Comment on Lev I. Verkhovsky's 'Memoir on the Theory of Relativity and Unified Field Theory'

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Abstract

In his remarkable 'Memoir on the Theory of Relativity and Unified Field Theory', Lev I. Verkhovsky has reanimated the scaling factor which occurs in pioneering articles on the Lorentz transformation by Lorentz 1904, Einstein 1905, and Poincaré 1906. In this comment, we shortly look at their determination of the scaling factor and then show that his reanimation is not successful.

1 Introduction

The Lorentz transformation is the mathematical heart of the theory of special relativity. For this, it enters all representations of electromagnetism and all other theories which are compatible with the theory of special relativity. Accordingly, there is quite a huge variety of derivations of it. In his remarkable 'Memoir on the Theory of Relativity and Unified Field Theory' [1], Lev I. Verkhovsky has reanimated the scaling factor which occurs in pioneering articles on the Lorentz transformation by Lorentz 1904 [2], Einstein 1905 [3], and Poincaré 1906 [4], see formulas (2). His connection of the scaling factor with the Doppler effect is most original but—unluckily—erroneous. He has found a remarkable expression for the scaling factor which obeys the group property. However, the exponent r

he has added for correctly switching between sources, which moves toward and away from the detector, respectively, destroys the group property. This will be shown in detail. Moreover, he has constructed a contradiction which does not really exist.

2 Verkhovsky's contradiction does not exist

First we show that his motivation for evoking the Doppler effect is erroneous. He makes the following gedankenexperiment, see his Fig. 1. A light flash is shot toward a person B being at rest in frame S both from, (i), a person A being at rest in the same frame S and, (ii), a rocket N moving with velocity v toward B just as it passes A. The distance between B and A (and N at the time of flashing) is L. The time of flight from A and N to B equals $T = L/c$.[1] At time T, the rocket N flew the distance $vT = vL/c$ till point P, its distance to B now being $L - vL/c = (1 - v/c)L$.

Now, Verkhovsky claims that the speed of light in the rest system of N is not given by $L/T = c$ but by $(1 - v/c)L/T = c - v$. He evokes the longitudinal Doppler factor $D(v) = \sqrt{\frac{c+v}{c-v}}$ because $D(v)\gamma(v) = \frac{c}{c-v}$ ($\gamma(v)$ being the Lorentz factor).

However, the conclusion that the speed of light in the rest system of the rocket N contradicts Verkhovsky's own transformation formulas. The latter ones yields Einstein's [3] formula for the addition of velocities because the scaling factor $\eta(v)$ shortens in the same manner as the Lorentz factor $\gamma(v)$ does, see formulas (2). As a consequence, the speed of the light flash in the rest frame of N is the same as that in the rest frame of A and B, namely, equal c.

3 On the scale factor

Nevertheless, despite its incorrect foundation by Verkhovsky [1], the scale factor $\eta(v)$ proposed by him could be correct (or an inspiration for further developments).

[1] BTW, Verkhovsky writes that the independence of the speed of light of the motion of the emitter "was an experimentally established, albeit paradoxical fact". We would like to remark that that independence is by no means "paradoxical" but an immediate consequence of Maxwell's 1864 (!) theory of light [5], in which $c = 1/\sqrt{\varepsilon_0\mu_0}$.

3.1 Early treatments of the scaling factor

Nowadays, the special Lorentz transformation for a relative motion along the x-axis reads

$$x' = \gamma(v)\,(x - vt\,); \quad \gamma(v) := \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{1a}$$

$$y' = y \tag{1b}$$

$$z' = z \tag{1c}$$

$$t' = \gamma(v)\left(t - \frac{v\,x}{c^2}\right). \tag{1d}$$

In the original expressions of the Lorentz transformation mentioned in the Introduction [2][3][4], an additional scale factor $\eta(v)$ occurs.

$$x' = \eta(v)\gamma(v)\,(x - vt\,) \tag{2a}$$

$$y' = \eta(v)y \tag{2b}$$

$$z' = \eta(v)z \tag{2c}$$

$$t' = \eta(v)\gamma(v)\left(t - \frac{v\,x}{c^2}\right). \tag{2d}$$

All three authors obtained $\eta(v) \equiv 1$;

- Lorentz from the transformation properties of force and acceleration ([2] after formula (33));
- Einstein from, (i), comparing the transformation and its inverses, and, (ii), the argument that, by symmetry, the transformation of the coordinates perpendicular to the relative motion does not depend on the direction of that motion ([3] pp. 901f.);
- Poincaré from comparing the transformation and its inverse as well as its combination with a 180 degree rotation about the y-axis ([4] § IV, eqs. (1)...(3)).

