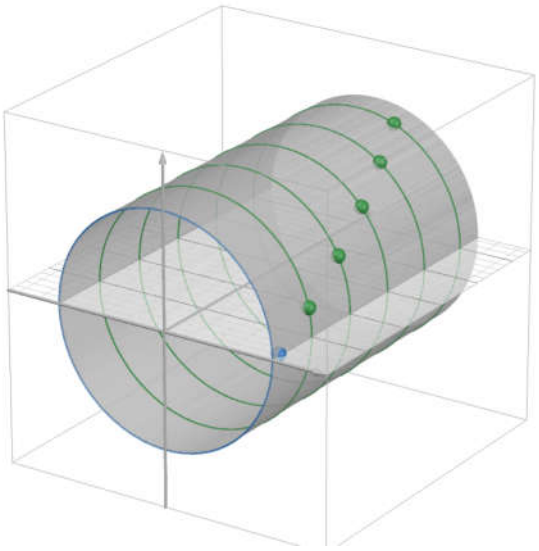

Figure 31 Positions of the Stable States at Time $t = \Delta t$

Figure 31 shows the situation some short interval of time later. A small movement around the orbit of the stationary reference frame produces a larger change in the base state by a factor of 137.036 and in all other states by 137.036*n. At first sight appears that electrons in higher orbits are moving faster than those in lower orbits, but this is not in fact the case, what is happening is that they are travelling shorter distances in the same interval of time, the distance being foreshortened by virtue of relativity, while the time, which is measured in the stationary reference frame is unaltered.

Figure 32 and Figure 33[5] take a slightly different perspective. Here the distance around the orbit is shown as being foreshortened due to relativity as it would be experienced by the moving electron. At time $t = 0$ the electrons are in alignment and remain in alignment over time as measured by a stationary observer.

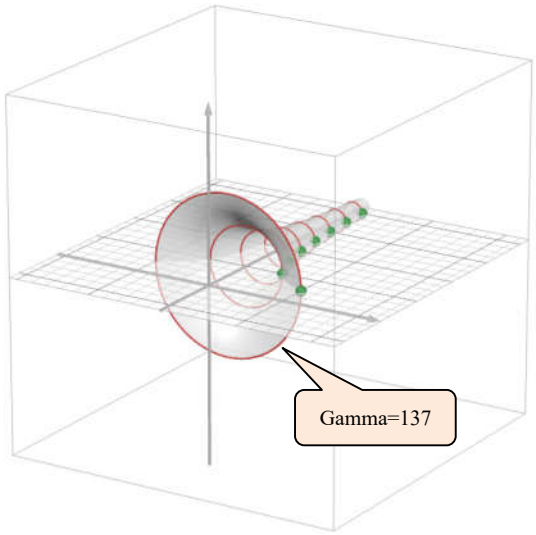

Figure 32 Stable States at Time t=0

[5] Note that the graph has been truncated so as not to show the position of the electron in the reference frame of the stationary user since this would be137 times the diameter of the base energy state.

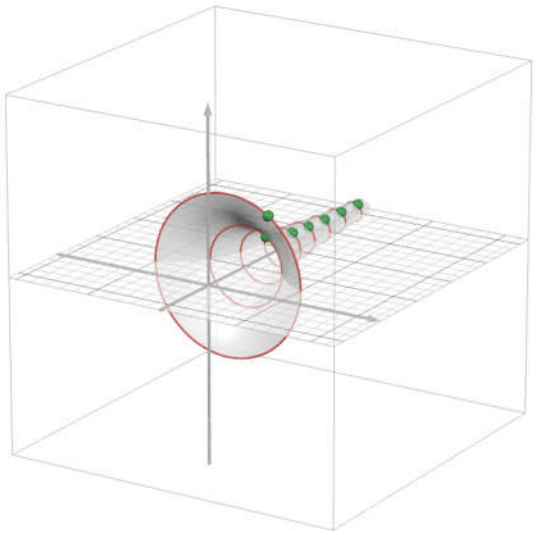

Figure 33 Stable States at Time t=Δy

In total there are four such representations of the interrelationships between the various variables involved in the dynamics of the atom. The hemisphere of Figure 12, the Lambertian projection of Figure 16 , the cylinder of Figure 30 and the trumpet of Figure 32. The transformations between these states are all simple linear transformations.

Transformations

Transformations from the Cylindrical representation to the Lambertian representation can be achieved by pre-multiplying the co-ordinates in the Cylindrical domain by a transformation matrix to obtain the corresponding co-ordinates in the Lambertian domain.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{\sqrt{1-\gamma^2}}{\gamma} \end{bmatrix} * \begin{bmatrix} x_c \\ y_c \\ g_c \end{bmatrix} \xrightarrow{yields} \begin{bmatrix} y_L \\ x_L \\ h_L \end{bmatrix}$$

To transform from the Lambertian to the Cylinder is achieved using the inverse matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{\sqrt{1-h^2}} \end{bmatrix} * \begin{bmatrix} x_L \\ y_L \\ h_L \end{bmatrix} \xrightarrow{yields} \begin{bmatrix} y_c \\ x_c \\ h_c \end{bmatrix}$$

To transform the Hemisphere into the Cylinder

$$\begin{bmatrix} \gamma & 0 & 0 \\ 0 & \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x_H \\ y_H \\ h_H \end{bmatrix} \xrightarrow{yields} \begin{bmatrix} y_c \\ x_c \\ g_c \end{bmatrix}$$

And its inverse

$$\begin{bmatrix} \frac{1}{\gamma} & 0 & 0 \\ 0 & \frac{1}{\gamma} & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x_c \\ y_c \\ h_c \end{bmatrix} \xrightarrow{yields} \begin{bmatrix} y_H \\ x_H \\ h_H \end{bmatrix}$$

These transformations are orthogonal which means they can be combined in any order to perform a transformation between any two of the domains mentioned.

Duality, Waves and Particles

Special relativity is unique among physical phenomena in seemingly providing two answers to the same question, a sort of natural duality. So for example there are two distances between points in space, one measured by a stationary observer and one by a moving observer, equally there are two time intervals and, where cyclic behavior is observed there are two frequencies and two wavelengths. Special relativity addresses the problem of dual solutions outlined by de Broglie directly and would therefore seem to be a natural place to start when looking for any form of duality.

In any reference frame there are a set of measurements such as distance, time, mass etc. which are those experienced by a stationary observer. For an object which is moving with respect to that reference frame those same measurements have different values which are related to the first through relativity. Distance is foreshortened; time is dilated, and mass is increased. For objects which are moving relatively slowly these two sets of measurements are very similar. At higher speeds these two sets of measurements begin to diverge quite markedly. A particularly significant point of divergence occurs when the speed is such that the Lorentz factor, Gamma, has a value of 137.036 or a multiple thereof. This particular speed is associated with the stable orbits of electrons within the hydrogen atom. For objects in orbit certain of these measurements take on multiple values all of which are valid at the same time, and it is this that leads to the discrete energy levels that are found within the atom.

In all this the wave characteristics of the electron derive directly from its orbital motion. The wave is an attribute of the particle, brought about by its motion rather than something which is integral to the particle. Whether it is viewed from the perspective of the stationary observer or from that of the moving electron, the relationship between the wavelength, frequency and velocity is consistent with classical theory within its respective reference frame.

For the stationary observer:

$$\omega = \frac{c}{R}$$

$$\lambda = \frac{c}{T}$$

For the moving electron

$$\Omega = \frac{cn}{R\alpha}$$

$$\Lambda = \frac{c\alpha}{Tn}$$

It is more sensible therefore to speak of a wave/particle identity in which frequency, wavelength, amplitude and phase are all related within their respective reference frames with orbital radius, circumference and period in this conventional way. The duality exists between the two reference frames, hence there are two frequencies, one in the reference frame of the electron and one in the reference frame of the stationary observer and there are two orbital path lengths, one in the reference frame of the electron and one in the reference frame of the stationary observer. And, it is argued here, there are two velocities. It is better to describe this as a wave duality and a separate but related particle duality.

Wave Equations

When Erwin Schrödinger developed his wave equation he was forced to base it around a travelling wave. This is because the notion of de Broglie's waves is that they travel around the orbit of the electron and that it is only at certain frequencies that the wavelengths of these waves add up to the orbital circumference and become standing waves. These standing waves correspond to the stable energy states of the atom. Schrödinger is thus forced to adopt a canonical form for his equation which includes two partial derivatives, one for time and one for distance, rather than the one derivative term needed here. This necessarily complicates the Schrödinger equation, unlike here, where the orbital motion can be described as simple harmonic oscillator in which the distance is the displacement with respect to some mean value.

Schrödinger faced a second constraint and that is that he assumed that changes in the energy level of the atom were accompanied by a change in potential energy of the electron, in effect acknowledging the existence of the quantum leap. This forced Schrödinger to postulate a solution to his equation which can only be described in the complex plane. The measurement problem then arises as physicists try to coerce this complex equation to have real solutions. The electron is said to exist in a state of superposition, where its exact position and momentum not only cannot be determined but does not even exist. When observed however the electron 'collapses' from this indeterminate state as a wave front to exist as a real particle. The so-called Measurement Problem is a question of how and whether the wave front collapse occurs. The question defies rational explanation and has confounded both physicists and philosophers alike.

With the introduction of Relativistic Velocity, the need for a quantum leap disappears. The electron is seen as an idealized entity with defined mass and no physical extent confined to a precise physical location in space at any given moment . The electron orbits at a constant radius, irrespective of the energy level and consequently there is no change in potential energy between the various energy states, only the kinetic energy of the electron changes. The result is a simple planetary model for the electron, albeit one which involves relativity. This greatly simplifies any wave equation used to describe the position of the electron as a function of time. There is no need to introduce the idea of a travelling wave and no change in orbital radius. The equation is that of a simple undamped second order system.

