Fizeau Experiment revisited and physical meaning of the refractive index

Non-ad-hoc classic explanation of the Fizeau Experiment and association of refractive index with molecule's distances

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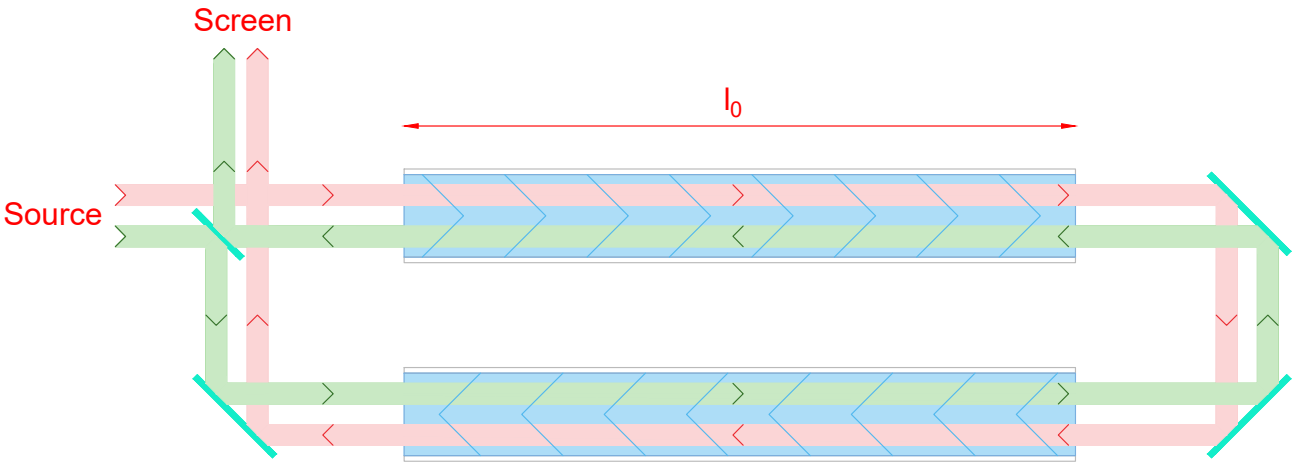
Abstract

The Fizeau experiment was a pivotal milestone on the long road to the discovery of Relativity. In this paper, we identify a fundamental flaw in Fizeau's Fresnel-based interpretation: the exclusion of different wavelengths of light in vacuum and water from the calculation. We present a physically robust explanation of the effect using classical physics. As a byproduct, the derived formulae clarify the physical meaning of the refractive index and address why refractive indices usually, but not always, correlate with matter density. Furthermore, they enable the calculation of the relationship between molecular diameter and the spacing between molecules in a medium, derived from the refractive index.

1. Description and critic on the experiment

Fizeau [1] relied on Fresnel's work [2] on the reduction of the speed of light in refracting media to explain his experiment's results. The experimental setup is as follows:



Fizeau's formula [1] aligns with Einstein's formula [3] when the wavelength within the medium is applied:

$$\boxed{\Delta l = 4 \cdot l_0 \cdot \left(\frac{v_{Medium}}{c_{Vacuum}}\right) \cdot (n^2 - 1)} \qquad (1)$$

with:

Δl = change in length
l_0= length of one tube of the lightpath at rest
v_{Medium}= movement speed of medium
c_{Vacuum}= light speed in vacuo
n = refractive index