However, Lorentz's transformations of velocity and charge density [2] are *not* special-relativistic ones as noted directly by Tyapkin ([6] fn. on p. 70) and indirectly by Poincaré ([4] after formulas (4a) and (7)). Einstein [3] and Poincaré [4] implicitly assume reciprocity, i.e. $1/\eta(v) = \eta(-v)$ without careful justification. Moreover, Poincaré's ([4] § IV) formulas (3) are actually not the inverse of his formulas (1) as they also transform from system S to S'. In view of that, Verkhovsky's attempt to reconsider the scaling factor is justified.

3.2 Group property of the longitudinal Doppler factor

Verkhovsky correctly states that the group property of the Lorentz transformation (2) is guaranteed, if

$$\eta(v')\,\eta(v) = \eta(v'')\,, \tag{3}$$

where v, $v´$, and $v´´$ are the corresponding relative velocities of the three frames involved. Remarkably enough, he shows that this is the case for the longitudinal Doppler factor, i.e. for $\eta(v) = D(v) = \sqrt{\frac{c+v}{c-v}}$ ([1] p. 9/p. 6, eq. (1.3)).

$$\eta(v´)\,\eta(v) = \sqrt{\frac{c+v´}{c-v´}}\sqrt{\frac{c+v}{c-v}} = \sqrt{\frac{c+\frac{v´+v}{1+\frac{v´v}{c^2}}}{c-\frac{v´+v}{1+\frac{v´v}{c^2}}}} = \sqrt{\frac{c+v´´}{c-v´´}} = \eta(v´´) \qquad (4)$$

As a matter of fact, that is true even for the more general case $\eta(v) = \left(\frac{c+v}{c-v}\right)^{\rho}$, where $\rho = \text{const.}$

$$\eta(v´)\,\eta(v) = \left(\frac{c+v´}{c-v´}\right)^{\rho}\left(\frac{c+v}{c-v}\right)^{\rho} = \left(\frac{c+\frac{v´+v}{1+\frac{v´v}{c^2}}}{c-\frac{v´+v}{1+\frac{v´v}{c^2}}}\right)^{\rho} = \left(\frac{c+v´´}{c-v´´}\right)^{\rho} = \eta(v´´) \qquad (5)$$

Now, eq. (5) applies if and only if the exponent ρ is independent of the frames involved. Unfortunately, this condition is *not* fulfilled by Verkhovsky's transformation as we will show next.

3.3 Violation of the group property

Contrary to $\rho \overset{!}{=} \text{const.}$, Verkhovsky sets $\rho = r/2$, where $r = \text{sgn}(x - vt)$. "The exponent r is required because the Doppler factor change[s] its value to the inverse on transition from approach to removing and vice versa... It is the change in the parameter r that ensures the equality $\eta(v) = \eta(-v)$ in the particular case that Einstein considered." (p. 6)

Unfortunately, the exponent r destroys the group property (3).

To prove that, we set

$$r = \text{sgn}(x - vt)\,, \quad r' = \text{sgn}(x' - v't')\,. \qquad (6)$$

Then, using the transformation formulas (2) and omitting the factor $\eta(v)\gamma(v) > 0$, we have

$$r' = \text{sgn}\left(x - vt - v'\left(t - \frac{xv}{c^2}\right)\right) = r\ \text{sgn}\left(1 - \frac{v'(t - vx/c^2)}{x - vt}\right). \qquad (7)$$

Here, the value of the factor after r is *rather arbitrary.* For instance, for $x \ll vt$, we obtain (approximately)

$$r' = r\ \text{sgn}(1 + v/v'). \qquad (8)$$

If $1 + v/v' < 0$, we have $r' = -r$, i.e.

$$\eta(v´)\,\eta(v) = \left(\frac{c+v´}{c-v´}\right)^{-r/2}\left(\frac{c+v}{c-v}\right)^{r/2} \neq \left(\frac{c+\frac{v´+v}{1+\frac{v´v}{c^2}}}{c-\frac{v´+v}{1+\frac{v´v}{c^2}}}\right)^{r/2}. \qquad (9)$$

It is obvious that this violation of the group property holds for a *finite* area in the x-t-plane (the size of which depends on the velocities v and v').

4 Summary and conclusions

Lev I. Verkhovsky's 'Memoir on the Theory of Relativity and Unified Field Theory' represents an original and interesting attempt to use the scaling factor $\eta(v)$ occurring in early articles on the Lorentz transformation [2][3][4] for including the Doppler effect from the very beginning. Unfortunately, his attempt is not successful.

1. His determination of the speed of light in the rest frame of the rocket N contradicts his own transformation formulas.

2. Generally speaking, his transformation formulas do *not* obey the group property because the exponent $r := x - v\,t$ is not constant.

Summa summarum, Lev I. Verkhovsky's transformation formulas are physically not valid.

After not having agreed about this issue, Lev I. Verkhovsky's has proposed us to present our criticism to the physics community.

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