ETUDE ON NONTRIVIAL COLLATZ CYCLES

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Nontrivial Collatz cycles correspond to the expression (1.1) or (1.2):

\[
\frac{(3a+1)(3b+1)(3c+1)}{abc} = 2^h \quad (1.1) \quad \frac{(A+1)(B+1)(C+1)}{ABC} = \frac{2^h}{3^m} \quad (1.2)
\]

The central part of transformation (2) from expression (1.2) corresponds to the sequence \((3n+1)\) for negative numbers.

\[
\frac{(A'+1)(B'+1)(C'+1)}{A'B'C'} = \frac{ABC}{(A-1)(B-1)(C-1)} \equiv \frac{3^m}{2^n} \quad (2)
\]

The sequence \((3n+1)\) for negative numbers actually has nontrivial cycles, accordingly of the central part of expression (2) correlates with the series \(\frac{3^m}{2^n}\). The left part of expression (2) correspond to the Collatz sequence for even numbers.

\[
\frac{3^m}{2^n} \equiv \frac{(A'+1)(B'+1)(C'+1)}{A'B'C'} \not\equiv \frac{ABC}{(A+1)(B+1)(C+1)} \quad (3)
\]

Replacing the left-hand side of the expression (2) of even numbers with odd ones (Collatz sequence) leads to non performing of correlation with the series \(\frac{3^m}{2^n}\) and the absence of non-trivial cycles (3).