Erratum of "Further Geometric and Lyapunov Characterizations of Incrementally Stable Systems on Finsler Manifolds"

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Abstract

A technical mistake of the paper $\left[\mathrm{Wu2021}\right]$ is pointed out and corrected.

In the proof of [Wu2021, Theorem 4], the function $\tilde{\beta}(r,s) := c\beta'(0,s)r$ is not necessarily a class \mathcal{KL} function even if β has the assumed properties of the theorem. For example, $\beta(r,s) = \frac{r}{rs+1}$, then $\beta'(0,s) = 1$ and $\tilde{\beta}(r,s) = cr$ is not of class \mathcal{KL} . To guarantee the correctness of the theorem, one may further assume that the limit $\lim_{h\to 0+} \frac{\beta(h,t)}{h} = \beta'(0,t)$ is uniform in $t \in \mathbb{R}$ since in this case

$$\lim_{s \to \infty} \beta'(0,s) = \lim_{s \to \infty} \lim_{h \to 0+} \frac{\beta(h,s)}{h} = \lim_{h \to 0+} \lim_{s \to \infty} \frac{\beta(h,s)}{h} = 0.$$

But this seems to be a strong assumption which makes the theorem less interesting compared to the integral form [Angeli2002, Theorem 1]. However, since Theorem 4 is a rather independent result, the other contributions of this paper are not affected.

References

- [Wu2021] D. Wu and G.-R. Duan, "Further Geometric and Lyapunov Characterizations of Incrementally Stable Systems on Finsler Manifolds," in *IEEE Transactions on Automatic Control*, vol. 67, no. 10, pp. 5614-5621, Oct. 2022
- [Angeli2002] D. Angeli, "A Lyapunov approach to incremental stability properties," in *IEEE Transactions on Automatic Control*, vol. 47, no. 3, pp. 410-421, March 2002.