This is readily solved by first postulating a solution of the form $x = R\cos(\omega t)$ from which

$$\frac{dy}{dx} = -R\omega \sin(\omega t) \qquad \text{Equation 38}$$

And

$$\frac{d^2x}{dt^2} = -R\,\omega^2 \cos(\omega t) \qquad \text{Equation 39}$$

Hence

$$x = R\cos(\omega t) \qquad \text{Equation 40}$$

And the resulting equation is

$$x = -\frac{R}{\omega^2}\frac{d^2y}{dx^2}$$ Equation 41

From the force balance equation

$$\frac{Kq^2}{R^2} = \frac{mv^2}{R\gamma}$$ Equation 42

From this an expression for v^2 can be developed

$$v^2 = \frac{Kq^2\gamma}{Rm}$$ Equation 43

Multiplying both sides by $1/c^2$

$$\frac{v^2}{c^2} = \frac{Kq^2\gamma}{Rmc^2}$$ Equation 44

Recognizing that $\hbar = mRc$ and that $\alpha = \frac{Kq^2}{\hbar c}$

$$\frac{v^2}{c^2} = \alpha\gamma$$ Equation 45

From the Lorentz formula

$$\frac{v^2}{c^2} = \frac{\gamma^2 - 1}{\gamma^2}$$ Equation 46

Eliminating the v^2/c^2term between Equation 45 and Equation 46 gives

$$\alpha\gamma^3 - \gamma^2 + 1 = 0$$ Equation 47

The numerical value[6] for α is 7.2973525693x10^{-3}. Substituting this and calculating the three roots for $n = 1$ gives:

γ = 137.028700954
γ = -0.99638422226
γ =1.00368235217

Of the three roots, the first is the one corresponding to the orbit of the electron. However, it falls slightly short of the value of $1/\alpha$ necessary for it to complete one orbit during the orbital period measured in the stationary reference frame. The reason for this is that the electron is travelling at slightly less than the speed of light. To a first approximation the velocity of the electron is given by

$$\frac{v}{c} = \sqrt{\frac{\gamma^2 - 1}{\gamma^2}} = 0.999973373968267$$ Equation 48

The situation is shown in Figure 34 this shows a graph of the cubic Equation 47 where the inset shows that the root does not exactly correspond with the value $1/\alpha$ on the x axis.

[6] CODATA - http://physics.nist.gov/cgi-bin/cuu/Value?alpha

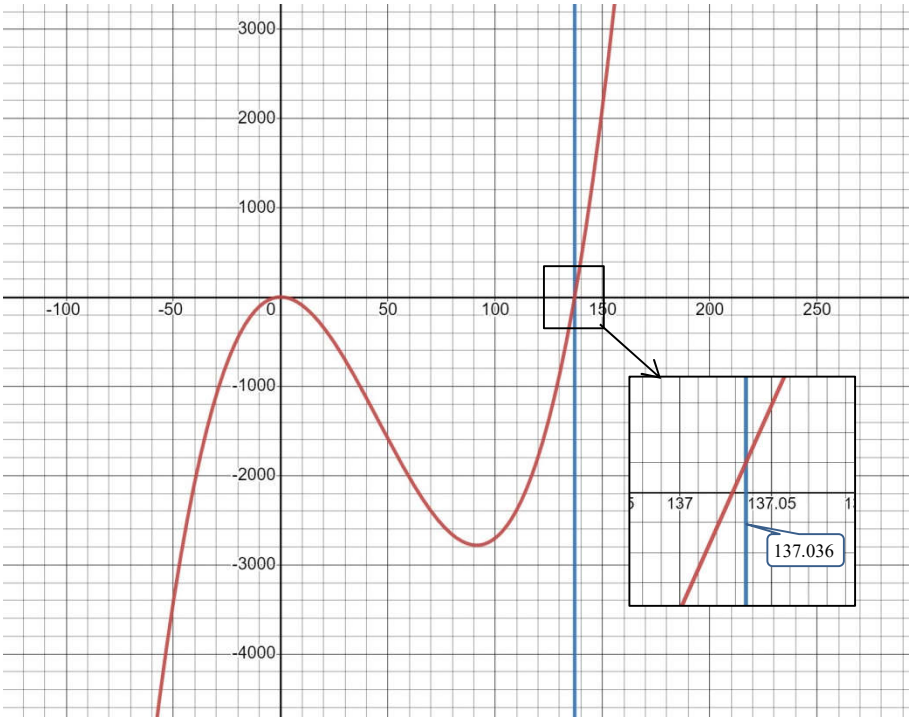

Figure 34 Graph of Cubic

The reason for this discrepancy concerns the value of Alpha, the fine structure constant. Alpha is a theoretical value predicated on the assumption that the election is travelling at exactly light speed when in fact it is travelling at slightly less than light speed. Consequently, using the reciprocal of Alpha as the scale factor of the orbital circumference leads to this small shortfall. We can correct for this by increasing the speed by a small amount. Such a change will of necessity change the value of Gamma which will far outweigh any change in the speed itself.

There are a number of ways in which to calculate this adjustment, but by far the simplest is iteration. Using Alpha and 1/Gamma as starting values, it is possible to calculate the error and apply this to obtain and adjusted value for Gamma and from this an adjusted value for Alpha. The process is convergent.

$Alpha$ =0.0072973525693
$Gamma$ = 137.035999083696

	Root of Cubic	Shortfall	Adjusted Gamma	Adjusted Alpha
Gamma 1	137.028700917629	0.00729816606648	137.043297249762	0.00729696395277
Gamma 2	137.035999508603	-0.00000042490720	137.043296824855	0.00729696397539
Gamma 3	137.035999083671	0.00000000002481	137.043296824880	0.00729696397539
Gamma 4	137.035999083696	0.00000000000000	137.043296824881	0.00729696397539

Table 3 Iterative Steps to Calculate Adjusted Gamma

From this the adjusted orbital velocity in the base state using Equation 46 is $v/c = 0.99997337680397$

Thus far the cubic Equation 47 deals with only the base energy state. All the other energy states can be dealt with by substituting n/α in place of $1/\alpha$ as the coefficient of the cube term and repeating the iterations for each energy state.

From Equation 45

$$\frac{v^2}{c^2} = \alpha\gamma = 0.99997337680397^2 = 0.99994675431674 \qquad \text{Equation 49}$$

Hence, the corrected equation to take account of the fact that $v < c$

$$\gamma = \frac{1}{\alpha}0.99997337680397^2 = \frac{1}{\alpha} * 0.99994675431674 \qquad \text{Equation 50}$$

Figure 35 shows the family of curves plotted for the first ten values of *n*. Where the curves each intersect the x axis at an integer multiple of $1/\alpha$.

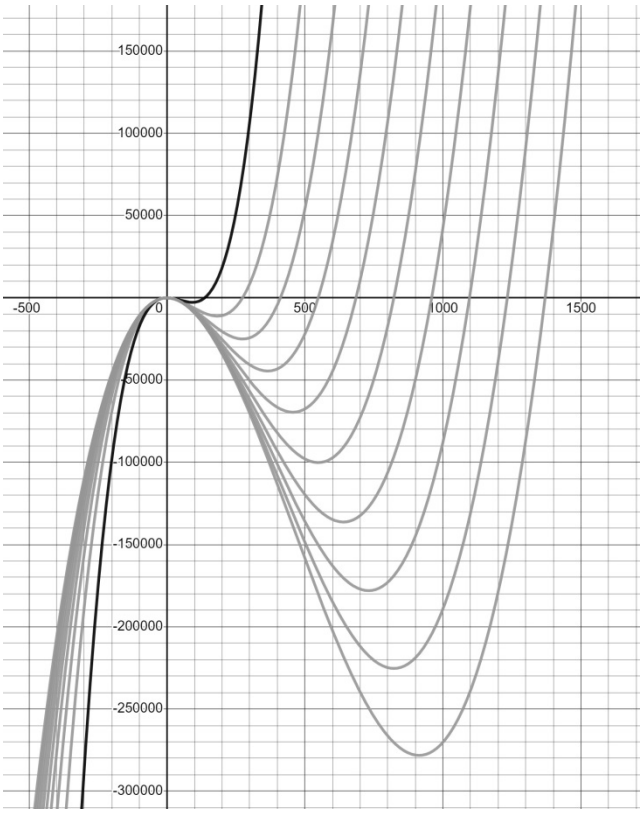

Figure 35 Graph of $y = \frac{\alpha}{n}\gamma^3 - \gamma^2 + 1$

From Equation 48 it is possible to calculate the orbital velocity in each energy state and from this the orbital frequency, the energy and the energy potential in each state as show in Table 4.

n	$v_{n\,/c}$	γ_n	ω_n	Energy eV	Energy Potential eV
1	0.9999733776803	137.0432965301540	7.7632391423E+20	255485.925	13.607
2	0.9999933437358	274.075646892332	7.7633941477E+20	255496.130	3.402
3	0.9999970416220	411.110429716811	7.7634228560E+20	255498.020	1.512
4	0.9999983359049	548.145820680228	7.7634329041E+20	255498.682	0.850
5	0.9999989349769	685.181454892904	7.7634375550E+20	255498.988	0.544
6	0.9999992603998	822.217210731314	7.7634400814E+20	255499.154	0.378
7	0.9999994566199	959.253036067305	7.7634416047E+20	255499.255	0.278
∞	1.0000000000000	0.000000000	7.763451838E+20	255499.532	0.000

Table 4 Orbital velocity, frequency and energy

The orbital velocity for $n = 1$ is 99.997337680397% of c and that for the infinite energy state is c itself, which means the dynamic range of orbital velocity is extremely small. This, together with the fact that the orbital radius is constant means that the morphology of the atom remains substantially the same for all energy states. This is important because the physical and chemical properties of the atom are independent of the energy state, and this is not likely to be the case if the morphology of the atom was significantly different from state to state.

Balance of Forces

As the Actual Velocity increases, in a very small dynamic range close to the speed of light, the Relativistic Velocity decreases. This causes the centrifugal force to decrease, the result is that the centrifugal force varies as the inverse of Gamma. In addition to this reduction in force with Gamma, the centrifugal force is subject to aliasing caused by sampling. Figure 36 shows a plot the family of curves of force against Gamma for the first 10 states together with a line representing the electrical force, which is not affected by Gamma. Each point where the curves intersect the line represents a stable state of the atom.