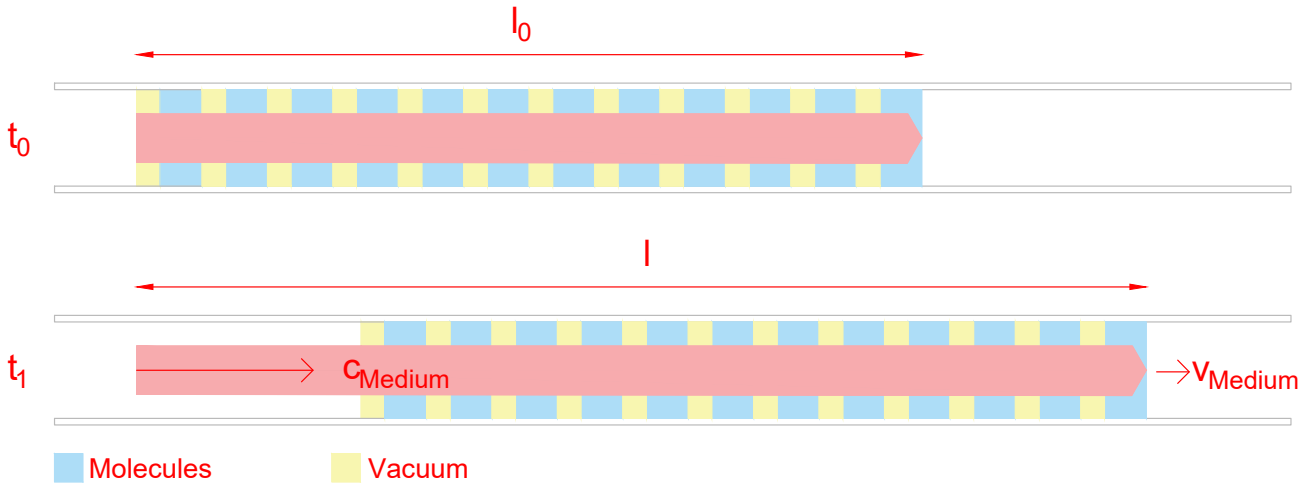
However, this formula is an ad hoc explanation, and current mathematics fails to derive the problem in a fully physically grounded manner.

2. Reinterpretation from the scratch

Our investigation assumes that the medium consists of molecules interspersed with vacuum. Light travels through vacuum regions at its vacuum velocity, but its velocity reduces near the molecules. Although the precise distances light travels in vacuum and near the molecules cannot be directly determined, this ambiguity for now proves irrelevant to our derivation.

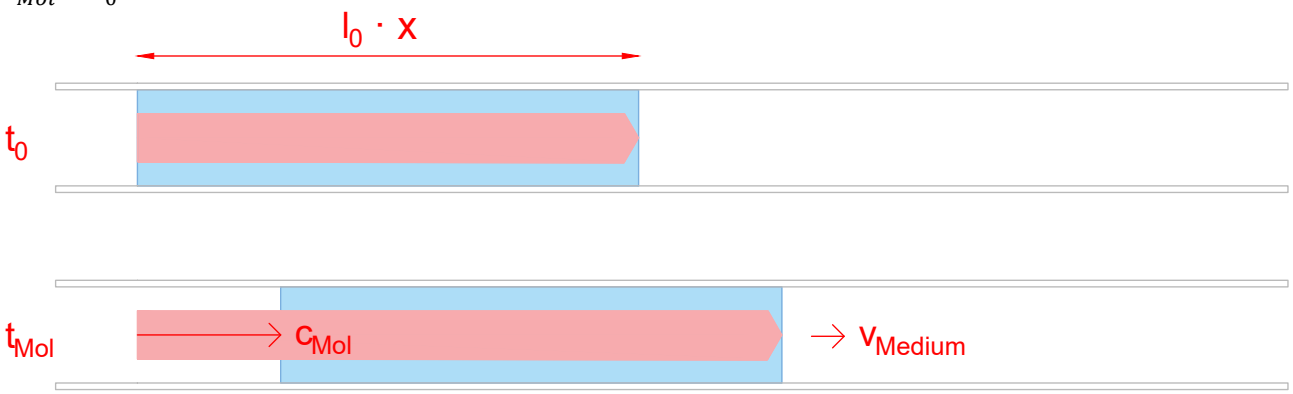
To simplify, we examine one direction of medium flow. Since the light rays move counterclockwise, second-order velocity addition effects are absent. Therefore, we consider a single segment of the medium's flow and later multiply results by four. Differentiating between the respective wavelengths within molecules and in vacuum is critical.

