de Sitter symmetry and neutrino oscillations

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Abstract

Although the phenomenon of neutrino oscillations is confirmed in many experiments, the theoretical explanation of this phenomenon in the literature is essentially model dependent and is not based on rigorous physical principles. We propose an approach where the neutrino is treated as a massless elementary particle in anti-de Sitter (AdS) invariant quantum theory. In contrast to standard Poincare invariant quantum theory, an AdS analog of the mass squared changes over time even for elementary particles. Our approach naturally explains why, in contrast to the neutrino, the electron, muon and tau lepton do not have flavors changing over time and why the number of solar neutrinos reaching the Earth is around a third of the number predicted by the standard solar model.

1 Problem statement

The phenomenon of neutrino oscillations is discussed in a vast literature and, as noted by most authors, the theory of this phenomenon contains essential uncertainties (see e.g., [1, 2]). We first describe the crucial difference between this theory and standard particle theory.

The goal of standard Poincare invariant particle theory is to find the S-matrix describing transitions between different sets of elementary particles. By definition, an elementary particle is described by an irreducible representation (IR) of the Poincare algebra. If P_0 is the energy operator and $\mathbf{P}$ is the momentum operator then $W = P_0^2 - \mathbf{P}^2$ is the Casimir operator of the Poincare algebra, i.e., it commutes with all operators defining the representation of the algebra for the system under consideration. As follows from the Schur lemma, all elements in the representation space of an IR are eigenvectors of W with some eigenvalue w. The existence of tachyons (for which $w < 0$) is a problem while for known elementary particles, their mass m is defined as a positive number m such that $m^2 = w$.

In standard particle theory, all processes are described by Feynman diagrams involving only elementary particles. In particular, in standard theory of weak interactions, the electron, muon and tau neutrinos with different flavors $(\nu_e, \nu_\mu, \nu_\tau)$=$(f_1, f_2, f_3)$ are treated as elementary particles, and the processes of their creation/annihilation are described by Feynman diagrams involving the (W, e, ν_e), (W, μ, ν_μ) and (W, τ, ν_τ) vertices.

In standard particle theory, the electron, muon and tau lepton quantum numbers are conserved physical quantities. However, the phenomenon of neutrino oscillations shows that conservation of those quantum numbers can be only approximate. For explaining this phenomenon, a fundamentally new property was introduced into the theory: now the neutrinos with definite flavors are treated not as elementary particles but as direct superpositions of neutrino states (ν_1, ν_2, ν_3) such that ν_i (i=1,2,3) is an elementary particle with a definite mass m_i, i.e.,

$$f_i = \sum_{j=1}^{3} \oplus U_{ij}\nu_j \ (i = 1, 2, 3) \tag{1}$$

where U_{ij} are elements of a complex 3×3 matrix. The mathematical meaning of this expression is as follows. Each state ν_j belongs to a Hilbert space H_j of an IR with the mass m_j while f_i belongs to the direct sum of Hilbert spaces H_j. Although the principle of superposition in quantum theory does not prohibit the existence of states belonging to direct sums of IRs, such states have not been used in quantum theory till the discussion of neutrino oscillations.

The concept of direct sum of Hilbert spaces crucially differs from the concept of tensor product of Hilbert spaces which is a standard concept in quantum theory. In particular, if H_1 and H_2 are representation spaces for IRs with the masses m_1 and m_2 then the tensor product of H_1 and H_2 describes a system of two elementary particles while the direct sum of H_1 and H_2 describes one particle which is not elementary because it is a quantum superposition of two elementary particles.

The new treatment of neutrinos implies that in the Feynman diagrams involving the (W, e, ν_e), (W, μ, ν_μ) and (W, τ, ν_τ) vertices, the neutrinos are not elementary particles, but nontrivial direct sums defined by Eq. (1). The theory of such diagrams has been proposed in [3] and here the neutrinos are superpositions of not plane waves but wave packets. As noted in [3], in this approach several problems should be solved.

In view of the current status of the neutrino theory, the following questions arise:

- Why only neutrinos have such an unusual status while the other leptons in these vertices $(e^\pm, \mu^\pm, \tau^\pm)$ are still treated as elementary particles.

- Why, in contrast to all other particle theories, the theory involving neutrinos is formulated not in terms of Feynman diagrams involving the elementary particles (ν_1, ν_2, ν_3) but in terms of $(\nu_e, \nu_\mu, \nu_\tau)$ which are now treated not as elementary particles.

- It is not clear whether the numbers U_{ij} should be defined by a new theory or they can be defined only by fitting experimental data.

Let us stress again that Eq. (1) does not imply that each neutrino is a composite state of states (ν_1, ν_2, ν_3). This expression shows that each neutrino can be detected only *in one of the states* (ν_1, ν_2, ν_3) with the probabilities defined by the numbers U_{ij}. In theories of entanglement, it is explained that, as follows from the principle of the wave function collapse, a measurement of the first entangled particle automatically reduces the wave functions of other particles entangled with the first one, even if they are very far from the first particle. Analogously, after a neutrino takes part in any interaction, the remaining wave function cannot be a superposition Eq. (1) anymore and only one of the wave functions ν_j (if any) can survive.

In view of the fact that, in the processes of neutrino oscillations, neutrinos can travel long distances during a long time, a question arises why, in spite of the phenomenon of the wave function collapse, those neutrinos remain superpositions of the states (ν_1, ν_2, ν_3) during all this time. Even if those neutrinos do not interact with matter, they necessarily interact with the background gravitational field, and, as follows from the principle of the wave function collapse, the wave functions of such neutrinos must collapse even after very weak interactions.

The above discussion poses a problem whether the phenomenon of neutrino oscillations can be explained if neutrinos are still treated as elementary particles. We believe that standard particle theories are not quite natural in view of the following. Those theories are based on Poincare symmetry where elementary particles are described by IRs of the Poincare group or its Lie algebra. In his famous paper "Missed Opportunities" [4] Dyson notes that de Sitter (dS) and anti-de Sitter (AdS) theories are more general (fundamental) than Poincare one even from pure mathematical considerations because dS and AdS groups are more symmetric than Poincare one. The transition from the former to the latter is described by a procedure called contraction when a parameter R (see below) goes to infinity. At the same time, since dS and AdS groups are semisimple, they have a maximum possible symmetry and cannot be obtained from more symmetric groups by contraction.

The paper [4] appeared in 1972 (i.e., more than 50 years ago) and, in view of Dyson's results, a question arises why general theories of elementary particles (QED, electroweak theory and QCD) are still based on Poincare symmetry and not dS or AdS ones. Probably, physicists believe that, since the parameter R is much greater than even sizes of stars, dS and AdS symmetries can play an important role only in cosmology and there is no need to use them for describing elementary particles. We believe that this argument is not consistent because usually more general theories shed a new light on standard concepts. The discussion in our publications and, in particular, in this paper is a good illustration of this point.