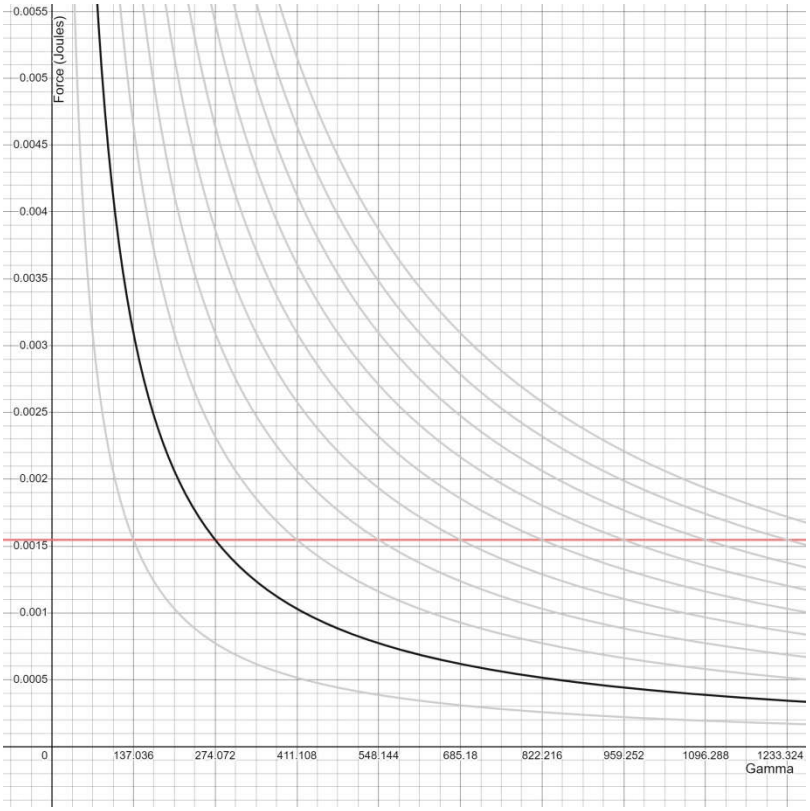

Figure 36 Family of Curves of Force vs Gamma

The vertical separation between these curves gives an indication of the force necessary to cause a change in energy level. From this it will be seen that the force required to cause an increase in energy level is always more than the force required to cause a decay between any pair of states.

It follows that if there are two separate but related time domains that it is possible to write a second wave equation in the domain of τ; that is the domain of time in the reference frame of the moving electron.

$$\tau = \gamma t$$

Frequency is multiplied by Gamma in the reference frame of the electron and $\gamma = \frac{n}{\alpha}$ in the stable states of the atom. The orbital frequency in the stationary reference frame is ω and so the orbital frequency in the electron's reference frame is $\frac{\omega n}{\alpha}$. It is thus possible to write a second order differential equation for an undamped system using the canonical form:

$$\frac{d^2x}{d\tau^2} = -\frac{R}{\omega^2 n^2}x \qquad \text{Equation 51}$$

And hence

$$x = R \sin(\omega n \tau) \qquad \text{Equation 52}$$

Or

$$x = R \sin\left(\frac{\omega n}{\alpha} t\right) \qquad \text{Equation 53}$$

Probability

While it is not possible to determine the exact position of the electron owing to the ambiguity caused by the sampling process, it is possible to determine the probability that the electron will be at a particular place, however such a probability may depend on the energy state of the atom. There are three aspects to the probable location of the electron in its orbit. First is the radial probability; that is the likelihood of finding the electron at a particular radius. The second is the amplitude probability; that is the likelihood of finding the amplitude of the electrons *x* or *y* displacement at a particular value. Finally, there is the angular or phase probability; that is the likelihood of finding the electron at a particular position around its orbit.

Radial probability

Since the electron is orbiting at a constant radius across all energy states, it follows that the radial probability is zero everywhere along the radial except where $R = \hbar/mc$ where it will have a value of one. Radial probability is independent of the energy level of the atom as shown in Figure 37

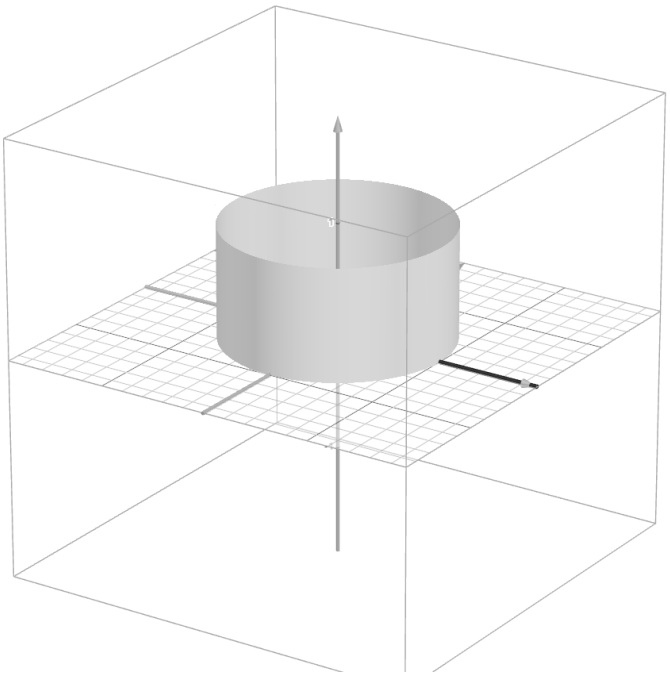

Figure 37 Radial Probability

Amplitude probability

The electron is following one of an infinite number of possible paths which means that during one orbital period it may be considered as having completed some unknown number of orbits around the nucleus as shown in.

However, the probability that the electron will have a particular amplitude is independent of the orbital frequency and depends solely on the shape of the wave, which in this case is a pure sin wave, Figure 38 . The orbital velocity of the electron is constant in any stable state and so the probability of finding it at any particular point along the orbital circumference is $\frac{1}{2\pi}$.

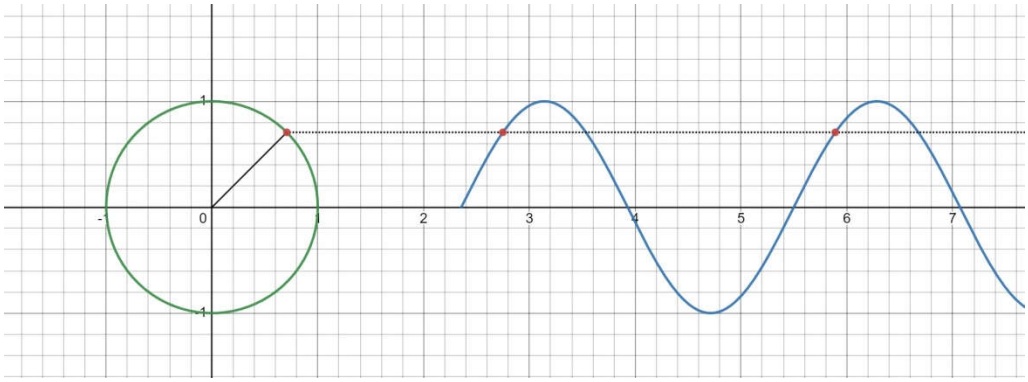

Figure 38 x=cos (ωt)

The calculation can be simplified somewhat by considering just half of the orbit since the other half will have the same probability. The radius can also be normalized to unity. Taking any one of the sinusoids, in this case the displacement in the y axis, as shown in Figure 38 the probability that the amplitude has a particular value can be found.

When viewed as a wave from the direction of the x axis for example, the change in amplitude is faster when close to the axis than near the peak of the of the wave and so the electron is more likely to be found at or near the peak rather than near the axis.

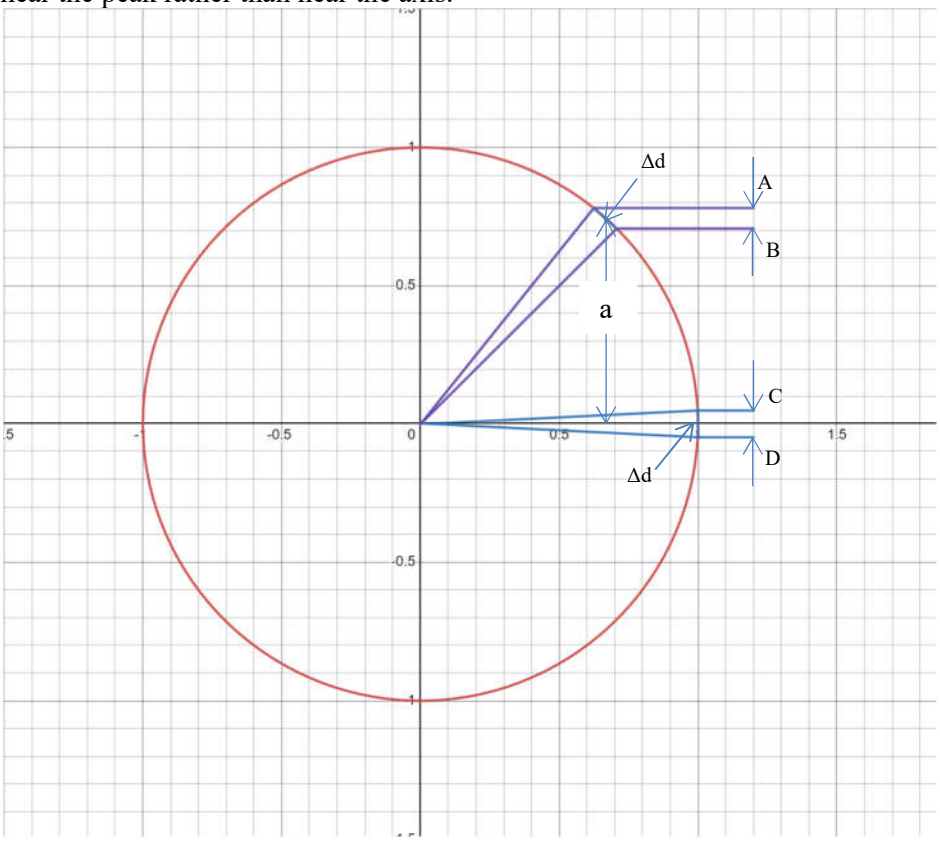

Figure 39 Amplitide probability

In any particular energy state, the electron is travelling at constant velocity, which means that the probability distribution of its position over half of the orbit is uniform.

$$p(\theta) = \frac{1}{\pi}$$

For $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

During a short interval Δt the electron will have travelled a short disance Δd but the change in amplitude is much less as Theta gets closer to 90 degrees. That is $AB < CD$ in Figure 39. The situation is shown in more detail in Figure 40. The figure shows that the extent of this change in probability is related to the angle Theta and varies as the reciprocal of the cosine of Theta.

