The classical theory of dark matter and dark energy

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ABSTRACT

Using the principle of least action, we have obtained the four dimensional tensor of mass. Diagonal elements of mass tensor represent the usual mass, time components of that tensor represent components of dark matter, and space components are components of dark energy. According to the new theory, photons make a contribution to dark matter and dark energy.

Dark matter and dark energy are mysterious substances presented in our universe, and discovered and measured in astrophysical scale, but have never been observed on the earth. The origin of these matters is still unknown, and this poses a challenge to theoretical physics. Perhaps we are trying to investigate something wrong because we do not understand what dark matter and dark energy are. In that article we answer the most important question, existing in modern physics: what's the origin of dark matter and dark energy?

L.Landau [1] applies the principle of least action:

$$\delta S=\delta\int\left(-mcds-\frac{e}{c}A_i dx^i\right)=0 \qquad (1)$$

to derive a four-dimensional equation for the motion of a charged particle in an external electromagnetic field. Note that in this case the components of the four-potential A_i are functions of only four-dimensional coordinates x_k, i.e. $A_i(x_k)$,and does not depend on the four-dimensional velocity u_i. In this case, the four-dimensional equation of motion is written as:

$$mcw_i=\frac{e}{c}\left(\frac{\partial A_k}{\partial x^i}-\frac{\partial A_i}{\partial x^k}\right)u^k \qquad (2)$$

were the four-dimensional acceleration $w_i=\frac{du_i}{ds}$. The electromagnetic field tensor F_{ik} defined as: $F_{ik}=\frac{\partial A_k}{\partial x^i}-\frac{\partial A_i}{\partial x^k}$, and in matrix form can be written as:

$$F_{ik}=\begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & -H_3 & H_2 \\ -E_2 & H_3 & 0 & -H_1 \\ -E_3 & -H_2 & H_1 & 0 \end{pmatrix} \qquad (3)$$

were the electric field intensity components E_1, E_2, E_3 and the magnetic field intensity components H_1, H_2, H_3 by definition are:

$$E_1=F_{01}=\frac{\partial A_1}{\partial x^0}-\frac{\partial A_0}{\partial x^1}, E_2=F_{02}=\frac{\partial A_2}{\partial x^0}-\frac{\partial A_0}{\partial x^2}, E_3=F_{03}=\frac{\partial A_3}{\partial x^0}-\frac{\partial A_0}{\partial x^3} \qquad (4)$$

$$H_1=F_{32}=\frac{\partial A_2}{\partial x^3}-\frac{\partial A_3}{\partial x^2},H_2=F_{13}=\frac{\partial A_3}{\partial x^1}-\frac{\partial A_1}{\partial x^3},H_3=F_{21}=\frac{\partial A_1}{\partial x^2}-\frac{\partial A_2}{\partial x^1} \tag{5}$$

For the sake of convenience, we define the four dimensional tensor of mass $M_{ik}=m\delta_{ik}$,were the unit tensor $\delta_{ik}=\begin{cases}1, if\ i=k\\0, if\ i\neq k\end{cases}$. Then the moving equation (2) can be rewritten in a brief form:

$$cM_{ik}w^k=\frac{e}{c}F_{ik}u^k \tag{6}$$

This is exactly the same equation as the equation (2), only instead of just mass m we use the tensor of mass M_{ik}. In fact, there is no such need now, because, the tensor consists only of diagonal elements, but later we will show that this is not entirely correct, and the tensor can contain off-diagonal elements, which will be dark matter and dark energy. The four dimensional equation of motion (6) written in matrix form looks like:

$$c\begin{pmatrix}m&0&0&0\\0&m&0&0\\0&0&m&0\\0&0&0&m\end{pmatrix}\begin{pmatrix}w^0\\w^1\\w^2\\w^3\end{pmatrix}=\frac{e}{c}\begin{pmatrix}0&E_1&E_2&E_3\\-E_1&0&-H_3&H_2\\-E_2&H_3&0&-H_1\\-E_3&-H_2&H_1&0\end{pmatrix}\begin{pmatrix}u^0\\u^1\\u^2\\u^3\end{pmatrix} \tag{7}$$

To find dark matter and dark energy we consider the motion of a charged particle in external and self consistent electromagnetic field. In this case the components of the four-potential A_{ik} are functions of four-dimensional coordinates x_k and four-dimensional velocity u_k, i.e. $A_i(x_k,u_k)$. The principle of least action is the same as equation (1), we only add the dependence of the four-potential A_{ik} on the four-dimensional velocity u_{ik}. It is easy to verify, that in this case the four dimensional equation of motion is

$$mcw_i-\frac{e}{c}\left(\frac{\partial A_k}{\partial u^i}-\frac{\partial A_i}{\partial u^k}\right)w^k=\frac{e}{c}\left(\frac{\partial A_k}{\partial x^i}-\frac{\partial A_i}{\partial x^k}\right)u^k \tag{8}$$

Compared to equation (2) on the left hand side of equation (8) we have an additional term, the reason for this is that we included in the consideration the self-consistent electromagnetic field, that is, the field generated by the moving charged particle itself. Now we introduce a new four dimensional tensor of mass:

$$M_{ik}=m\delta_{ik}-\frac{e}{c^2}\left(\frac{\partial A_k}{\partial u^i}-\frac{\partial A_i}{\partial u^k}\right) \tag{9}$$

