ABSTRACT

Through solving the Collatz conjecture problem, 
I think about the problem of the modern mathematics.

Without the modern mathematics,

Through Pythagorean triangle
How to find a large Pythagorean triangle, and Primitive Pythagorean triangle.

Through Primitive Pythagorean triangle, 
How to find a large Pythagorean triangle, and Prime number.

Through the interpretation of Fermat's Last Theorem, 
About what 'his surprising method of proof' is,

INTRODUCTION

The Modern mathematics
  is thinking that mathematics is universal, 
  is becoming more and more complex, 
  is forgetting about the mathematics.

This paper is written in mathematics to think about the meaning of numbers.

The Mathematics can contain all things, but that does not mean universal.
THEOREM 1. Pythagorean triangle

Diophantus
Quaestio VIII.

Propositum quadratum dividere in duos quadratos.
To divide a given square into a sum of two squares.

(1)
Imperatum sit 16. dividatur in duos quadratos.
To divide 16 into a sum of two squares.

(2)
Ponatur primus 1Q. Oportet igitur 16. - 1Q. aequales esse quadrate.
Let the first summand be $x^2$, and thus the second $16 - x^2$.
The latter is to be a square.

(3)
Fingo quadratum a numeris quotquot libuerit, cum defectu tot unitatum quod contineat latus ipsius 16.
I form the square of the difference of an arbitrary multiple of $x$ diminished by the root [of] 16, that is, diminished by 4.

(4)
esto a $2N. - 4$.
I form, for example, the square of $2x - 4$.

(5), (6)
ipse igitur quadratus erit $4Q. + 16. - 16N. haec aequabuntur unitatibus 16. - 1Q$.
It is $4x^2 + 16 - 16x$. I put this expression equal to $16 - x^2$.

(3), (5), (6)
Communis adiciatur utrimque defectus, et a similibus auferantur similia, fient 5Q. aequales 16N. et fit 1N. 16/5.
I add to both sides $x^2 + 16x$ and subtract 16. In this way I obtain $5x^2 = 16x$, hence $x = 16/5$.

(6)
Thus one number is 256/25 and the other 144/25. The sum of these numbers is 16 and each summand is a square.
THEOREM 2. Primitive Pythagorean triples and Prime number

**Using Mathematics.** Define Diophantus Quaestio VIII.

\[ f(n) = \sqrt{m^2 + 4m^2n(n+1)} \]

\[ n = \mathbb{N} \{1, 2, 3, 4, \ldots\}, \quad m = \text{Odd} \{3, 5, 7, \ldots\} \]

**NOTE 1.** In this paper, the case that \( m \) is an all number is excluded, because 1, 2 is complicated to explain.

The outermost Primitive Pythagorean triples is, \( p \) is prime number

\[ m = p, \quad n = \frac{(m-1)}{2}, \quad (p \geq 3) \]

\[ \{p, p\times n+n, p\times n+n+1\} \]

is always Outermost Primitive Pythagorean triples.

**Conversely,**

Since the Primitive Pythagorean triples always contains prime numbers that can be obtained by excluding composite numbers.

To find the prime numbers up to an arbitrary number \( R \),

Using Primitive Pythagorean triples.

\[ p_n \geq \sqrt{R}, \quad P = \{p_1, p_2, p_3, \ldots p_n\}, \quad P = \{3, 5, 7, \ldots p_n\} \]

\[ n_n \geq \frac{R}{6} \quad N = \{n_1, n_2, n_3, \ldots n_n\}, \quad N = \{1, 2, 3, \ldots n_n\} \]

In addition,

\[ f(n) = p^2, \quad \text{when} \quad n = \frac{p-1}{2}, \]

So,

\[ F(N), \quad N = \left\{ \frac{P-1}{2}, \frac{P-1}{2}+1, \frac{P-1}{2}+2, \ldots \frac{R}{6} \right\} \]

Subtract the resulting composite number from the odd numbers up to \( R \).

Then only the prime number remains.

**NOTE 2.** The above formula finds prime numbers by excluding ONLY using \( p_n \geq \sqrt{R} \),

There more ways to reduce the number of calculations but do not fit this paper and no blank spaces.
THEOREM 3. THE FERMAT’S LAST THEOREM

Observatio domini Petri de Fermat.

Cubum autem in duos cubos, aut quadratoquadratum in duos quadratoquadratos & generaliter nullam in infinitum ultra quadratum potestatem in duos eiusdem nominis fas est dividere cuius rei demonstrationem mirabilem sane detexi. Hanc marginis exiguitas non caperet.

It is impossible to separate a cube into two cubes, or a fourth power into two fourth powers, or in general, any power higher than the second, into two like powers. I have discovered a truly marvelous proof of this, which this margin is too narrow to contain.