In [5] it has been proposed the following

Definition: *Let theory A contain a finite nonzero parameter and theory B be obtained from theory A in the formal limit when the parameter goes to zero or infinity. Suppose that, with any desired accuracy, theory A can reproduce any result of theory B by choosing a value of the parameter. On the contrary, when the limit is already taken, one cannot return to theory A, and theory B cannot reproduce all results of theory A. Then theory A is more general than theory B and theory B is a special degenerate case of theory A.*

As argued in [5, 6], in contrast to Dyson's approach based on Lie groups, the approach to symmetry on quantum level should be based on Lie algebras. Then it has been proved that, on quantum level, dS and AdS symmetries are more general (fundamental) than Poincare symmetry, *and this fact has nothing to do with the comparison of dS and AdS spaces with Minkowski space*. It has been also proved that classical theory is a special degenerate case of quantum one in the formal limit $\hbar \to 0$ and nonrelativistic theory is a special degenerate case of relativistic one in the formal limit $c \to \infty$. In the literature the above facts are explained from physical considerations but, as shown in [5, 6], they can be proved mathematically by using properties of Lie algebras.

The goal of the present paper is to investigate whether the phenomenon of neutrino oscillations can be explained in the framework of the AdS quantum theory by treating neutrinos as elementary particles. In Sec. 2 we describe problems in treating neutrino oscillations in the framework of current approaches. In Sec. 3 we explain why quantum theory based on de Sitter symmetries is more general and natural than standard quantum theory. In Secs. 4 and 5 we describe a formalism for describing the massless neutrino as an elementary particle in AdS invariant quantum theory. In Sec. 6 we discuss how the problem of neutrino flavors is treated in our approach and explain that the solar neutrino problem has a natural explanation.

2 Direct sum of Hilbert spaces in neutrino oscillations

In the present treatments of neutrino oscillations, it is assumed that all the masses m_j are different. Then the states ν_j are mutually orthogonal even without the requirement that they belong to different Hilbert spaces H_j. So, the states ν_j (j=1,2,3) can be chosen as a basis in a full Hilbert space H, and the sign $\oplus$ in Eq. (1) is obsolete. The states ν_j are not characterized by flavor and they cannot have different quantum numbers except m_j because in that case their superpositions will be prohibited.

Since the neutrino flavor quantum numbers are not conserved anymore, there are no quantum numbers distinguishing the states $(\nu_e, \nu_\mu, \nu_\tau)$ and they differ only because they are defined by different decompositions in Eq. (1). The phenomenon of neutrino oscillations shows that the states $(\nu_e, \nu_\mu, \nu_\tau)$ can transform to each other and those states do not have definite masses. The existing quantum theory does not specify physical criteria which should be used for constructing the matrix U, and currently the numbers U_{ij} are defined only from fitting the experimental data. Wikipedia says: "*$U_{\alpha i}$ represents the Pontecorvo–Maki–Nakagawa–Sakata matrix (also called the PMNS matrix, lepton mixing matrix, or sometimes simply the MNS matrix). It is the analogue of the CKM matrix describing the analogous mixing of quarks. If this matrix were the identity matrix, then the flavor eigenstates would be the same as the mass eigenstates. However, experiment shows that it is not.*" In [2] it is discussed in detail which numbers U_{ij} are well defined by experimental data, which of them are defined with uncertanties and which of those numbers are not known yet.

Since there should be a one-to-one relation between the states $(\nu_e, \nu_\mu, \nu_\tau)$ and (ν_1, ν_2, ν_3), the matrix U should be invertable and then, as follows from Eq. (1),

$$\nu_i = \sum_{j=1}^{3} (U^{-1})_{ij} f_j \ (i = 1, 2, 3) \tag{2}$$

There are no physical criteria requiring that each ν_j in Eq. (1) has the same momentum but usually it is assumed that the components (ν_1, ν_2, ν_3) are ultrarelativistic, their energies E_j can be written as $E_j = (p_j^2 + m_j^2)^{1/2} \approx p + m_j^2/2p$, where p is the same for all the components. Suppose that all the ν_j are moving along the x axis. Then the dependence of the wave function of ν_j on t and x can be written as $\nu_j(t,x) = exp[-i(p^2 + m_j^2)^{1/2} t + ipx]\nu_j$. Then, since each neutrino state is strongly ultrarelativistic, it follows from Eq. (1) that

$$f_j(t,x) = exp[-i(t-x)p] \sum_{k=1}^{3} exp(-i\frac{m_k^2}{2p}t) U_{jk} \nu_k \ (j = 1, 2, 3) \tag{3}$$

Finally, as follows from this expression and Eq. (2), in units $\hbar = c = 1$

$$f_j(t,x) = exp[-i(t-x)p] \sum_{k,l=1}^{3} exp(-i\frac{m_k^2}{2p}t) U_{jk} (U^{-1})_{kl} f_l \ (j = 1, 2, 3) \tag{4}$$

If all the m_k are the same and equal m, it follows from Eq. (4) that $f_j(t,x) = exp[-i(t-x)p - i\frac{m^2}{2p}t]f_j$, i.e., f_j is moving as a particle with the mass m and neutrino oscillations do not take place. However, if the masses m_k are different, the oscillations are possible and their quantitative details depend on the model for U_{jk}.

The problem of the mass-squared differences $\Delta m_{jk}^2 = m_j^2 - m_k^2$ has been discussed in many publications and typical fitting procedures give for those quantities values of the order of $10^{-4}ev^2$ (see e.g., [2]). In view of this observation, consider the following problem.

Neutrinos emitted by distant stars fly to Earth for many years. What are typical states of those neutrinos when they reach the Earth, i.e., to what extent their interaction with the interstellar medium is important? An analogous problem was widely discussed for photons emitted by stars. The approach where the interaction of such photons with the interstellar medium is important was called "tired light". However, after numerous discussions, physicists have come to the conclusion that the phenomenon "tired light" does not explain the available data, and that photons reaching the Earth practically do not interact with the interstellar medium. Since the interaction of neutrinos emitted by stars with the interstellar medium is much weaker than the interaction of the photons, the major part of those neutrinos practically do not interact with the interstellar medium.

Then the following problem arises. Since the masses (m_1, m_2, m_3) are different and the momentum p of those masses is the same, the velocities of those masses are different. Since the masses are ultrarelativistic, the differences of velocities are $\Delta v_{jk} = \Delta m_{jk}^2/2p^2$ and, if p is of the order of Mev/c then after a year, the distances between the masses will be of the order of $1m$. So, for example, neutrinos coming from Sirius (which is "only" 8 light years from the Earth) will be superpositions of elementary particles the distances between which will be of the order of $8m$ but the major part of neutrinos which reach the Earth will be quantum superpositions of elementary particles the distances between which can be of the order of kilometers or more. A problem arises whether interactions of such neutrinos with detectors on Earth can be still described in terms of $(\nu_e, \nu_\mu, \nu_\tau)$. This problem is of high theoretical interest, but the experimental investigation of this problem is problematic. Most of neutrinos detected by neutrino observatories are either solar neutrinos or neutrinos produced when energetic particles from space, crash into Earth's atmosphere. So, it is very difficult to select cases when a detected neutrino came to Earth from a distant star.