To begin with, we will show how one can picture the principle outlined before:

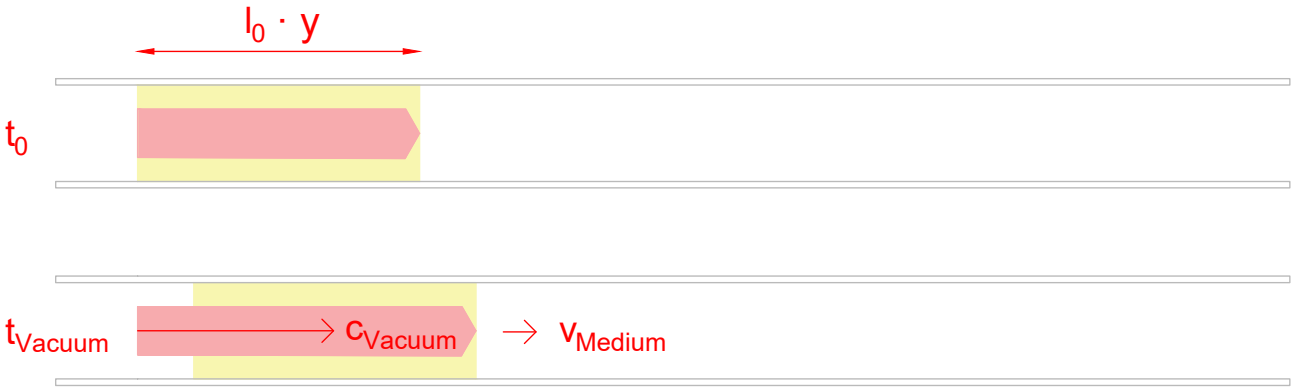


Light travels a fraction of the total distance of l_0 through vacuum and another fraction through molecules, where the combined distances equal l_0. Summing these fractional distances across the entire length l_0, we represent the division l_0 into molecular (x) and vacuum (y) fractions, with $x + y = 1$:

$l_{Mol} = l_0 \cdot x$



$l_{Vacuum} = l_0 \cdot y$



Molecular Propagation:

We focus now on the time required for light to traverse the molecular fraction at molecular light speed c_{Mol}, while accounting for the medium's movement at velocity v_{Medium} . As a reference, first we find the time that light would need to cover the distance $l_0 \cdot x$ if the medium was not moving:

$$t_{0,Mol} = \frac{l_0 \cdot x}{c_{Mol}}$$

And now the time required when the light follows the moving medium (the movement of the source being irrelevant as per classic wave theory):

$$t_{Mol} = \frac{l_0 \cdot x}{c_{Mol} - v_{Medium}}$$

We have now the difference in time caused by the moving medium:

$$\Delta t_{Mol} = \frac{l_0 \cdot x}{c_{Mol} - v_{Medium}} - \frac{l_0 \cdot x}{c_{Mol}}$$

From these, we determine the corresponding difference in distance, based on molecular light speed:

$$\Delta s_{Mol} = \left(\frac{l_0 \cdot x}{c_{Mol} - v_{Medium}} - \frac{l_0 \cdot x}{c_{Mol}}\right) \cdot c_{Mol}$$

Finally, for fringe shift calculations, the molecular wavelength is used:

$$\Delta fringe_{Mol} = \frac{\left(\frac{l_0 \cdot x}{c_{Mol} - v_{Medium}} - \frac{l_0 \cdot x}{c_{Mol}}\right) \cdot c_{Mol}}{\lambda_{Mol}} = x \cdot l_0 \cdot \frac{c_{Mol}}{\lambda_{Mol}} \cdot \left(\frac{1}{c_{Mol} - v_{Medium}} - \frac{1}{c_{Mol}}\right)$$

Rearranging yields:

$$\Delta fringe_{Mol} = x \cdot \frac{l_0}{\lambda_{Mol}} \cdot \frac{v_{Medium}}{(c_{Mol} - v_{Medium})} \qquad (\mathbf{2})$$

Vacuum Propagation:

For the vacuum portion, the same method applies. Also here, the vacuum moves at v_{Medium}, and the vacuum wavelength is used to calculate the fringe shift difference.