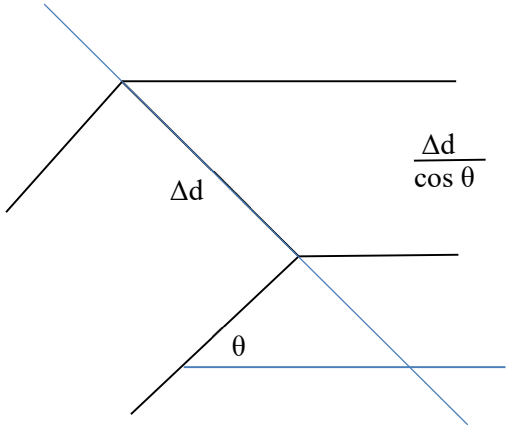

Figure 40 Enlarged view

From this the probability of the amplitude being equal to *a* is given as Δd tends to zero and can be written:

$$P(a) = \frac{1}{\pi \cos\theta}$$ Equation 54

For $-1 < a < 1$ or $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

But from Figure 39

$$\mathrm{a} = \sin\theta$$ Equation 55

And

$$\cos\theta = \sqrt{1 - \sin^2\theta}$$ Equation 56

And so

$$P(a) = \frac{1}{\pi\sqrt{1 - a^2}}$$ Equation 57

We can carry out a similar calculation for P(b) to derive the amplitude probability in the *x* axis to obtain

$$P(b) = \frac{1}{\pi\sqrt{1 - b^2}}$$ Equation 58

But from Pythagoras

$$b^2 = 1 - a^2$$ Equation 59

And so

$$p(b) = \frac{1}{\pm\sqrt{a^2}}$$ Equation 60

Since probability can only ever be positive, only the positive value of the square root is of concern

$$p(b) = \frac{1}{\pi|a|}$$ Equation 61

Similarly

$$p(a) = \frac{1}{\pi|b|}$$ Equation 62

Thisd probability density function is shown in Figure 41

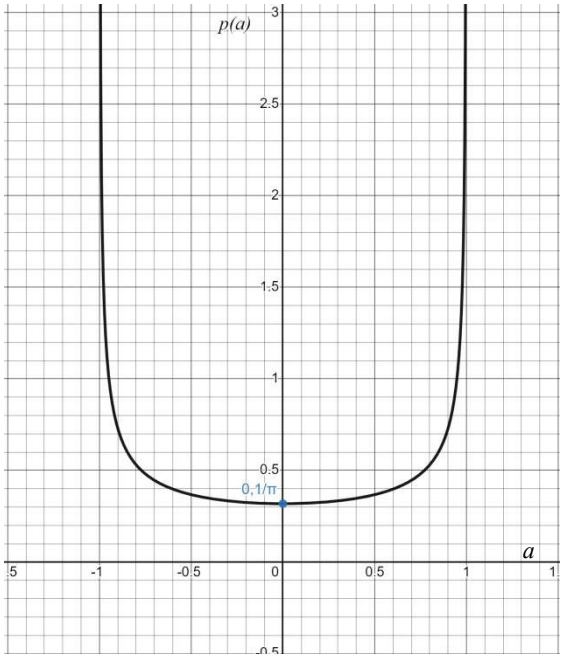

Figure 41 Amplitude Probability Density Function

The probability of finding the amplitude to be zero is $1/\pi$ while the probability of finding it to be *R* is infinite. At first sight this would appear to be impossible, however the time it spends with a value of *R* is infinitesimal and what matters is that the total probability is equal to unity[7]. That is to say that

$$\int_{-1}^{1} \frac{1}{\pi\sqrt{1-a^2}}\,dx = 1$$ Equation 63

Which is in fact the case.

Angular or phase probability

This is the likelihood that the electron will be found at a particular place around its orbit. It is important to understand that this can refer to either the *x* component or the *y* component of the electrons position and not to its position in a polar plot.

[7] The mathematics treats the electron as having zero size and so it is possible to consider such an electron having infinite probability of being at radius R for zero time. However, in the real world will have a finite radius which would alter this curve slightly.

The x component of the electron's position with τ is given by:

$$x = R\cos\omega\tau \qquad \text{Equation 64}$$

And the y component is given by

$$y = R\sin\omega\tau \qquad \text{Equation 65}$$

However, the radius can be normalized to one unit of $R = \hbar/mc$.

Born's probability rule can be invoked to find the probability density function. In its simplest form, this states that the probability density of finding a system is proportional to the square of the amplitude of the system's wave function at that state. The Born rule is most often quoted as

$$p(\Psi) = |\Psi^2| \qquad \text{Equation 66}$$

Where ψ is the wave function.

Here the wave function is either the x component of the electron's position with time τ or the y component of the electron's position with time τ and so the wave function and its corresponding probability density functions for the x and y in base energy state are as shown in Figure 42 and Figure 43 respectively.

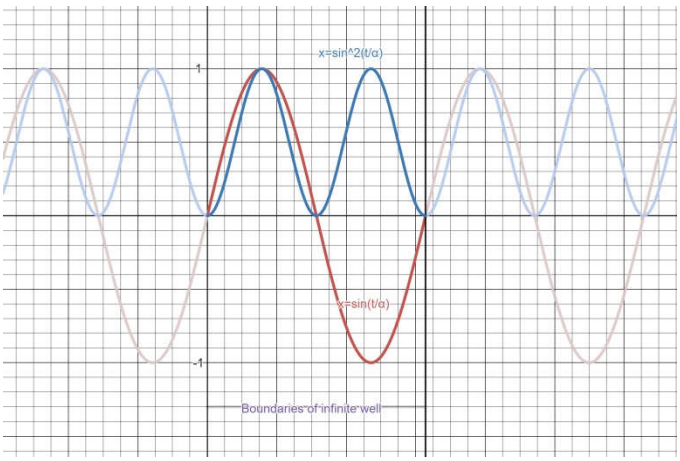

Figure 42 Probability density y component

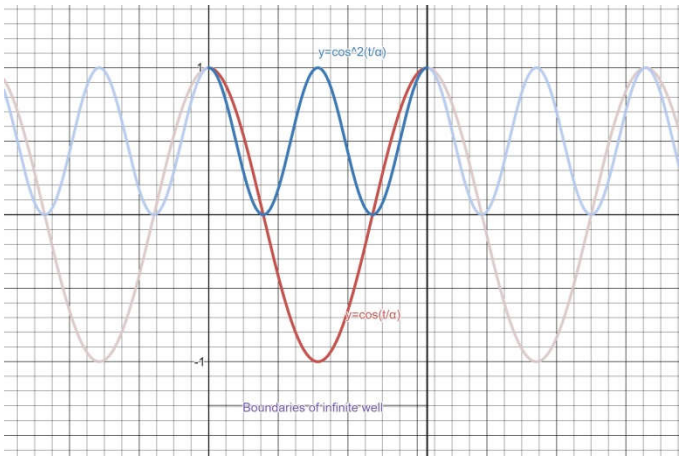

Figure 43 Probability density xy component

It should be noted that since both *x* and *y* are real variables, the squared term means that the modulus is not necessary since the square of a real number is always going to be positive, irrespective of whether it is positive or negative.

This exercise can be repeated for the second and subsequent energy states and is shown in Figure 44

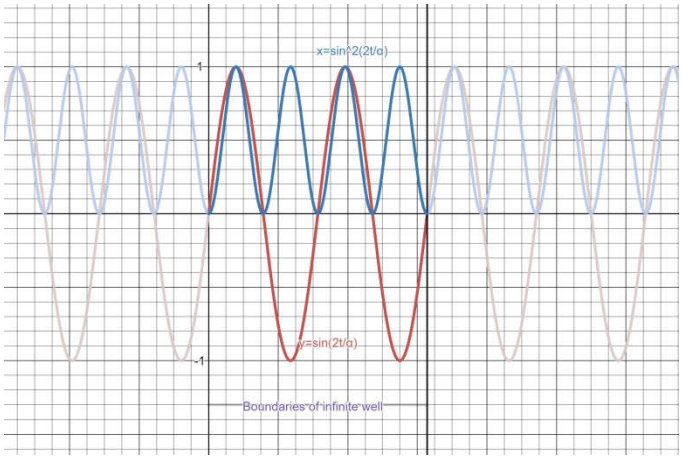

Figure 44 Probability density for the second energy state

The probability density function is cyclic and so it can be wrapped around the orbit for *x* and *y* components in the base state as shown in Figure 45 and Figure 46 respectively. This gives a combined view of the radial and angular probability functions.