We conclude that, although the phenomenon of neutrino oscillations is well confirmed in many experiments, the discussion in Sec. 1 and in this section shows that the theoretical explanation of this phenomenon in the literature is essentially model dependent and is not based on rigorous physical principles.

3 Poincare invariance and de Sitter invariance

As already noted, the main goal of the present paper is to investigate whether the phenomenon of neutrino oscillations can be explained in the framework of de Sitter invariant particle theory.

Standard particle theory is based on Poincare symmetry where elementary particles are described by IRs of the Poincare group or its Lie algebra. The representation operators of the Poincare algebra commute according to the commutation relations

$$[P^\mu, P^\nu] = 0, \quad [P^\mu, M^{\nu\rho}] = -i(\eta^{\mu\rho}P^\nu - \eta^{\mu\nu}P^\rho),$$
$$[M^{\mu\nu}, M^{\rho\sigma}] = -i(\eta^{\mu\rho}M^{\nu\sigma} + \eta^{\nu\sigma}M^{\mu\rho} - \eta^{\mu\sigma}M^{\nu\rho} - \eta^{\nu\rho}M^{\mu\sigma}) \tag{5}$$

where $\mu, \nu = 0, 1, 2, 3$, P^μ are the operators of the four-momentum, $M^{\mu\nu}$ are the operators of Lorentz angular momenta and $\eta^{\mu\nu}$ is such that $\eta^{00} = -\eta^{11} = -\eta^{22} = -\eta^{33} = 1$ and $\eta^{\mu\nu} = 0$ if $\mu \neq \nu$.

Although the Poincare group is the group of motions of Minkowski space, the description in terms of relations (5) does not involve Minkowski space at all. It involves only representation operators of the Poincare algebra, and *those relations can be treated as a definition of relativistic invariance on quantum level* (see the discussion in [5, 6]). In particular, the fact that $\eta^{\mu\nu}$ formally coincides with the metric tensor in Minkowski space does not imply that this space is involved.

Since W is the Casimir operator of the Poincare algebra, the mass m of an elementary particle remains the same during the lifetime of this particle, and, since the momentum, energy and angular momentum operators commute with the Hamiltonian, for any free elementary particle (i.e., the particle which does not interact with other particles and with external fields), these physical quantities are conserved.

By analogy with relativistic quantum theory, the definition of quantum dS symmetry should not involve dS space. If M^{ab} ($a, b = 0, 1, 2, 3, 4$, $M^{ab} = -M^{ba}$) are the operators describing the system under consideration, then, *by definition of dS symmetry on quantum level*, they should satisfy the commutation relations *of the dS Lie algebra* so(1,4), *i.e.*,

$$[M^{ab}, M^{cd}] = -i(\eta^{ac}M^{bd} + \eta^{bd}M^{ac} - \eta^{ad}M^{bc} - \eta^{bc}M^{ad}) \tag{6}$$

where η^{ab} is such that $\eta^{00} = -\eta^{11} = -\eta^{22} = -\eta^{33} = -\eta^{44} = 1$ and $\eta^{ab} = 0$ if $a \neq b$. The *definition* of AdS symmetry on quantum level is given by the same equations but $\eta^{44} = 1$.

The procedure of contraction from dS and AdS symmetries to Poincare one is defined as follows. If we *define* the operators P^μ as $P^\mu = M^{4\mu}/2R$, where R is a parameter with the dimension *length*, then in the formal limit when $R \to \infty$, $M^{4\mu} \to \infty$ but the quantities P^μ are finite, Eqs. (6) become Eqs. (5). This procedure is the same for the dS and AdS symmetries.

By analogy with relativistic quantum theory, in theories with dS and AdS symmetries, elementary particles can be defined as objects described by IRs of the dS and AdS algebras. The contraction procedure shows that M^{40} is the dS or AdS analog of the relativistic Hamiltonian and the operators M^{4i} ($i = 1, 2, 3$) are the de Sitter analogs of the relativistic momentum operators $\mathbf{P}^i$. However, as follows from Eq. (6), the operators M^{4i} do not commute with M^{40}. Therefore, if, by analogy with the relativistic case, the evolution over time is defined by M^{40} then the physical quantities corresponding to the M^{4i} are not conserved even for free particles. Also, since the operator $\tilde{W} = M_{04}^2 - \sum_{i=1}^3 M_{0i}^2$ is not the Casimir operator for the dS and AdS algebras, the eigenvalues of this operator cannot be treated as the mass squared which remains the same during the lifetime of the free elementary particle. On the other hand, since the operators (M_{12}, M_{31}, M_{23}) commute with M^{40}, the angular momentum operators in de Sitter theories are conserved.

The Casimir operator of the second order for the representation of the so(2,3) algebra is

$$I_2 = \frac{1}{2}\sum_{ab} M_{ab}M^{ab} \tag{7}$$

Therefore, states of elementary particles in the AdS invariant theory are eigenvectors of I_2 with the eigenvalues m_{AdS}^2, i.e., m_{AdS} is the AdS analog of m and remains the same during the lifetime of the particle.

The procedure of contraction has a physical meaning only if R is rather large. In that case, m_{AdS} and m are related as $m_{AdS} = 2Rm$, and the relation between the AdS and Poincare energies is analogous. Since AdS symmetry is more general (fundamental) then Poincare one then m_{AdS} is more general (fundamental) than m. In contrast to the Poincare masses and energies, the AdS masses and energies are dimensionless. From cosmological considerations (see e.g., [5]), the value of R is usually accepted to be of the order of $10^{26}m$. Then the AdS masses of the electron, the Earth and the Sun are of the order of 10^{39}, 10^{93} and 10^{99}, respectively. The fact that even the AdS mass of the electron is so large might be an indication that the electron is not a true elementary particle. In addition, the accepted upper level for the photon mass is $10^{-17}ev$. This value seems to be an extremely tiny quantity. However, the corresponding AdS mass is of the order of 10^{16}, and so, even the mass which is treated as extremely small in Poincare invariant theory might be very large in AdS invariant theory. As noted in Sec. 1, it has been proved in [4, 5, 6] that quantum theory based on the dS and AdS symmetries is more general (fundamental) than quantum theory based on Poincare symmetry: the latter is the special degenerate case of the former in the formal limit $R \to \infty$.

4 Massless neutrino in AdS theory

Before the discovery of neutrino oscillations, the neutrino was treated as the massless elementary particle with the spin 1/2. In this section we discuss how massless elementary particles should be described in AdS theory. Our consideration is based on the results of [5, 7, 8, 9].