$$\Delta fringe_{Vacuum} = y \cdot \frac{l_0}{\lambda_{Vacuum}} \cdot \frac{v_{Medium}}{(c_{Vacuum} - v_{Medium})} \qquad (\mathbf{3})$$

Combined Propagation:

Subtracting vacuum (3) from molecular contributions (2) gives the net fringe shift, multiplied by four to account for the rays' paths:

$$\Delta fringe = 4\left(x \cdot \frac{l_0}{\lambda_{Mol}} \cdot \frac{v_{Medium}}{(c_{Mol} - v_{Medium})} - y\frac{l_0}{\lambda_{Vacuum}} \cdot \frac{v_{Medium}}{(c_{Vacuum} - v_{Medium})}\right)$$

Rearranging yields a precise formula without approximations:

$$\boxed{\Delta fringe = 4 \cdot l_0 \cdot v_{Medium}\left(\frac{x}{\lambda_{Mol} \cdot (c_{Mol} - v_{Medium})} - \frac{y}{\lambda_{Vakuum} \cdot (c_{Vakuum} - v_{Medium})}\right)} \qquad (\mathbf{4})$$

Now since v_{Medium} should be minor in the denominator, at non-relativistic speeds the approximation holds:

$$\Delta fringe = 4 \cdot l_0 \cdot v_{Medium}\left(\frac{x}{\lambda_{Mol} \cdot c_{Mol}} - \frac{y}{\lambda_{Vakuum} \cdot c_{Vakuum}}\right)$$

We reduce all quantities to molecular parameters:

$$\Delta fringe = 4 \cdot l_0 \cdot v_{Medium}\left(\frac{x}{\lambda_{Mol} \cdot c_{Mol}} - \frac{y}{\lambda_{Mol} \cdot n \cdot c_{Mol} \cdot n}\right)$$

$$\Delta fringe = 4 \cdot \frac{l_0}{\lambda_{Mol}} \cdot \frac{v_{Medium}}{c_{Mol}}\left(\frac{x}{1} - \frac{y}{n \cdot n}\right)$$

And we finally obtain:

$$\boxed{\Delta fringe = 4 \cdot \frac{l_0}{\lambda_{Mol}} \cdot \frac{v_{Medium}}{c_{Mol}}\left(x - \frac{y}{n^2}\right)} \qquad (\mathbf{5})$$

Initially, the results deviate from Fizeau's formula due to x and y. However, the following correlation between x and y, based on the refractive index n, ensures consistency:

$$\boxed{x = \frac{1}{\frac{1}{n^2} + 1}} \qquad (\mathbf{6})$$

$$\boxed{y = \frac{1}{n^2 + 1}} \qquad (\mathbf{7})$$

Where $x + y = 1$

Substituting x and y with (6) and (7) into formula (5), we derive for the bracket $\left(x - \frac{y}{n^2}\right)$:

$$\left(x - \frac{y}{n^2}\right) = \left(\frac{1}{\frac{1}{n^2} + 1} - \frac{1}{n^2(n^2 + 1)}\right) = \left(\frac{n^2}{n^2 + 1} - \frac{1}{n^2(n^2 + 1)}\right) = \left(\frac{n^2 n^2}{(n^2 + 1)n^2} - \frac{1}{n^2(n^2 + 1)}\right) =$$

$$= \left(\frac{n^4 - 1}{(n^2 + 1)n^2}\right) = \left(\frac{(n^2 - 1)(n^2 + 1)}{(n^2 + 1)n^2}\right) = \left(\frac{n^2 - 1}{n^2}\right) = \left(1 - \frac{1}{n^2}\right)$$

Therefore we obtain for (5):

$$\boxed{\Delta fringe = 4 \cdot \frac{l_0}{\lambda_{Mol}} \cdot \frac{v_{Medium}}{c_{Mol}}\left(1 - \frac{1}{n^2}\right)} \qquad (\mathbf{8})$$

Comparison with Fizeau's formula:

Up to this point, we have derived a formula that exclusively contains quantities related to the molecules. To facilitate a direct comparison with Fizeau's formula, we now substitute λ_{Mol} with λ_{Vakuum} and c_{Mol} with c_{Vakuum}. This yields:

$$\Delta fringe = 4 \cdot \frac{l_0}{\lambda_{Vakuum} \cdot \frac{1}{n}} \cdot \frac{v_{Medium}}{c_{Vakuum} \cdot \frac{1}{n}} \left(1 - \frac{1}{n^2}\right)$$

$$\Delta fringe = 4 \cdot \frac{l_0}{\lambda_{Vakuum}} \cdot \frac{v_{Medium}}{c_{Vakuum}} (n^2 - 1)$$

And thus for the difference in length, multiplying with λ_{Vakuum}:

$$\boxed{\Delta l = 4 \cdot l_0 \cdot \frac{v_{Medium}}{c_{Vacuum}} \cdot (n^2 - 1)} \qquad (\mathbf{9})$$

This matches Fizeau's formula (1) exactly, showing it as a valid approximation derived from solid physical principles.

The parameters x and y represent the proportional contribution of empty space and molecular dimension within a medium. This directly links the refractive index to the molecular spacing and diameter.

Since the above terms yield experimentally accurate results, it is evident that x and y represent inherent properties of the material, directly derivable from the refractive index. In other words, the refractive index is fundamentally determined by these intrinsic properties.

Validation test:

To validate our findings, we perform a test by calculating the proportion of the diameter of a water molecule to the distance between two molecules at room temperature, using known dimensions:

Mass of water molecule:

$m_{H_2O} = 3.07 \cdot 10^{-26} kg$

Number of molecules z in one liter of water:

$z = 3.07 \cdot 10^{25}$

Volume occupied by one molecule (including the space to the next molecule):

$vol = 3.07 \cdot 10^{-2}\ m^3$

Equivalent total of molecular diameter and intermolecular distance d_{H_2O}+ s_{H_2O}:

$d_{H_2O} + s_{H_2O} = 313.189 pm$

Known diameter of a water molecule d_{H_2O}:

$d_{H_2O} \approx 200pm = 0.63859$ of $d_{H_2O} + s_{H_2O}$

Therefore, the distance between molecules is:

$s_{H_2O} = 113.189pm = 0.36140$ of $d_{H_2O} + s_{H_2O}$

Now, we use our derived formula (6) with the refractive index of water n=1.333, to calculate the molecular diameter:

$$d_{H_2O} = \frac{1}{\frac{1}{n^2} + 1} = \frac{1}{\frac{1}{1.333^2} + 1} = 0.63988$$

And according to formula (7), we calculate the distance between molecules:

$$s_{H_2O} = \frac{1}{n^2 + 1} = \frac{1}{1.333^2 + 1} = 0.36011$$

The calculated values closely match the empirical ones, with a deviation of only 2‰ (2 parts per thousand). This minimal deviation demonstrates the accuracy of our model and its alignment with physical reality.

3. Conclusion

We provide a classical interpretation of the Fizeau experiment [1], rooted in fundamental physical principles. Additionally, we establish a relationship between molecular dimensions and intermolecular spacing, explaining the refractive index's origin. Although alternative liquids with complex molecules and indices could further validate this model, existing data for water molecules already demonstrate a match within 2‰ deviation. We hope this work inspires further exploration into the refractive index's material dependence and molecular properties.

References and Acknowledgments (in order of appearance):

[1] H. Fizeau, Sur les hypothèses relatives à l'éther lumineux., Comptes Rendus. 33, 1851, S. 349–355.

[2] Fresnel, Augustin, Lettre d'Augustin Fresnel à François Arago sur l'influence du mouvement terrestre dans quelques phénomènes d'optique, Bde. %1 von %2Oeuvres complètes d'Augustin Fresnel / publiées par MM. Henri de Senarmont, http://visualiseur.bnf.fr/CadresFenetre?O=NUMM-91937&I=633&M=tdm, Gallica, 1818.

[3] Einstein, Albert, Zur Elektrodynamik bewegter Körper, Annalen der Physik und Chemie. 17, 1905, S. 891–921, 1905.