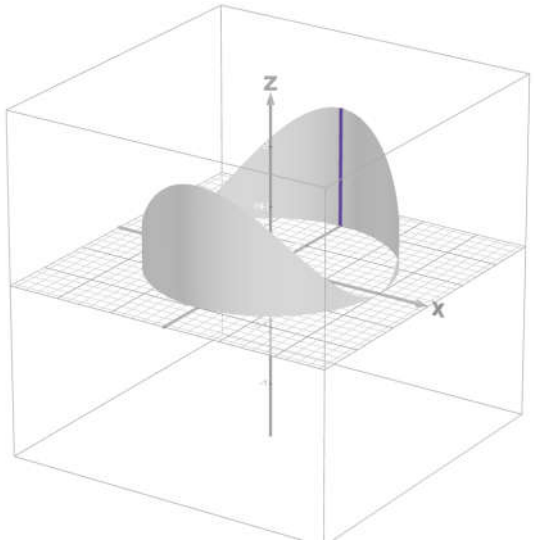

Figure 45 Probability density *x(τ)* n=1

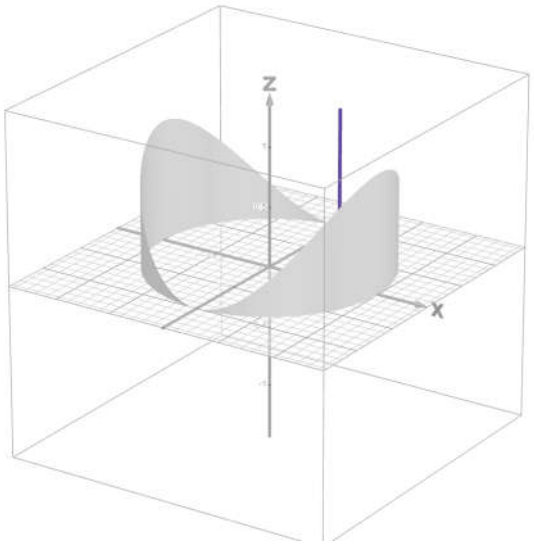

Figure 46 Probability density *y(τ)* n=1

We can do this for higher energy states to obtain Figure 47 for the second energy state.

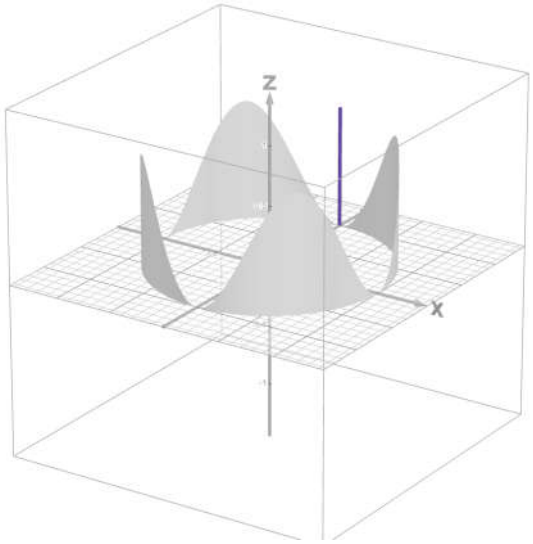

Figure 47 Probability density for $y(\tau)$ $n = 2$

https://www.desmos.com/3d/zlx284ezyu

Note that for each energy level the sum of its x and y probability density functions is a unit cylinder with radius *R*, consistent with the electron having a fixed orbital radius as per Figure 37

The Absence of Synchrotron Radiation

When an electrically charged particle, such as an electron or an ionized atom follows a curved path, it emits a type of radiation called synchrotron radiation. This is the expected norm, but the electron orbiting the hydrogen nucleus does not do so, despite it following a circular path. For an isolated particle, such as an atomic nucleus or even an isolated electron, the curved path occurs because of interactions between the particle and the surrounding atoms. In the case of the atom, however, the forces involved act within the atom itself.

The dynamics within the atom are different to those of an atom interacting with other atoms, the laws are the same, but the circumstances are different. The electron within the atom is orbiting at near light speed and so subject to the effects of relativity. We have argued that the atom is stable if the effective orbital velocity is seen to be affected by relativity and scaled by a factor of 1/Gamma and it is the

presence of this Gamma term in the denominator of the term for centrifugal force which leads to the absence of synchrotron radiation.

The forces acting on the electron are the electrical force of attraction and the centrifugal force. These must be equal for the electron to be in balance. Consider what happens if there is a small change in the orbital radius while the orbital angular velocity remains constant as shown in Figure 48

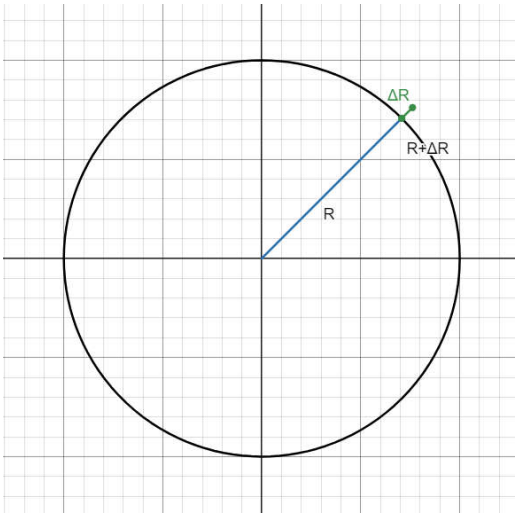

Figure 48 A small change in orbital radius

From Equation 31 the total force acting on the electron is given by

$$f = \frac{m\omega^2 R}{\gamma} - \frac{Kq^2}{R^2}$$ Equation 67

However, Gamma is a function of both R and ω. And when this is correctly inserted into the equation for the total force acting on the electron becomes

$$f = \frac{m\omega^2 R\sqrt{c^2 - \omega^2 R^2}}{c} - \frac{Kq^2}{R^2}$$ Equation 68

A small change in the orbital radius R at constant angular velocity will lead to a corresponding change in the velocity term ωR this in turn will lead to a change in the value of Gamma. Gamma is extremely sensitive to changes in the velocity and so this will lead to a relatively large change in the value of Gamma and, since Gamma is in the denominator of the term for centrifugal force, this will in turn lead to a large change in the centrifugal force. The change is such that if the radius is increased then the centrifugal force is reduced and vice versa. This change in the value of centrifugal force far outweighs any change in the electrical force due to the small change in radius and so the result is that any deviation from the orbital radius results in a strong force acting to restore the orbital radius of the electron to its equilibrium value.

We can assess the extent of this force by calculating the partial derivative of the force in Equation 68 with respect to R while the angular velocity remains constant.

The first step is to recognize that

$f = A + B$

Where $A = \frac{m\omega^2}{c} R\sqrt{c^2 - \omega^2 R^2}$ and $B = -\frac{Kq^2}{R^2}$ and so $\frac{\partial f}{\partial R} = \frac{\partial A}{\partial R} + \frac{\partial B}{\partial R}$

$$\frac{\partial B}{\partial R} = \frac{2Kq^2}{R^3}$$ Equation 69

To find the partial derivative of A with respect to R the chain rule can be invoked $\frac{\partial f}{\partial R} = \frac{\partial f}{\partial P}\frac{\partial P}{\partial R}$

$$A = \frac{m\omega^2}{c}(R^2c^2 - \omega^2R^4)^{\frac{1}{2}}$$ Equation 70

Let $P = R^2c^2 - \omega^2R^4$

Then $A = \frac{m\omega^2}{c}P^{\frac{1}{2}}$

So $\frac{\partial A}{\partial P} = \frac{P^{-\frac{1}{2}}}{2} = \frac{m\omega^2}{c}\frac{1}{2R\sqrt{c^2-\omega^2R^2}}$

And $\frac{\partial P}{\partial R} = 2(c^2R - 2\omega^2R^3) = 2R(c^2 - 2\omega^2R^2)$

But $\frac{\partial A}{\partial R} = \frac{\partial A}{\partial P}\frac{\partial P}{\partial R}$

Hence

$$\frac{\partial A}{\partial R} = \frac{m\omega^2}{c}\frac{\mathrm{c}^2 - 2\omega^2\mathrm{R}^2}{\sqrt{c^2 - \omega^2R^2}}$$ Equation 71

And for the total residual force Equation 69 and Equation 71 can be combined to give

$$\frac{\partial f}{\partial R} = \frac{m\omega^2}{c}\frac{(\mathrm{c}^2 - 2\omega^2\mathrm{R}^2)}{\sqrt{c^2 - \omega^2R^2}} + \frac{2Kq^2}{R^3}$$ Equation 72

However, since $\omega^2R^2 \approx c^2$ this can be substituted in Equation 72 to give

$$\frac{\partial f}{\partial R} = -m\omega^2\frac{c}{\sqrt{c^2 - \omega^2R^2}} + \frac{2Kq^2}{R^3}$$ Equation 73

$$\frac{\partial f}{\partial R} = -m\omega^2\gamma + \frac{2Kq^2}{R^3}$$ Equation 74

The size of this rate of change of force with orbital radius at the orbital radius is thus

$\frac{\partial f}{\partial R} = -7.524 * 10^{13}$ **N/m**

The negative sign indicates that the residual force always acts to drive the electron back to orbital equilibrium.

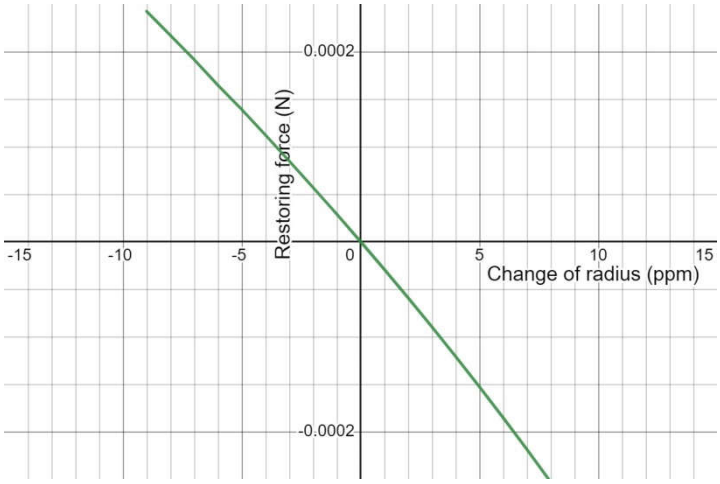

Figure 49 Restoring force vs Change of Radius

The situation is shown graphically in Figure 49 which plots the restoring force in the region of the orbital radius. Compared to the mass of the electron and to the electrical and centrifugal forces when these are in balance, this is a huge force. The change in the total restoring force is some 7000 times larger than the change in the electrical force alone. A change of just 80 ppm in the orbital radius would lead to a doubling of the centrifugal force if that change were inwards and to a complete elimination of the centrifugal force if it were in the other direction. To all intents and purposes the electron is orbiting around a solid surface, which it cannot penetrate and from which it cannot deviate.