FIRST

Simplified the formula.

$$X^n + Y^n = Z^n, \quad X < Z \land Y < Z$$

NOTE 3. Simplification of equations is very important, When using the INFINITE DESCENT METHOD. If you can, DO NOT USE REAL NUMBER, If you need real number, ONLY USE 1, 2 OR OTHERS.

SECOND

Divide $X, Y, Z$ into odd or even numbers,

When $n$ is any Natural Number,

A) When $x$ is ODD, $y$ is ODD, $x^n + y^n$ is EVEN, $z$ is EVEN
B) When $x$ is ODD, $y$ is EVEN, $x^n + y^n$ is ODD, $z$ is ODD
C) When $x$ is EVEN, $y$ is ODD, $x^n + y^n$ is ODD, $z$ is ODD
D) When $x$ is EVEN, $y$ is EVEN, $x^n + y^n$ is EVEN, $z$ is EVEN

However since,

D) is divided by 2, so it is equivalent to A), B) or A), C).

Fermat prove A)

So, ONLY prove ‘B’ or ‘C’ in this paper.

THIRD

B) $X^n + Y^n = Z^n, \quad X < Z \land Y < Z, \quad$ ODD $\{X, Z\}, \quad$ EVEN $\{Y\}$
C) $X^n + Y^n = Z^n, \quad X < Z \land Y < Z, \quad$ ODD $\{Y, Z\}, \quad$ EVEN $\{X\}$

NOTE 4. If you can, remove odd number rule, Because all odd prime numbers are over 3.
Imaging,
When \( n \) is 1, Imagining the **LINE**.
When \( n \) is 2, Imagining the **SURFACE**.
When \( n \) is 3, Imagining the **MASS**.

**NOTE 4.** When \( n \) is over 4,
We can’t imagining without Mathematics.
Putting in the numbers at **LINE**, **SURFACE**, **MASS**, with using Mathematics, **CALCULATE** rate of change.
(like differentiation)

**NOTE 5.** Approach with mathematics.
Create a formula and approach the shape.

\[
\begin{align*}
X^n + Y^n &= Z^n, \quad X < Z \land Y < Z, \quad \text{ODD} \{ X, Y \}, \ \text{EVEN} \{ Z \} \\
X^n + Y^n &= Z^n, \quad X < Z \land Y < Z, \quad \text{ODD} \{ X, Z \}, \ \text{EVEN} \{ Y \} \\
X^n + Y^n &= Z^n, \quad X < Z \land Y < Z, \quad \text{ODD} \{ Y, Z \}, \ \text{EVEN} \{ X \}
\end{align*}
\]

\[
X^n + Y^n = Z^n \quad \Leftrightarrow \quad \frac{X^n}{Z^n} + \frac{Y^n}{Z^n} = 1
\]

\[
\frac{X^n}{m} + \frac{Y^n}{m} = \frac{Z^n}{m} \quad \Leftrightarrow
\]

\[
\left( \frac{X}{m} \right)^n + \left( \frac{Y}{m} \right)^n = \left( \frac{Z}{m} \right)^n
\]

\[
\frac{X^n}{Z^{n-1}} + \frac{Y^n}{Z^{n-1}} = Z
\]

**NOTE 6.** If There’s no more way.
Then **CALCULATE** with using **REAL NUMBER**.

**NOTE 7.** **FIND RULE AND PROVE.**
PROVE

This formula

\[ X^n + Y^n = Z^n, \quad X < Z \land Y < Z, \quad \text{ODD}\{X, Z\}, \text{EVEN}\{Y\} \]
\[ X^n + Y^n = Z^n, \quad X < Z \land Y < Z, \quad \text{ODD}\{Y, Z\}, \text{EVEN}\{X\} \]
\[ n \geq 3, \quad n \text{ is ODD} \]

Divide both sides by \( m \).

\[ \frac{X^n}{m} + \frac{y^n}{m} = \frac{Z^n}{m}, \quad n \geq 3, \quad n \text{ is ODD} \]

When \( m = 2 \)

\[ \text{ODD}^n/2 \]
\[ 4a + 1 \quad \Rightarrow \quad \left[ \text{EVEN}' + 1/2 \right] \]
\[ 4a + 3 \quad \Rightarrow \quad \left[ \text{ODD}' + 1/2 \right] \]

\[ \text{EVEN}^n/2 \]
\[ 4a \quad \Rightarrow \quad \left[ \text{EVEN}' \right] \]
\[ 4a + 2 \quad \Rightarrow \quad \left[ \text{EVEN}' \right] \]