Let (a_j', a_j'', h_j) $(j = 1, 2)$ be two independent sets of operators satisfying the commutation relations for the sp(2) algebra

$$[h_j, a_j'] = -2a_j', \quad [h_j, a_j''] = 2a_j'', \quad [a_j', a_j''] = h_j \tag{8}$$

The sets are independent in the sense that for different j they mutually commute with each other.

Since the AdS algebra is 10-dimensional, as well as the Poincare algebra, in addition to the six operators (a_j', a_j'', h_j), we should also define four operators which we denote as b', b'', L_+, L_-. The operators $L_3 = h_1 - h_2, L_+, L_-$ satisfy the commutation relations of the su(2) algebra

$$[L_3, L_+] = 2L_+, \quad [L_3, L_-] = -2L_-, \quad [L_+, L_-] = L_3 \tag{9}$$

and the other commutation relations are as follows

$$[a_1', b'] = [a_2', b'] = [a_1'', b''] = [a_2'', b''] = [a_1', L_-] = [a_1'', L_+] =$$

$$[a'_2, L_+] = [a''_2, L_-] = 0, \quad [h_j, b'] = -b', \quad [h_j, b''] = b''$$
$$[h_1, L_\pm] = \pm L_\pm, \quad [h_2, L_\pm] = \mp L_\pm, \quad [b', b''] = h_1 + h_2$$
$$[b', L_-] = 2a'_1, \quad [b', L_+] = 2a'_2, \quad [b'', L_-] = -2a''_2, \quad [b'', L_+] = -2a''_1$$
$$[a'_1, b''] = [b', a''_2] = L_-, \quad [a'_2, b''] = [b', a''_1] = L_+$$
$$[a'_1, L_+] = [a'_2, L_-] = b', \quad [a''_2, L_+] = [a''_1, L_-] = -b'' \qquad (10)$$

At first glance these relations might seem rather chaotic but in fact they are very natural in the Weyl basis of the so(2,3) algebra.

The relation between the above set of ten operators and M_{ab} is

$$M_{10} = i(a''_1 - a'_1 - a''_2 + a'_2), \quad M_{14} = a''_2 + a'_2 - a''_1 - a'_1$$
$$M_{20} = a''_1 + a''_2 + a'_1 + a'_2, \quad M_{24} = i(a''_1 + a''_2 - a'_1 - a'_2)$$
$$M_{12} = L_3, \quad M_{23} = L_+ + L_-, \quad M_{31} = -i(L_+ - L_-)$$
$$M_{04} = h_1 + h_2, \quad M_{34} = b' + b'', \quad M_{30} = -i(b'' - b') \qquad (11)$$

and therefore the sets are equivalent.

We work in the system of units $\hbar/2 = c = 1$. Then $s = 1$ for particles with spin 1/2 in the usual units where $\hbar = 1$. We use the basis in which the operators (h_j, K_j) $(j = 1, 2)$ are diagonal. Here K_j is the Casimir operator

$$K_j = h_j^2 - 2h_j - 4a''_j a'_j = h_j^2 + 2h_j - 4a'_j a''_j \qquad (12)$$

for the algebra $(a'_j, a_j", h_j)$. For constructing IRs we need operators relating different representations of the sp(2)×sp(2) algebra. By analogy with [5, 7, 8, 9], one of the possible choices is:

$$A^{++} = b''(h_1 - 1)(h_2 - 1) - a''_1 L_-(h_2 - 1) - a''_2 L_+(h_1 - 1) + a''_1 a''_2 b'$$
$$A^{+-} = L_+(h_1 - 1) - a''_1 b', \quad A^{-+} = L_-(h_2 - 1) - a''_2 b', \quad A^{--} = b' \qquad (13)$$

We consider the action of these operators only on the space of minimal sp(2)×sp(2) vectors, i.e., such vectors x that $a'_j x = 0$ for $j = 1, 2$, and x is the eigenvector of the operators h_j. If x is a minimal vector such that $h_j x = \alpha_j x$ then $A^{++}x$ is the minimal eigenvector of the operators (h_1, h_2) with the eigenvalues $(\alpha_1 + 1, \alpha_2 + 1)$, $A^{+-}x$ - with the eigenvalues $(\alpha_1 + 1, \alpha_2 - 1)$, $A^{-+}x$ - with the eigenvalues $(\alpha_1 - 1, \alpha_2 + 1)$, and $A^{--}x$ - with the eigenvalues $(\alpha_1 - 1, \alpha_2 - 1)$.

In the theory of IRs of Lie algebras, it is known that each nonzero vector e_0 in the space of IRs is cyclic, i.e., that any vector in the representation space can be obtained by acting by representation operators on e_0 and taking all possible linear combinations of the results. We choose as e_0 the vector e_0 satisfying the conditions

$$a'_j e_0 = b' e_0 = L_+ e_0 = 0, \quad h_j e_0 = q_j e_0 \quad (j = 1, 2) \qquad (14)$$

where q_j are positive integers. Then, as shown in [5], for the massless IR with the spin $s = 1$, $q_2 = 1$ and $q_1 = q_2 + s = 2$. The reasons why such IRs are called massless will be explained below.

As follows from Eqs. (8-11)

$$I_2 = 2(h_1^2 + h_2^2 - 2h_1 - 4h_2 - 2b''b' + 2L_-L_+ - 4a_1''a_1' - 4a_2''a_2') \quad (15)$$

Then, in the IR characterized by (q_1, q_2), all the nonzero elements of the representation space are the eigenvectors of the operator I_2 with the eigenvalue

$$I_2 = 2(q_1^2 + q_2^2 - 2q_1 - 4q_2) \quad (16)$$

In particular, for the massless neutrino, $I_2 = -6$. One of the reasons why such an IR is called massless, is that, as follows from the contraction procedure, the Casimir operator W in the Poincare invariant theory is a formal limit of $I_2/4R^2$ when $R \to \infty$ and this limit obviously equals zero.

As follows from Eqs. (8) and (10), the operators (a_1', a_2', b') reduce the AdS energy (h_1+h_2) by two units. Therefore e_0 is an analog of the state with the minimum energy which can be called the rest state. In standard classification [7], the massive case is characterized by the condition $q_2 > 1$ and the massless one — by the condition $q_2 = 1$.

We define the vectors

$$e(0,0,1) = [b"(h_1-1) - a_1"L_-]e_0,\ f(0,0,0) = L_-e_0,\ f(0,0,1) = [b''(h_2-1) - a_2''L_+]f_0 \quad (17)$$

Then, as shown in Chapter 8 of [5], the basis of the massless IR of the so(2,3) algebra with $s = 1$ consists of e_0, the vectors defined by Eq. (17), the vectors

$$e(n_1,n_2,n) = (a_1")^{n_1}(a_2")^{n_2}e(0,0,n),\ f(n_1,n_2,n) = (a_1")^{n_1}(a_2")^{n_2}f(0,0,n) \quad (18)$$

if $n <= 1$ and the vectors

$$\begin{aligned} e(n_1,n_2,n) &= (a_1")^{n_1}(a_2")^{n_2}(A^{++})^{n-1}e(0,0,1) \\ f(n_1,n_2,n) &= (a_1")^{n_1}(a_2")^{n_2}(A^{++})^{n-1}f(0,0,1) \end{aligned} \quad (19)$$

if $n > 1$. Here n_1 and n_2 are any positive integers.