The above calculation is for the base energy state where Gamma has a nominal value of 137.036 since Gamma has a higher value at higher energy levels, the force acting on the electron becomes even higher.

Uncertainty

Werner Heisenberg formulated his uncertainty principle based on analysis of the dynamics of the hydrogen atom. He arranged the terms for momentum and position of the electron in a grid or matrix, but when he came to manipulating them, he discovered (or rediscovered) a quirk of matrix arithmetic: that multiplication is not commutative; in other words [A]*[B] ≠ [B]*[A]. The difference he ascribed to uncertainty; the idea that it is not possible to measure both momentum and position at the same time to an arbitrary degree of accuracy. There is always a tradeoff between these two measurements.

Eventually Niels Bohr adopted this idea to try to circumvent the problem he had encountered with the quantum leap, arguing that uncertainty was somehow intrinsic to the electron. That it does not exist as a particle in the classical sense, but somehow is spread around in multiple places at once and that it is only when it is observed that it is transformed into a particle having both position and velocity. The electron is said to exist as a wave front in a state of quantum uncertainty or superposition, where it is deemed not to be located at a single point, but to occupy a region in space. When it is subject to an observing process, the state of uncertainty or non-locality ‘collapses’ such that the electron manifests itself as a particle in a particular location. The measurement problem is a question of whether and how such a wave front collapse occurs.

The Observer Effect

To gain a complete understanding of uncertainty and the uncertainty it is first necessary to consider the so-called ‘observer effect’. When making a measurement, it is essential for the tools being used to make the measurement do not affect the measurement. The normal way to get around this problem is to ensure that the resolution of the measuring tool is much finer than the tolerance to which the measurement is being made. Unfortunately, on the scale of the electron there are no such tools

available. The only tools available are other electrons and photons and these are of the same order of magnitude as the electron being measured. The observer effect confounds any practical attempt to make measurements on this scale and it is often confused with the underlying problem of uncertainty, which is different to the observer effect. While it is not practical to make measurements on this scale, it is possible to imagine what is happening to the various particles were it possible to do so.

Shortly before Heisenberg published his findings on uncertainty, Erwin Schrödinger had developed an equation which described the particle in terms of a wave. At the time there was an element of competition between Heisenberg and Schrödinger. The uncertainty principle gave Heisenberg a clear lead, however eventually Schrödinger was able to show that his wave equation could be used to derive the same expression for uncertainty as that of Heisenberg – and that in fact the two methods are equivalent.

Uncertainty

While the observer effect is inescapable, it is not the cause of uncertainty. Uncertainty arises because it is impossible to know or measure the position (and velocity) of the electron between sample instants. This is true for all states except the base energy state. Figure 50 shows the Infinite Well for the atom in the base energy state. The period of the wave defines the boundaries of the Infinite Well and is also equal to the sample interval measured in the reference frame of the electron. If the position of the electron is known at some time, $\tau = 0$ then it can be determined at any time up to $\tau = \mathbf{T}$ by interpolation.

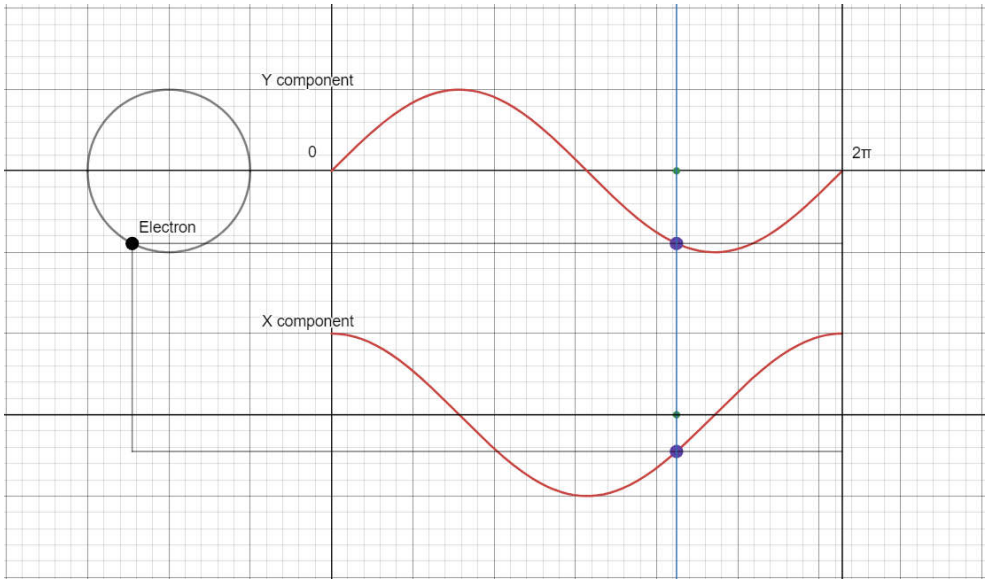

Figure 50 Uncertainty in the base energy state

Figure 51 shows the situation in the second energy state. Here the position of the electron is ambiguous and cannot be determined by simple interpolations since it could be in either of two possible positions within the Infinite Well during the sample interval. The probability of determining which of these two locations represents the position of the electron is therefore 50%.

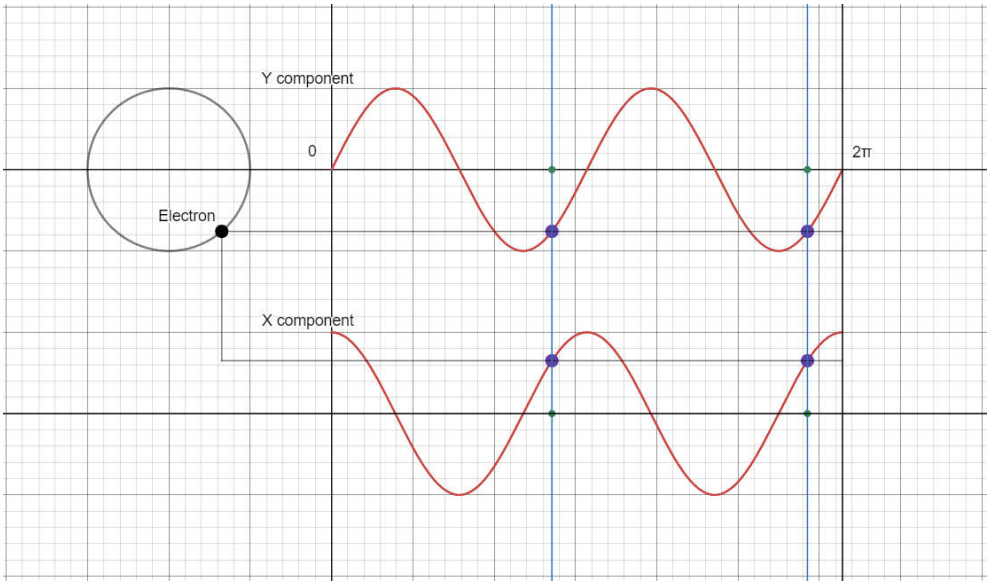

Figure 51 Uncertainty in the second energy state

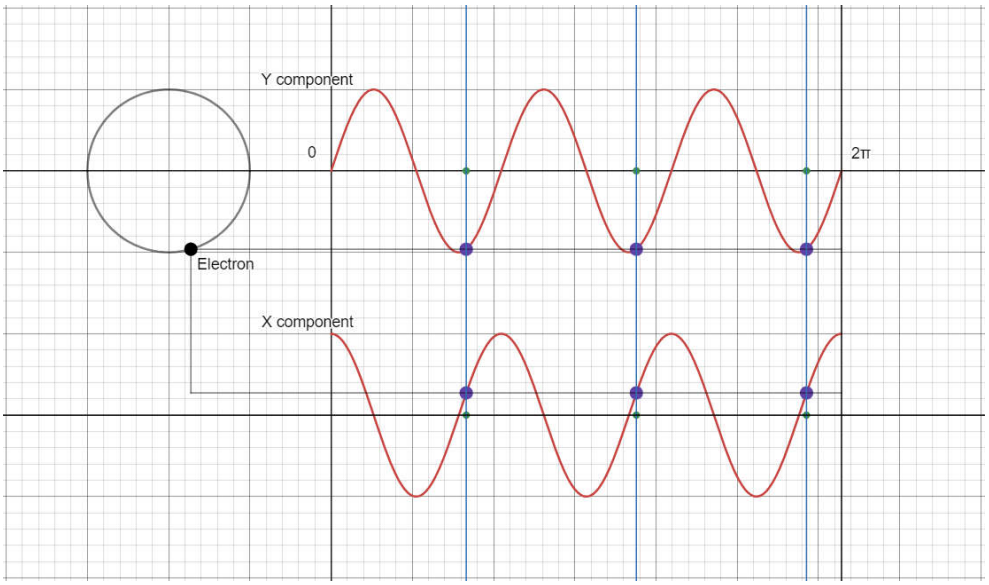

Figure 52 Uncertainty in the third energy state

https://www.desmos.com/calculator/olitpwjgeu

Figure 52 shows the situation in the third energy state. Here the ambiguity has increased to three possible positions during the sampled interval. The probability of finding the correct one of these three positions is now 33.33%. This pattern repeats for higher energy states and so in general the probability of determining the position of the electron is the reciprocal of its energy state.

When Heisenberg first set out the uncertainty principle, he ascribed uncertainty entirely to the observer effect. Later he was persuaded that uncertainty was somehow intrinsic to the electron. That is, the electron did not exist as a discrete particle. The two effects are inseparable. The state of the electron is unknown until it is involved in some sort of interaction. Such an interaction necessarily involves an observation process and incurs the observer effect and that the state of the electron must change in some way because of the interaction. But even without the observer effect the state of the electron is uncertain because of the ambiguity that occurs due to the position and velocity being sampled.

Superposition and the Measurement Problem

The requirement for the position of the electron to be a single valued function of time means that it always has a precise position, but that position cannot be determined because of the ambiguity associated with the sampling process.