When \( m = 4 \)

\[ \text{ODD}^n/4 \]
\[ 8a + 1 \quad \Rightarrow \quad \left[ \text{EVEN}' + 1/4 \right] \]
\[ 8a + 3 \quad \Rightarrow \quad \left[ \text{EVEN}' + 3/4 \right] \]
\[ 8a + 5 \quad \Rightarrow \quad \left[ \text{ODD}' + 1/4 \right] \]
\[ 8a + 7 \quad \Rightarrow \quad \left[ \text{ODD}' + 3/4 \right] \]

\[ \text{EVEN}^n/4 \]
\[ 8a \quad \Rightarrow \quad \left[ \text{EVEN}' \right] \]
\[ 8a + 2 \quad \Rightarrow \quad \left[ \text{EVEN}' \right] \]
\[ 8a + 4 \quad \Rightarrow \quad \left[ \text{EVEN}' \right] \]
\[ 8a + 6 \quad \Rightarrow \quad \left[ \text{EVEN}' \right] \]

Using INFINITE DESCENT METHOD 1.

When \( m = 2^b \land b < n \)

\[ m = 2^b \land b < n \]
\[ \begin{cases} \text{ODD}\{2m+\text{ODD}\}, & n(\text{ODD}) = m \\ \text{EVEN}\{2m+\text{EVEN}\}, & n(\text{EVEN}) = m \end{cases} \]
AND MORE DIVIDE

When \( m = 3 \)

\[
\begin{align*}
ODD^n/3 & \quad 6a+1 \Rightarrow [EVEN+1/3] \quad (1) \\
& \quad 6a+3 \Rightarrow [ODD] \quad (2) \\
& \quad 6a+5 \Rightarrow [ODD+2/3] \quad (3) \\
EVEN^n/3 & \quad 6a \Rightarrow [EVEN] \quad (4) \\
& \quad 6a+2 \Rightarrow [EVEN+2/3] \quad (5) \\
& \quad 6a+4 \Rightarrow [ODD+1/3] \quad (6)
\end{align*}
\]

\[
(Small)\ ODD^n+EVEN^n=(Large)' ODD^n
\]

\[
A \left( \begin{array}{ccc}
(1)+(4)=(1)' \\
(2)+(4)=(2)' \\
(3)+(4)=(3)'
\end{array} \right)
\]

\[
B \left( \begin{array}{ccc}
(1)'-(1)=(4) \\
(2)'-(1)=(5) \\
(3)'-(1)=(6)
\end{array} \right)
\]

\[
(Large)\ ODD^n-(Small) ODD^n=EVEN^n
\]

\[
A \cap B = \left[ \begin{array}{c}
(1)+(4)=(1)' \\
(2)+(4)=(2)' \\
(3)+(4)=(3)'
\end{array} \right]
\]

Intersection A and B.
AND MORE MORE DIVIDE
When \( m=6, \ (2^b \times 3) \land b=1 \)

\[
\begin{align*}
ODD^{n}/6 & \quad 12a+1 \Rightarrow [EVEN '+1/6] \quad (1-1) \\
12a+3 & \Rightarrow [EVEN '+3/6] \quad (2-1) \\
12a+5 & \Rightarrow [EVEN '+5/6] \quad (3-1) \\
12a+7 & \Rightarrow [ODD '+1/6] \quad (1-2) \\
12a+9 & \Rightarrow [ODD '+3/6] \quad (2-2) \\
12a+11 & \Rightarrow [ODD '+5/6] \quad (3-2)
\end{align*}
\]

\[
\begin{align*}
EVEN^{n}/6 & \quad 12a \Rightarrow [EVEN '] \quad (4-1) \\
(6a+2) & \quad 12a+2 \Rightarrow [ODD '+2/6] \\
(6a+4) & \quad 12a+4 \Rightarrow [EVEN '+4/6] \\
& \quad 12a+6 \Rightarrow [EVEN '] \quad (4-2) \\
(6a+2) & \quad 12a+8 \Rightarrow [ODD '+2/6] \\
(6a+4) & \quad 12a+10 \Rightarrow [EVEN '+4/6]
\end{align*}
\]

Using INFINITE DESCENT METHOD 2.
When \( m=2^b \times 3 \land b<n \)

\[
\begin{align*}
m=3 \times 2^b \\
(ODD \{2m+ODD\ldots\}, \ n(ODD)=m) \\
(EVEN \{2m, 2m \times 1.5\}, \ n(EVEN)=2)
\end{align*}
\]

Using INFINITE DESCENT METHOD 3.
When \( m=2^b \times 3 \land b \leq n \)

\[
\begin{align*}
m=3 \times 2^c \\
(ODD \{m \over 2, 3m \over 4, 2m+ODD\ldots\}, \ n(ODD)=m+2) \\
(EVEN \{\}, \ n(EVEN)=0)
\end{align*}
\]

QED.

COMMENTARY
It’s working only when \( n \geq 3, \ n \text{ is ODD} \)
So, Fermat have to prove when \( n=4 \)

REFERENCES
KWANGSUN SONG, The Proof of the Collatz Conjecture
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