One might think that, as follows from the definition of the operators A^{+-} and A^{-+}, $A^{+-}e(0,0,n)$ should be proportional to $f(0,0,n)$ and $A^{-+}f(0,0,n)$ should be proportional to $e(0,0,n)$. However, a direct calcullation using Eq. (10) shows that, in the massless case, $A^{+-}e(0,0,n) = 0$ and $A^{-+}f(0,0,n) = 0$.

In Poincare invariant theory without spatial reflections, massless particles are characterized by the condition that they have a definite helicity. The operator M^{04} is the AdS energy, and its minimum eigenvalue for massless IRs with positive energy is $E_{min} = 3$ for the neutrino. In contrast to the situation in Poincare invariant theory, where massless particles cannot be in the rest state, the massless particles in the AdS theory do have rest states and, for $s = 1$, the values of the z projection of the spin can be -1 or 1. However, since $A^{+-}e(0,0,n) = 0$ and $A^{-+}f(0,0,n) = 0$ for $n \geq 1$, for any value of the energy greater than E_{min}, the spin state is characterized only by helicity, which can take the values either 1 or -1, i.e., we have the same result as in Poincare invariant theory. Note that, in contrast to IRs of the Poincare algebra, IRs describing particles in AdS invariant theory belong to the discrete series of IRs and the energy spectrum in them is discrete: $E = E_{min}, E_{min}+2, ...\infty$.

Therefore, strictly speaking, the rest states do not have measure zero as in Poincare invariant theories.

Nevertheless, although the probability that the energy is exactly E_{min} is extremely small, as a consequence of existence of rest states, one IR now contains states with both helicities, 1 and -1. Indeed, the states $e(0,0,n)$ for $n\geq 1$ have the energy $E=E_{min}+2n$ and helicity 1. By acting by the operator $(b')^n$ on $e(0,0,n)$ we obtain the element proportional to the rest state e_0. Then L_-e_0 is the rest state $f(0,0,0)$. Finally, as explained above, one can obtain $f(0,0,1)$ from $f(0,0,0)$ and, if $n>1$, $f(0,0,n)$ can be obtained as $(A^{++})^{n-1}f(0,0,1)$. This state has the energy $E=E_{min}+2n$ and helicity -1. Therefore, as a consequence of existence of rest states, states with the same energies but opposite helicities belong to the same IR.

A known case in Poincare invariant theory is that if neutrino is massless then neutrino and antineutrino are different particles with opposite helicities. However, in AdS theory, neutrino and antineutrino can be only different states of the same particle. In experiment they manifest as different particles because the probability to be in the rest state is extremely small and in weak reactions only states with definite helicities can take part. Note also that in Poincare invariant theory, the photon is a massless elementary particle which does not have a definite helicity because it is described by an IR in the theory where, in addition to Poincare invariance, invariance under spatial reflections is required.

5 Matrix elements of representation operators

The states $e(n_1,n_2,n)$ are not normalized to one. A direct calculation using Eqs. (10,13,14,17) gives that the norm of the state $e(n_1,n_2,n)$ is

$$N(n_1,n_2,n)=(e(n_1,n_2,n),e(n_1,n_2,n))=n_1!n_2!(1+n+n_1)!(n+n_2)![n!(n-1)!]^2 \quad (20)$$

Therefore, the basis vectors normalized to one are $\tilde{e}(n_1,n_2,n)=N(n_1,n_2,n)^{-1/2}e(n_1,n_2,n)$. Then, as follows from Eqs. (10,13,14,17), the matrix elements of the representation operators in the normalized basis are given by

$$\begin{aligned}
b'\tilde{e}(n_1,n_2,n) &= (n_1n_2)^{1/2}\tilde{e}(n_1-1,n_2-1,n+1)+\\
&\quad [(n+n_1+1)(n+n_2)]^{1/2}\tilde{e}(n_1,n_2,n+1)\\
b"\tilde{e}(n_1,n_2,n) &= [(n+n_1+2)(n+n_2+1)]^{1/2}\tilde{e}(n_1,n_2,n+1)+\\
&\quad [(n_1+1)(n_2+1)]^{1/2}\tilde{e}(n_1+1,n_2+1,n-1)\\
a_1'\tilde{e}(n_1,n_2,n) &= [n_1(n+n_1+1)]^{1/2}\tilde{e}(n_1-1,n_2,n)\\
a_1"\tilde{e}(n_1,n_2,n) &= [(n_1+1)(n+n_1+2)]^{1/2}\tilde{e}(n_1+1,n_2,n)\\
a_2'\tilde{e}(n_1,n_2,n) &= [n_2(n+n_2+1)]^{1/2}\tilde{e}(n_1,n_2-1,n)\\
a_2"\tilde{e}(n_1,n_2,n) &= [(n_2+1)(n+n_2+2)]^{1/2}\tilde{e}(n_1,n_2+1,n)\\
L_+\tilde{e}(n_1,n_2,n) &= [n_2(n+n_1+2)]^{1/2}\tilde{e}(n_1,n_2-1,n+1)+\\
&\quad [(n_1+1)(n+n_2)]^{1/2}\tilde{e}(n_1+1,n_2,n-1)\\
L_-\tilde{e}(n_1,n_2,n) &= [n_1(n+n_2+1)]^{1/2}\tilde{e}(n_1-1,n_2,n+1)+\\
&\quad [(n+n_1+1)(n_2+1)]^{1/2}\tilde{e}(n_1,n_2+1,n-1)\\
h_1\tilde{e}(n_1,n_2,n) &= (2+n+n_1)\tilde{e}(n_1,n_2,n),\quad h_2\tilde{e}(n_1,n_2,n)=(1+n+n_2)\tilde{e}(n_1,n_2,n) \quad (21)
\end{aligned}$$

At the present stage of the universe, the value of R is very large. Therefore, in situations where Poincare limit is valid with a high accuracy, the quantum numbers (n_1, n_2, n) are very large since in the formal limit $R \to \infty$ the quantities $(n_1/2R, n_2/2R, n/2R)$ become continuous momentum variables. For this reason, we can neglect the probability for the neutrino to be in the rest state. Then the wave function of the neutrino with positive helicity can be written as

$$\Psi = \sum_{n_1 n_2 n} c(n_1, n_2, n)\tilde{e}(n_1, n_2, n) \tag{22}$$

where the minimum value of n is very large. The wave function of the neutrino with the negative helicity can be defined analogously.