The orbit of the electron can be resolved into separate x and y components which appear as sinusoidal waves.
The situation is shown in Figure 53. where the electron is always seen to be following one of an infinite number of possible trajectories (of which only the first six are shown), sometimes referred to as aliases, appropriate to its energy level, but it is not known which one it is following. The sampling frequency cannot be increased because it is dictated by the way in which the orbital path length can be observed and so it is meaningless to try to relate the distance travelled to the time taken except over a complete orbital period of the lowest orbital frequency, the base energy state.

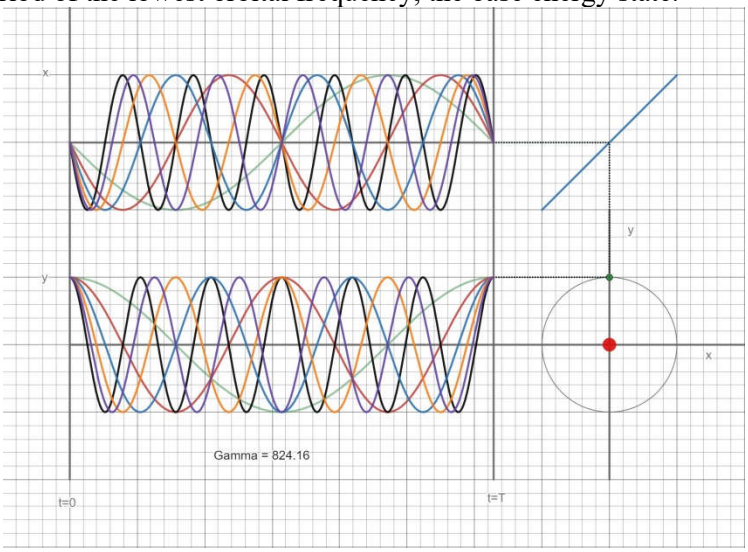

Figure 53 Trajectories for the first six energy states

The density of these infinite number of trajectories is such that the entire space is covered by such possible trajectories, and it is this that has led to the idea that the electron can be everywhere at the same time. However, this is not the case, it is constrained to lie on one of these infinite number of possible trajectories. While there are an infinite number of such trajectories, they do not represent all the space, in much the same way as the infinite number of integers does not represent all of the possible numbers within a space or domain.

This gives a slightly different view of the state of quantum indeterminacy or superposition. Here superposition is when the electron is in this state of it not being known which trajectory is being followed rather than it being everywhere. When subject to an observing process there are no structural changes to the electron; instead, it is the properties of the electron, such as speed, spin etc., that are affected by the observing process due to the observer effect but the electron itself remains unaltered. The changes that take place are to the properties of the electron and to the state of knowledge of the observer, not to the electron itself. Hence the so-called measurement problem does not really exist as such.

Duality

The duality of quantum theory is a duality of form. That is to say the electron is able to take on one of two forms, either as a particle or as a wave. In its particle guise it is the electron of the Bohr atom, orbiting at the Bohr whose orbits are described in Equation 5 and Equation 6 above. It is somehow able to transform itself into a wave. The wave into which it is transformed is that of Louis de Broglie.

This wave had no physical significance. It exists only as a mathematical entity, in a medium which doesn't exist at a frequency which exceeds that of anything that exists in the particulate form of the atom. The wave is such that a whole number of its wavelengths fit in the corresponding Bohr orbital circumference. Precisely how this transformation takes place remains a mystery.

Here the situation is somewhat different. Here the duality is a duality of measurement and is directly attributable to special relativity. Relativity inherently provides two values for every measurement, so there are two values for the orbital path length, one seen by the stationary observer, and one seen by the moving electron. There are two values for orbital period and two values for orbital frequency. One of these frequencies is more or less constant for all energy states while the other forms a harmonic series. And it is argued here that there are two values for orbital velocity. One, close to light speed, is seen in the reference frames of both the stationary observer and that of the moving electron, the second exists only in the hinterland between these two reference frames and affects centrifugal and centripetal forces and angular momentum.

Here waves do have physical significance. It is the conventional understanding of a wave representing the position in one axis of an orbiting body. They exist in the absence of an external medium since the orbiting electron is in a sense the medium. Their frequency is the orbital frequency and their amplitude is the orbital radius.

The Fine Structure Constant

Niels Bohr was able to solve his equations to match the energy levels of the Rydberg formula and consequently was able to derive an analytic formula for the Rydberg constant. When Sommerfeld linked the Bohr velocity to the speed of light he opened the door to allow the Fine Structure Constant to be expressed in terms of other known physical values, but the reason why it should have that particular value remained a mystery.

From the foregoing it is evident that the Fine Structure Constant (Alpha) is the value by which Gamma must be scaled to reduce the effective orbital path length and in turn the effective orbital velocity to a point where the forces acting on the electron are in balance. This scaling is brought about by relativity as the actual orbital velocity approaches the speed of light.

The foreshortened orbital path length and the orbital period exist in two different reference frames and can only therefore be related to one another once per orbit. We can in some sense think of this as a third reference frame, one which exists in the hinterland between that of the electron and the nucleus, one in which the orbital path length is foreshortened by relativity, but the time is not dilated. This causes the velocity term to take on a series of aliases and these each result in an effective orbital velocity that results in a stable atom. Each of these stable states corresponds to a value of Gamma which is an integer multiple of the reciprocal of the Fine Structure Constant.

This constant is a recurring theme throughout atomic and nuclear physics and yet until now it has remained one of the great unsolved mysteries of physics. Richard P Feynman said of the Fine Structure Constant:

"It has been a mystery ever since it was discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it. Immediately you would like to know where this number for a coupling comes from: is it related to pi or perhaps to the base of natural logarithms? Nobody knows. It's one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man. You might say the "hand of God" wrote that number, and "we don't know how He pushed his pencil." We know what kind of a dance to do experimentally to measure this number very accurately, but we don't know what kind of dance to do on the computer to make this number come out, without putting it in secretly!" [v]

It is frequently described as *Coupling Constant*, an appropriate epithet in the light of the above, since it couples a measurement of distance in the reference frame of the moving electron with that of time measured in the reference frame of a stationary observer.

There is nothing mysterious or special about the fact that the Fine Structure Constant is a pure number, lacking units or dimensions. It simply means that it derives from the ratio of two values which have the same dimensions, in much the same way as π is dimensionless because it is the ratio of two lengths.

The introduction of Relativistic Velocity solves the mystery and shows that it is indeed the ratio of two quantities. We can think of it as the ratio of two lengths, although it is equally valid to describe it as a ratio of two times or even two masses. In terms of length, it is the ratio of the orbital circumference measured by a stationary observer to that measured by the moving electron in the hydrogen atom in its base energy state. It also occurs in each stable energy state as one of the multivalued orbital circumferences that occur due to the phenomenon of aliasing.

Alpha can be thought of as being the ratio of two times, the period of the orbit as seen by the stationary observer and the period as seen by the moving electron in the base energy state. Similarly, it can be viewed as the ration of two frequencies, the orbital frequency as seen by the stationary observer to that seen by the moving electron in the base energy state.

Zero Point Energy

The model also provides an explanation for the hitherto mysterious Zero Point Energy. Debate has raged about the existence and the nature of Zero-Point Energy since the concept was first introduced by Planck in 1911. With at least one interpretation showing that the atom is possessed of energy even when it is cooled to absolute zero. When an atom is cooled to absolute zero it ceases to have Brownian motion and therefore has zero kinetic energy. However, the orbiting electron still has kinetic energy. The electron is orbiting at close to the speed of light and so has energy equal to $1/2mc^2$ exactly in line with prediction.

The Morphology of the Atom

The model for the atom presented here has an orbital radius which is constant for all energy states. The orbital velocity varies between 99.9973376% of the speed of light and a theoretical upper limit of the speed of light which means that atoms of different energy levels have the same overall shape and are almost indistinguishable from one another in shape and form. This is consistent with an atom whose physical and chemical properties are the same in all energy states.

Planck's constant

Bohr's model for the atom was based on the idea that Planck's constant was the fundamental unit of angular momentum, and that each energy level was associated with a integer multiple of this basic unit. That assumption was carried over into all subsequent models within quantum theory. The assumption has been shown to be false. Angular momentum is not quantized; instead it is the Lorentz factor, Gamma, which is quantized. Here Planck's constant is still seen as the orbital angular momentum of the electron, but rather than being quantized, it is constrained by the dynamics of the atom to have a particular value for all energy states.

But why exactly is Planck's constant; constant at all? Why is the angular momentum forced to take in this particular value?

Planck's constant is a measure of the orbital angular momentum of the electron orbiting the hydrogen nucleus. It is the product of three variables, the mass of the electron, its tangential velocity and its orbital radius.

If angular momentum were to be quantized it would require a complex interplay between these three variables plus that of the energy level of the atom such that when the velocity changes so too does the orbital radius and vice versa. The velocity must somehow be cognizant of the change in the orbital radius so as to comply with the quantization requirement or the radius must cognizant of the change in the velocity. The problem with this is that there is no mechanism which could cause this to happen. There is no causal link between changes in orbital radius and orbital velocity. The only link is that they must do this to comply with the quantization requirement, which is rather like saying it is quantized because it is quantized.

The idea that Planck's constant is equal to the orbital angular momentum of the electron is not unreasonable since it has the units of angular momentum. Only now let's consider what happens when orbital velocity is seen as being affected by relativity.

$$\hbar = m\gamma\, R\frac{c}{\gamma} \qquad \text{Equation 75}$$

Equation 75 is that for the orbital angular momentum as seen from the point of view of a stationary observer located at the atomic nucleus. For such an observer the electron is moving at near light speed and so both its mass and its velocity are affected by relativity but in the opposite sense. The equation links Planck's constant to the orbital radius, the mass and the velocity of the orbiting electron, but it also includes the factor Gamma, only here it occurs twice, once in the numerator, where it acts to modify the rest mass of the moving electron in line with relativity, and once in the denominator where it acts to modify the orbital speed to create the Relativistic Velocity term.