Then the equation of motion (8) is written as $cM_{ik}w^k=\frac{e}{c}F_{ik}u^k$, which is absolutely equivalent to the equation (6), with the only difference that the four dimensional tensor of mass M_{ik} have off-diagonal elements. Now, similarly to the definitions of the components of the electric field intensity (4) and the magnetic field intensity (5), we define the components of dark matter dm_1, dm_2, dm_3 and dark energy de_1, de_2, de_3:

$$dm_1=M_{01}=\frac{-e}{c^2}\left(\frac{\partial A_1}{\partial u^0}-\frac{\partial A_0}{\partial u^1}\right),dm_2=M_{02}=\frac{-e}{c^2}\left(\frac{\partial A_2}{\partial u^0}-\frac{\partial A_0}{\partial u^2}\right),$$
$$dm_3=M_{03}=\frac{-e}{c^2}\left(\frac{\partial A_3}{\partial u^0}-\frac{\partial A_0}{\partial u^3}\right) \tag{10}$$

$$de_1 = M_{32} = \frac{-e}{c^2}\left(\frac{\partial A_2}{\partial u^3} - \frac{\partial A_3}{\partial u^2}\right), de_2 = M_{13} = \frac{-e}{c^2}\left(\frac{\partial A_3}{\partial u^1} - \frac{\partial A_1}{\partial u^3}\right),$$
$$de_3 = M_{21} = \frac{-e}{c^2}\left(\frac{\partial A_1}{\partial u^2} - \frac{\partial A_2}{\partial u^1}\right) \tag{11}$$

then the four dimensional tensor of mass M_{ik} can be written in matrix form as:

$$M_{ik} = \begin{pmatrix} m & dm_1 & dm_2 & dm_3 \\ -dm_1 & m & -de_3 & de_2 \\ -dm_2 & de_3 & m & -de_1 \\ -dm_3 & -de_2 & de_1 & m \end{pmatrix} \tag{12}$$

and the four dimensional equation of motion as:

$$c\begin{pmatrix} m & dm_1 & dm_2 & dm_3 \\ -dm_1 & m & -de_3 & de_2 \\ -dm_2 & de_3 & m & -de_1 \\ -dm_3 & -de_2 & de_1 & m \end{pmatrix}\begin{pmatrix} w^0 \\ w^1 \\ w^2 \\ w^3 \end{pmatrix} = \frac{e}{c}\begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & -H_3 & H_2 \\ -E_2 & H_3 & 0 & -H_1 \\ -E_3 & -H_2 & H_1 & 0 \end{pmatrix}\begin{pmatrix} u^0 \\ u^1 \\ u^2 \\ u^3 \end{pmatrix} \tag{13}$$

At first glance, it seems that the differences between equations (2) and (8) are not so big, but they are not, they are big. We well try to explain it as simply as possible. All components of equation (2) are derivatives of some function with respect to coordinates, as are almost all equations of modern physics. Unlike equation (2), equation (8) contains dark matter and dark energy components that are derivatives of the four-potential with respect to four-dimensional velocity, something that does not occur in physics. This leads us to the conclusion that the expressions for dark matter (10) and dark energy (11) are the first necessary step toward new physics, both theoretical and experimental. Also, people have been using mass for thousands of years assuming that it is a scalar quantity, but according to our theory, it is a tensor quantity, which makes physics more complicated.

What about the mass tensor of a massless particle like a photon? Substituting $m=0$ into the expression (12), we obtain the photon mass tensor:

$$M_{ik} = \begin{pmatrix} 0 & dm_1 & dm_2 & dm_3 \\ -dm_1 & 0 & -de_3 & de_2 \\ -dm_2 & de_3 & 0 & -de_1 \\ -dm_3 & -de_2 & de_1 & 0 \end{pmatrix} \tag{14}$$

The photon mass tensor is not equal to the zero tensor. It consists only of components of dark matter and dark energy. If we consider these components in the framework of the special theory of relativity, flat spacetime, then they all become equal to zero and there is no dark matter or dark energy. But in curved spacetime (the general theory of relativity), dark matter and dark energy components are not equal to zero, and therefore a photon can be a source of dark matter and dark energy.

Dark matter and dark energy are considered "dark" because we don't know the origin of it. But according to our new theory, that dark matter and dark energy are not "dark" anymore, and each component of the four dimensional tensor of mass has an exact physical meaning. The diagonal elements of the tensor (13) are the usual mass. Off-diagonal elements of the first row and first column of the four dimensional tensor of mass are called the time components of the tensor, and therefore we name them as components of the time mass. All other off-diagonal components of the tensor are called space components, and hence we call them as space mass components. We change names of dark matter components dm_1, dm_2, dm_3 to time mass components tm_1, tm_2, tm_3 , and names of dark energy components de_1, de_2, de_3 to space mass components sm_1, sm_2, sm_3. For the sake of completeness, we will call the mass m as the baryonic mass bm. Then the four dimensional tensor of mass can be written as:

$$M_{ik} = \begin{pmatrix} bm & tm_1 & tm_2 & tm_3 \\ -tm_1 & bm & -sm_3 & sm_2 \\ -tm_2 & sm_3 & bm & -sm_1 \\ -tm_3 & -sm_2 & sm_1 & bm \end{pmatrix} \quad (15)$$

and each component of this tensor has a physical meaning.

CONCLUSION

We developed the theory of dark matter and dark energy. The mass of a moving charged particle is not a scalar quantity, it is a tensor quantity, the four dimensional tensor of mass. The diagonal elements of this tensor are the baryonic mass, time components represent dark matter and space components are dark energy. Photons create dark matter and dark energy in curved spacetime.

REFERENCES

1. L. D. Landau and E. M. Lifshitz, The Classical Theory of Fields, 3rd ed. (Pergamon Press, 1971), Vol. 2.