Since the current theory of weak interactions is based on Poincare invariance, the form of the neutrino wave function in Eq. (22) is a problem. In standard particle theory, particles in Feynman diagrams have definite four-momenta and such state states are not even normalized. When it is said that a neutrino has the momentum $\mathbf{p}_0$, it is assumed that the state of the neutrino is described by a wave function $\int c(\mathbf{p})|\mathbf{p} > d^3\mathbf{p}$ where $c(\mathbf{p})$ has a sharp maximum at $\mathbf{p}_0$ and the width of the maximum is much less than $|\mathbf{p}_0|$. As noted above, in the approach proposed in [3] the neutrinos are described by wave packets.

However, in the AdS case, the situation is more complicated. The AdS analogs of the momentum operators are M^{4i} $(i = 1, 2, 3)$ and these operators do not commute with each other. In addition, IRs of the AdS algebra belong to the discrete series. However, when the quantum numbers (n_1, n_2, n) are very large, one might expect that semiclassical approximation will work with a high accuracy because, as follows from Eqs. (21), the operators of the IR of the AdS algebra can change these numbers only by one.

A typical form of the semiclassical wave function is

$$c(n_1, n_2, n) = a(n_1, n_2, n)exp[i(n_1\varphi_1 + n_2\varphi_2 + n\varphi)] \tag{23}$$

where the amplitude $a(n_1, n_2, n)$ has a sharp maximum at semiclassical values of (n_1, n_2, n). Since these numbers are very large, when some of them change by one, the major change of $c(n_1, n_2, n)$ comes from the rapidly oscillating exponent and in semiclassical approximation the change of $a(n_1, n_2, n)$ can be neglected. As a consequence, in semiclassical approximation, each representation operator becomes the operator of multiplication by a function and, as follows from Eqs. (11) and (21), if $\varphi_1 = \pi + \chi_1$ then

$$\begin{gathered}
M_{04} = 2(n + n_1 + n_2), \quad M_{14} = 2[n_1(n + n_1)]^{1/2}cos\chi_1 + 2[n_2(n + n_2)]^{1/2}cos\varphi_2 \\
M_{24} = -2[n_1(n + n_1)]^{1/2}sin\chi_1 + 2[n_2(n + n_2)]^{1/2}sin\varphi_2 \\
M_{34} = 2[(n + n_1)(n + n_2)]^{1/2}cos\varphi - 2(n_1 n_2)^{1/2}cos(\chi_1 + \varphi_2 - \varphi) \\
M_{10} = -2[n_1(n + n_1)]^{1/2}sin\chi_1 - 2[n_2(n + n_2)]^{1/2}sin\varphi_2 \\
M_{20} = -2[n_1(n + n_1)]^{1/2}cos\chi_1 + 2[n_2(n + n_2)]^{1/2}cos\varphi_2 \\
M_{30} = -2[(n + n_1)(n + n_2)]^{1/2}sin\varphi + 2(n_1 n_2)^{1/2}sin(\chi_1 + \varphi_2 - \varphi) \\
M_{12} = 2(n_1 - n_2), \quad M_{31} = 2[n_1(n + n_2)]^{1/2}sin(\chi_1 - \varphi) - 2[n_2(n + n_1)]^{1/2}sin(\varphi_2 - \varphi) \\
M_{23} = 2[n_1(n + n_2)]^{1/2}cos(\chi_1 - \varphi) + 2[n_2((n + n_1)]^{1/2}cos(\varphi_2 - \varphi)
\end{gathered} \tag{24}$$

$$\tilde{W} = 4\{[(n + n_1)(n + n_2)]^{1/2}sin\varphi + (n_1 n_2)^{1/2}sin(\chi_1 + \varphi_2 - \varphi)\}^2$$

$$\mathbf{N}^2 = M_{10}^2 + M_{20}^2 + M_{30}^2 =$$
$$4n_1^2 + 4n_2^2 + 4n(n_1+n_2) - 8[n_1n_2(n+n_1)(n+n_2)]^{1/2}cos(\chi_1+\varphi_2-2\varphi) +$$
$$4\{[(n+n_1)(n+n_2)]^{1/2}sin\varphi + (n_1n_2)^{1/2}sin(\chi_1+\varphi_2-\varphi)\}^2$$
$$\mathbf{M}^2 = M_{12}^2 + M_{31}^2 + M_{23}^2 =$$
$$4n_1^2 + 4n_2^2 + 4n(n_1+n_2) - 8[n_1n_2(n+n_1)(n+n_2)]^{1/2}cos(\chi_1+\varphi_2-2\varphi) \tag{25}$$

Since $I_2 = \tilde{W} - \mathbf{N}^2 + \mathbf{M}^2$, it follows from Eq. (25) that $I_2 = 0$. This is in agreement with the exact result $I_2 = -6$ obtained from Eq. (16) because in semiclassical approximation, the numbers (n_1, n_2, n) are much greater than 1.

At the present stage of the universe, the value of R is very large and Poincare approximation works with a very high accuracy. Then, as noted in Sec. 3, the four-momentum $P_\nu = M_{\nu 4}/2R$ should be finite in the formal limit $R \to \infty$ and then, as follows from Eq. (24), the numbers (n_1, n_2, n) should be very large because $(n_1/R, n_2/R, n/R)$ should be finite in this limit. At the same time, even when R is very large, the Lorenz algebra operators M_{ab} $(a, b \neq 4)$ should be finite because they should be the same as in Poincare approximation. Then, as follows from the expression for M_{12} in Eq. (24), the numbers n_1 and n_2 should be close to each other because $n_1 - n_2$ should be finite in the formal limit $R \to \infty$ and, as follows from other expressions for M_{ab}, the angles $(\chi_1, \varphi_2, \varphi)$ should be very small because they should be of the order of $O(1/R)$.

In Poincare approximation, i.e., when R is very large, $\tilde{W}/4R^2$ should become m^2. Since we treat neutrino as a particle which in Poincare approximation becomes massless then $\tilde{W}/4R^2$ should become zero in the formal limit $R \to \infty$, and as follows from the first expression in Eq. (25) and from the above remarks about the quantities (n_1, n_2, n) and $(\chi_1, \varphi_2, \varphi)$, this is the case. Since $n_1 \approx n_2$ and $(\chi_1, \varphi_2, \varphi) \ll 1$, the expression for $\tilde{W}$ in Eq. (25) can be written as

$$\tilde{W} = 4(n_1\chi_1 + n_2\varphi_2 + n\varphi)^2 \tag{26}$$

In Poincare theory, the evolution is defined by the operator $exp(-iHt)$ where H is the Hamiltonian and t is time. Since M_{04} is the AdS analog of the Poincare Hamiltonian, and $M_{04}/2R$ should become H when R is very large, the evolution in AdS theory is defined by $exp(-iM_{04}\tau)$ where $\tau = t/2R$. Then, as follows from Eq. (23) and the first expression in Eq. (24), the evolution of the angles $(\chi_1, \varphi_2, \varphi)$ is defined by

$$\chi_1(t) = \chi_{10} - t/R, \quad \varphi_2(t) = \varphi_{20} - t/R \quad \varphi(t) = \varphi_0 - t/R \tag{27}$$

and then, as follows from Eq. (26)

$$\tilde{W} = 4(n_1\chi_{10} + n_2\varphi_{20} + n\varphi_0 - Et)^2$$

because, as follows from the first expression in Eq. (24), the Poincare energy E equals $(n_1 + n_2 + n)/R$. Experimental data on neutrino oscillations are usually described in terms of the neutrino energy and the oscillation distance $l = ct$. Hence, inserting $\hbar$ and c, we finally get

$$\tilde{W} = 4(A - \frac{El}{\hbar c})^2 \tag{28}$$

where

$$A = A(n_1, n_2, n, \chi_{10}, \varphi_{20}, \varphi_0) = n_1\chi_{10} + n_2\varphi_{20} + n\varphi_0 \tag{29}$$