As the orbital velocity increases, so does the value of Gamma. This causes the effective mass of the electron to increase, but at the same time the orbital path length, and therefore the Relativistic Velocity decreases in the same measure. This can be thought of rather like a mathematical see-saw, as one goes up the other goes down, with Planck's constant at the fulcrum between these two. The two Gamma terms balance one another out but at the same time they force the angular momentum of the orbiting electron to maintain its constant value. One cannot alter without the other altering and the sensitivity to any such change increases with Gamma. The higher the value of Gamma, the more tightly the angular momentum is constrained to be a constant value. Since the rest mass and the speed of light are both constants, this means that the orbital radius is also constrained to have a fixed value. As to angular momentum itself, it is not quantized but is continuous and, in general, can take on any value. However, the effects of relativity combined with an orbital velocity near light speed constrain it to have a particular value in the context of the atom.

Viewing the angular momentum from within the reference frame of the electron, the electron possesses its rest mass and its orbital velocity is near light speed, which in this context is seen as being invariant with respect to relativity. The radius is the same as for the stationary observer and so the orbital angular momentum is given by the formula

$$\hbar = mRc \qquad \text{Equation 76}$$

Strictly speaking then Planck's constant is not a constant at all, it is invariant with respect to velocity close to the speed of light and it is invariant between the reference frames of the stationary and moving observers.

The Laws of Dynamics

All the above implies that the laws of dynamics as they are currently understood are incorrect and will need to be modified. Newtonian dynamics fails to account for the discrete energy levels of the atom. The only question is what form that modification should take. The pioneers of quantum theory assumed that the necessary modification was that angular momentum could only take on discrete

values which were an integer multiple of Planck's constant. However, this leads directly to the physically impossible quantum leap and indirectly to numerous incomprehensible phenomena and so cannot be true.

Here the modification to Newton's laws is that certain velocity which straddle the boundary between an orbiting object and a stationary observer are affected by relativity.

Put simply, the problem facing Bohr and the pioneers of quantum theory was to discover a means by which to include such a multiplier in the Newtonian equations that describe the dynamics of the atom. The solution they chose was to simply and arbitrarily associate it with angular momentum. It seemed to work, at least in so far as it returned energy levels that matched those of the Rydberg formula. Closer examination however reveals that it is not the Bohr model, nor indeed that of de Broglie or Schrödinger or any of the models that use Nicholson's assumption that leads to this result, but the presence of the integer multiplier 'n' that leads to this success.

This applies not just to the energy levels of the atom, but a number of values that spring from this calculation. Values such as the Sommerfeld Fine Structure Constant, the Bohr radius and Bohr velocity, the Compton wavelength and even Planck's constant itself, form the building blocks of quantum mechanics. There is nothing about the derivation of these values that requires specifically that angular momentum is quantized. It is the presence of the factor *n* that leads to these building blocks and their numerical values that is ultimately responsible for the accuracy of quantum mechanics and not the structure of the model from which they derive.

Unification of Quantum and Classical Dynamics

The unification of observable phenomena into one overarching law is one of the primary goals of physics. In the 17th century Newton unified gravity with astronomy by formulating the inverse square law, while in the 19th century Maxwell and Lorentz between them unified electricity and magnetism into a single electro-magnetic force and Clausius together with Maxwell unified heat with mechanical energy with the kinetic theory of gases.

The key to the unification of quantum and classical dynamics is to understand how velocity is the relationship between distance and time each measured in different reference frames and that distance can only be meaningfully established over one or more complete orbits. This leads to the idea of a velocity term where distance and time can only be related to one another at certain discrete times. The combined effects of this sampling process and of special relativity create a mechanism that causes the electron to only be able to orbit at certain discrete energy levels.

Relativistic Velocity asserts that certain orbital velocity terms are affected by relativity and comes about because these terms are composed of the orbital path length foreshortened by relativity divided by the orbital period which is unaffected by relativity. One constraint on the existence of such a velocity term is that the orbital path length can only be measured or experienced over a whole orbit or whole number of orbits. This in turn introduces a sampling process whereby the orbital circumference is sampled in time with a sampled frequency equal to the orbital period. The consequence of this sampling is to create aliases for the orbital path length which in turn creates aliases for the orbital velocity and centrifugal force.

It is the idea that as the actual velocity increases, the effective velocity reduces, which leads to the stable states of the atom. The first stable state occurs when the effective orbital velocity is scaled down due to relativity by a when Gamma has a value of 137.036 and subsequent stable states correspond to values of Gamma which are an integer multiple of this base value.

The fact that Gamma appears in the denominator of the term for the centrifugal force acting on the electron means that any deviation from its prescribed orbit is opposed by a massive force relative to any other forces acting on the electron and so relativity causes the electron to orbit the atom without the emission of synchrotron radiation.

All of this takes place in the classical domain using a set of equations that would be recognizable to Newton, as modified by Einstein to take account of special relativity. There is just one simple and plausible extension regarding the nature of velocity and that is to assert when considering objects in orbit such orbital velocity is affected by relativity. The causal relationship between the realms of classical mechanics and quantum mechanics is thus a series of links, a chain of causality which effectively unifies the quantum and classical domains.

Appendix 1 Proof of Aliasing

The proof of aliasing was first carried out by Claude Shannon in 1949[vi]. Versions of this proof can be found in most standard texts on digital signal processing.

Aliasing is the name given to the phenomenon where two distinct signals $x_1(t)$ and $x_2(t)$ give the same set of values $x[n]$ when sampled at a fixed rate f_s. We shall consider the case where x_1 and x_2 are both sinusoids.

$$x[n] = \cos\left(2\pi f \frac{n}{f_s} + \phi\right) \qquad \text{Equation A1}$$

Two signals will be aliases of one another if they meet the condition

$$f_2 = f_1 + kf_s \qquad \text{Equation A2}$$

Where f_s is the sampling frequency and k is an integer.

Sampling the two sinusoidal waves, x_1 and x_2, at a rate of f_s will produce the following sequences

$$x_1[n] = A\cos\left(2\pi f_1 \frac{n}{f_s} + \phi\right) \qquad \text{Equation A3}$$

$$x_2[n] = A\cos\left(2\pi f_2 \frac{n}{f_s} + \phi\right) \qquad \text{Equation A4}$$

The signals will be aliases of one another if $x_1[n] = x_2[n]$ for all $n = 1,2,3,\ldots$

The proof of aliasing relies on the identity $\cos(\theta + 2\pi k) = \cos(\theta)$ for all integer k.

Substituting for f_2 in Equation A4

$$x_2[n] = A\cos\left(2\pi f_1 + 2\pi k f_s \frac{n}{f_s} + \phi\right) \qquad \text{Equation A5}$$

$$x_2[n] = A\cos\left(2\pi f_1 \frac{n}{F_s} + 2\pi k n + \phi\right) \qquad \text{Equation A6}$$

$$x_2[n] = A\cos\left(2\pi f_1 \frac{n}{f_s} + \phi\right) = x_1[n] \qquad \text{Equation A7}$$

This is the special case where $F_s = F_1$ and so

$$x_1[n] = x_2[n] = A\cos(\phi) \qquad \text{Equation A8}$$

And

$$f_2 = nf_1 \qquad \text{Equation A9}$$

In other words the orbital frequencies form a harmonic series with base frequency F_1.

Appendix 2 Constants and Formulae

Constant	Symbol	Value	Units
Gravitational constant	G	6.6743×10^{-11}	$m^3\ kg^{-1}\ s^{-2}$
			$Kg\ m^3\ s^{-2}$
Electrical constant	K	$8.9875517923 \times 1^{09}$	C^{-2}
Charge on electron	q	$1.60217663 \times 10^{-19}$	C
Mass of Electron	m	$9.1093837015 \times 10^{-31}$	Kg
Mas of the proton	m_p	1.67262×10^{-27}	Kg
Planck's constant	h	$6.62607015 \times 10^{-34}$	J s
Planck's constant	ħ	$1.054571817... \times 10^{-34}$	J s
Planck's constant	h	4.14E-15	eV s
Planck's constant	ħ	$6.582119569... \times 10^{-16}$	eV s
Joules to eV	J-eV	$6.241\ 509\ 074... \times 10^{18}$	eV/J
Speed of light	c	299792458	m/s
Reduced Compton Wavelength	R	3.86159×10^{-13}	m
Fine Structure Constant	α	0.007297728	
	1/α	137.036	
Bohr velocity		2187803	m/s
Rydberg constant	R_H	10973731.568160	m^{-1}
Linear momentum	l	mv	$Kg\ m\ s^{-1}$
Angular momentum	L	mvr	$kg\ m^2\ s^{-1}$
Moment of inertia of body in orbit	I	mr^2	$kg\ m^2$
Kinetic energy	E	$\frac{1}{2}mv^2$	J
Kinetic energy	E	$\frac{1}{2}I\omega^2$	J
Electrostatic force	f	$\frac{Kq_1q_2}{d^2}$	N
Inertial force	f	ma	N
Lorentz factor	γ	$\frac{c^2}{\sqrt{c^2 - v^2}}$	
Reduced Compton Wavelength	R	$\frac{\hbar}{mc}$	m

Fine Structure Constant	α	$\frac{Kq^2}{\hbar c}$	
Planck's equation	E	$\hbar\omega$	J

[i] H. Nyquist, *Certain Topics in Telegraph Transmission Theory*, Trans. AIEE, vol 47, pp 617-644, April 1928

[ii] C. E Shannon, C*ommunication in the Presence of Noise*, Proc. Institute of Radio Engineers, vol 37 no. 1 pp 10-21, January 1949.

[iii] Bracewell, R.N. (1986), *The Fourier Transform and Its Applications (revised ed.)*, McGraw-Hill; 1st ed. 1965, 2nd ed. 1978

[iv] Knopp, K. *Theory of Functions Parts I and II, Two Volumes Bound as One.* New York: Dover, Part I p. 103 and Part II p. 93, 1996.

[vi] Shannon, Claude E. (January 1949). *"Communication in the presence of noise".* Proceedings of the Institute of Radio Engineers. **37** (1): 10–21.