6 Neutrino flavors in AdS theory

As noted in Sec. 3, in the famous Dyson's paper [4] and in our publications it has been proved that quantum theory based on dS and AdS symmetries is more general (fundamental) than quantum theory based on Poincare symmetry. We believe that it is rather strange that, although the paper [4] appeared more than 50 years ago, standard particle theories (QED, QCD and electroweak theory) are still based on Poincare symmetry, and possible reasons for such a situation have been mentioned in Sec. 3.

In Poincare invariant theory, the representation operators $M^{\mu\nu}$ $(\mu, \nu = 0, 1, 2, 3)$ of the Lorentz algebra are dimensionless (in units $\hbar = c = 1$) while the momentum operators P^μ have the dimension $1/length$. As noted in Sec. 3, the AdS invariant theory also contains the operators $M^{\mu\nu}$ but, instead of the four operators P^μ, it contains the dimensionless operators $M^{4\mu}$. Poincare invariant theory is a special degenerate case of AdS invariant theory such that the operators P^μ can be treated as a formal limit of $M^{4\mu}/2R$ when $R \to \infty$.

In Poincare invariant theory, the operator $W = (P^0)^2 - \mathbf{P}^2$ is the Casimir operator and elementary particles are eigenstates of W with eigenvalues m^2 where m is called the mass. In this theory, the energy, the momentum and the mass of a free particle do not change during the whole lifetime of the particle.

In AdS theory, there are two analogs of W — the Casimir operator I_2 defined by Eq. (7) and the operator $\tilde{W}$ defined in Sec. 3. States of elementary particles are eigenvalues of I_2 which remain the same over the whole lifetime of the particle but *even for a free particle, the spectrum of* $\tilde{W}$ *changes over time.*

The eigenvalues of $\tilde{W}$ are dimensionless and, in Poincare approximation, they are related to the eigenvalues of W as $W = \tilde{W}/4R^2$ when R is very large. As shown in Sec. 4, in semiclassical approximation, the neutrino is a state with the eigenvalue of $\tilde{W}$ given by Eq. (26), and this eigenvalue changes over time. As follows from Eq. (28), in general, the eigenvalues $\tilde{w}$ of $\tilde{W}$ are rather large numbers but, when $R \to \infty$, the formal limit of $\tilde{w}/4R^2$ equals zero, in agreement with treating the neutrino as the massless particle in Poincare approximation.

The phenomenon of neutrino oscillations shows that when free neutrinos fly a long distance, they can change their flavor. As noted above, in the current treatment of neutrino oscillations, such neutrinos are treated as quantum superpositions of elementary particles with different masses (m_1, m_2, m_3) and, depending on the coefficients of the superpositions and the neutrino helicities, the neutrinos are treated as one of the particles $(\nu_e, \nu_\mu, \nu_\tau)$ or their antiparticles. As noted in Sec. 2, such a treatment is highly problematic from the theoretical point view.

The goal of the present paper is to investigate whether AdS quantum theory can shed a new light on the neutrino oscillation phenomenon. We treat the neutrino not as a superposition of three elementary particles but as a particle which in AdS quantum theory is treated as elementary and massless, and the meaning of such a treatment is explained in Sec. 4.

As follows from Eq. (26), for neutrinos, eigenvalues of $\tilde{W}$ are rather large but a formal limit of the operator $\tilde{W}/4R^2$ when $R \to \infty$ is zero, in agreement with the fact that this limit equals m^2 and, in our approach, the Poincare neutrino mass equals zero. It will be clear from our discussion that the neutrino is a quantum superposition of states with essentially different eigenvalues $\tilde{w}$ of the operator $\tilde{W}$. *Our conjecture is that the neutrino flavor is* *defined by the most probable range of $\tilde{w}$ in the neutrino wave function.* As noted in Sec. 5, computing the neutrino wave function in the AdS theory is a problem and so, our conjecture will be proved or disproved when this theory is constructed.

However, for the electron, muon and tau lepton, the situation is drastically different. For example, for the electron, typical eigenvalues of $\tilde{W}$ are of the order of $4m_e^2R^2$ where m_e is the electron mass. As follows from cosmological data (see e.g., the discussion in [5]), the value of R is of the order of $10^{26}m$. Then all the eigenvalues of the operator $\tilde{W}$ are very close to $4m_e^2R^2 \sim 10^{77}$. For the muon and tau lepton, almost all the eigenvalues are close to $4m_\mu^2R^2 \sim 10^{81}$ and $4m_\tau^2R^2 \sim 10^{84}$, respectively. *Therefore, our conjecture naturally explains the fact that the electron, muon and tau lepton do not have flavors changing over time.*

The author of [10] summarizes the present experimental status of neutrino oscillations as follows:

- Atmospheric ν_μ and $\bar{\nu}_\mu$ disappear most lickely converting to ν_e and $\bar{\nu}_e$. The results show an energy and distance dependence perfectly described by mass-induced oscillations.

- Accelerator ν_μ and $\bar{\nu}_\mu$ disappear over distances of $\sim$ 200 to 800 *km*. The energy spectrum of the results show a clear oscillatory behavior also in accordance with mass-induced oscillations with wavelength in agreement with the effect observed in atmospheric neutrinos.

- Accelerator ν_μ and $\bar{\nu}_\mu$ appear as ν_e and $\bar{\nu}_e$ at distances $\sim$ 200 to 800 *km*.

- Solar ν_e convert to ν_μ and/or ν_τ. The observed energy dependence of the effect is well described by massive neutrino conversion in the Sun matter according to the MSW effect.

- Reactor $\bar{\nu}_e$ disappear over distances of $\sim 200km$ and $\sim 1.5km$ with different probabilities. The observed energy spectra show two different mass-induced oscillation wavelengths: at short distances in agreement with the one observed in accelerator ν_μ disappearance, and a long distance compatible with the required parameters for MSW conversion in the Sun.

Here, for example, the word "disappear" means not that all neutrinos in the given experiment disappear but only a part of them disappears, and the words "appear" and "convert" should be understood analogously. We will describe the results of experiments where the energies and flavors of initial and final neutrinos are known without model assumptions.

In the K2K experiment [11], 56 ν_μ neutrinos with the energies $E \approx 1.3Gev$ have been observed at Super-Kamiokande, the far detector at 250 km distance while the expectation was $80.6^{+7.3}_{-8.0}$. For this case, $El/(\hbar c) = 1.6 \cdot 10^{18}$.

In the KamLAND experiment [12], $\bar{\nu}_e$ reactor neutrinos with the energies in the range $(2.6-6)Mev$ flew the distance 180km and, although 365 events were predicted, only 258 were observed. For this case, if $E = 3Mev$ then $El/(\hbar c) \approx 2 \cdot 10^{15}$.

In the RENO experiment [13], $\bar{\nu}_e$ reactor neutrinos with the energies $E \approx 3Mev$ flew the distance 1183m. The ratio of observed to expected numbers of antineutrinos was $0.920 \pm 0.009(stat.) \pm 0.014(syst.)$. For this case, $El/(\hbar c) = 2 \cdot 10^{13}$.

In the NOvA experiment [14], ν_μ neutrinos with the energies in the range $(2-20)Gev$ flew from Fermilab 810km. 720 cases were expected without oscillations but only 126 ν_μ cases and 66 ν_e cases were observed. For this range of energies, $El/(\hbar c)$ is in the range $(0.8-8)\cdot 10^{19}$.

In the OPERA experiment [15], ν_μ neutrinos with the energies $E \approx 17Gev$ flew from CERN toward the Gran Sasso underground laboratory, 730 km away. 5603 neutrino interactions were fully reconstructed, 10 of them were identified as conversions to ν_τ and 35 — as conversions to ν_e. In this experiment, $El/(\hbar c) \approx 6.2\cdot 10^{19}$.

Those experiments confirm that the phenomenon of neutrino oscillation does take place. At the same time, in all those experiment except [14], most neutrinos did not change their flavor. In the framework of the approach described in Sec. 5, the description of the above experiments is model dependent because, in semiclassical approximation, the values of the angles $(\chi_{10}, \varphi_{20}, \varphi_0)$ in Eq. (29) are not known. We noted that those values should be very small. Since in the above experiments, the phenomenon of neutrino oscillation does take place, it follows from Eq. (28) and from the above values of the quantity $El/(\hbar c)$ that $\tilde{W}$ essentially depends on l. Therefore the quantity A is of the order of 10^{19} or less. Since the AdS neutrino energy equals $2(n_1+n_2+n)$ then, even if Poincare energy is of the order of Mev, the AdS energy is of the order of $2R\cdot Mev \sim 10^{39}$, i.e., the quantities (n_1, n_2, n) in A are of the order of 10^{39} and the angles $(\chi_{10}, \varphi_{20}, \varphi_0)$ are indeed very small.

As noted in Sec. 2, the current theory of neutrino oscillations cannot describe the data without fitting parameters in the Pontecorvo–Maki–Nakagawa–Sakata matrix. Analogously, as noted in Sec. 5, in the absence of AdS theory of weak interactions, the details of the initial neutrino wave function are not known. We proposed to consider this function in semiclassical approximation. Then one can treat $(n_1, n_2, n, \chi_{10}, \varphi_{20}, \varphi_0)$ as fitting fitting parameters which can be found from experimental values of the quantities $M^{\mu\nu}$ by using Eq. (24) but these quantities are not measured in neutrino experiments.

However, there is a case which can be described without fitting parameters. Historically, the problem of neutrino oscillations first appeared in view of the solar neutrino problem. In 2002, Ray Davis and Masatoshi Koshiba won part of the Nobel Prize in Physics for experimental work which found the number of solar neutrinos to be around a third of the number predicted by the standard solar model.

The result [16] of the Sudbury Neutrino Observatory is that the ν_e survival probability at $10Mev$ is $0.317 \pm 0.016(stat) \pm 0.009(syst)$, and this number is very close to 1/3. If we take for l the distance from Sun to Earth then for $E = 10Mev$, $El/(\hbar c) = 7.5\cdot 10^{21}$. Since the quantity A is of the order of 10^{19} or less, one can neglect A in Eq. (28) and get

$$\tilde{W} = 4(\frac{El}{\hbar c})^2 \tag{30}$$

Since most neutrinos created in the Sun are created in the reaction

$$p + p \to d + e^+ + \nu_e$$

and their typical energies are in the range $(1-20)Mev$, the values of $\tilde{W}$ are in the range $(2\cdot 10^{42} - 9\cdot 10^{44})$. These values should not be treated as anomalously large. Indeed, as noted above, even for the electron, which is the lightest massive particle, the AdS mass squared is $4m_e^2R^2$ and this quantity is of the order of 10^{77}.

As noted above, *our conjecture is that the neutrino flavor is defined by the most probable range of $\tilde{w}$ in the neutrino wave function.* So, if we assume that the range $(2 \cdot 10^{42} - 9 \cdot 10^{44})$ is partitioned into equal pieces such that each piece corresponds to one of the neutrino flavors, then the ν_e neutrinos created on the Sun can be detected on the Earth with equal probabilities 1/3 as ν_e, ν_μ or ν_τ. So, our conjecture naturally explains that the survival probability for ν_e equals 1/3.

7 Conclusion

In the current approach to neutrino oscillations, the neutrino is treated not as an elementary particle but as a superposition of three elementary particles with different masses. In Sec. 2 we note that this approach is essentially model dependent and is not based on rigorous physical principles.

We propose an approach where the neutrino is treated as a massless elementary particle in AdS invariant quantum theory. The fact that this theory is more general (fundamental) than standard Poincare invariant quantum theory, has been proved in the famous Dyson's paper "Missed Opportunities" [4] and in our publications [5, 6]. We believe that it is rather strange that, although the Dyson's paper has appeared more than 50 years ago, standard particle theories (QED, QCD and electroweak theory) are still based on Poincare symmetry, and in Sec. 1 we describe possible reasons for such a situation.

If P_0 and $\mathbf{P}$ are the operators of standard energy and momentum then, in standard theory, elementary particles are eigenstates of $W = P_0^2 - \mathbf{P}^2$ with the eigenvalue $w = m^2$ where m is called the mass. In this theory, the value of m remains the same during the whole lifetime of the particle. The operator $\tilde{W}$ defined in Sec. 3 is an AdS analog of W. In contrast to the situation in Poincare invariant theory, the spectrum of $\tilde{W}$ changes over time even for elementary particles. If the spectrum of $\tilde{W}$ consists of the values $\tilde{w}$ then *our conjecture is that the neutrino flavor is defined by the most probable range of $\tilde{w}$ in the neutrino wave function.*

In this approach, it becomes clear why, in contrast to the neutrino, the electron, muon and tau lepton do not have flavors changing over time. Since the current theory of weak interactions is based on Poincare invariance, the calculation of the most probable range of $\tilde{w}$ in the neutrino wave functions is a problem but, as explained in Sec. 6, the solar neutrino problem has a natural explanation.

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