abstract

This analysis shows Cantor's diagonal argument published in 1891, cannot form an additional sequence for an infinite set of infinite sequences M, that prevents a one to one correspondence with N the set of natural numbers.

the argument

Translation from Cantor's 1891 paper [1]:
Namely, let $m$ and $n$ be two different characters, and consider a set $\text{Inbegriff } M$ of elements

$$E = (x_1, x_2, \ldots, x_v, \ldots)$$

which depend on infinitely many coordinates $x_1, x_2, \ldots, x_v, \ldots$, and where each of the coordinates is either $m$ or $n$. Let $M$ be the totality $\text{Gesamtheit}$ of all elements $E$.
To the elements of $M$ belong e.g. the following three:

$$E_I = (m, m, m, m, \ldots),$$
$$E_{II} = (m, m, m, m, \ldots),$$
$$E_{III} = (m, m, m, m, \ldots).$$

I maintain now that such a manifold $\text{Mannigfaltigkeit } M$ does not have the power of the series $1, 2, 3, \ldots, v, \ldots$

This follows from the following proposition:
"If $E_1, E_2, \ldots, E_v, \ldots$ is any simply infinite $\text{einfach unendliche}$ series of elements of the manifold $M$, then there always exists an element $E_0$ of $M$, which cannot be connected with any element $E_v$.
"For proof, let there be

$$E_1 = (a_{1,1}, a_{1,2}, \ldots, a_{1,v}, \ldots)$$
$$E_2 = (a_{2,1}, a_{2,2}, \ldots, a_{2,v}, \ldots)$$
$$E_u = (a_{u,1}, a_{u,2}, \ldots, a_{u,v}, \ldots)$$

where the characters $a_{uv}$ are either $m$ or $n$. Then there is a series $b_1, b_2, \ldots, b_v, \ldots$, defined so that $b_v$ is also equal to $m$ or $n$ but is different from $a_{uv}$.
Thus, if $a_{uv} = m$, then $b_v = w$.
Then consider the element

$$E_0 = (b_1, b_2, b_3, \ldots)$$

of $M$, then one sees straight away, that the equation
E_0 = E_u
cannot be satisfied by any positive integer u, otherwise for that u and for all values of v.

b_v = a_{u,v}

and so we would in particular have

b_u = a_{u,u}

which through the definition of b_v is impossible. From this proposition it follows immediately that the totality of all elements of M cannot be put into the sequence [Reihenform]: E_1, E_2, ..., E_v, ... otherwise we would have the contradiction, that a thing [Ding] E0 would be both an element of M, but also not an element of M.

(end of translation)

**analysis**

![fig1](image)

Using the u,v coordinate system, Cantor defines a diagonal sequence D using one symbol with coordinates uu from each row. He forms a horizontal sequence E0 derived from D by interchanging all symbols m and w forming the negation of D as shown in fig.1. He
concludes E0 differs from D for all v in the set M and this additional sequence prevents a one to one correspondence of M to N the natural numbers. Fig.1 shows there is nothing prohibiting the diagonal D from being an existing horizontal sequence (E9) of M. Cantor would not be aware that E9 = D, since it cannot be detected with one comparison, and can occur anywhere in the list. In a true random list there are no dependencies of one sequence on another. The sequences are formed independently of each other and randomly entered in the list. There is no requirement for any type of order in the list.

**conclusion**

In the one dimensional case, no sequence interacts with another. In the two dimensional case, the property of orientation presents the potential for contradictions where two different symbols occupy the same location simultaneously as noted by C in fig.2. The diagonal sequence D prevents the appearance of its negation N in the range r2, effectively imposing a degree of order on an otherwise random list. If Cantor's diagonal method is applied to the remaining D in the infinite range r2, the corresponding N could not be contained in the finite range r1. The list would remain incomplete. Cantor's example is a special case using the first diagonal as D, thus eliminating the range r1, and excluding N from the list. The list is a random composite of horizontal sequences from two subsets. If the sequence begins with m, it is a member of Sm and its negation is a member of Sw. Sm and Sw are mutually exclusive, thus any sequence must be a member of one or the other, but not both. E0 being a member of Sw and not a member of Sm, is not a contradiction. The problem of the contradiction appears as the result of the contrived diagonal sequence. The problem is solved with the removal of D, which is redundant, as shown in fig.1. Cantor's relative comparison of D and its negation E0, only shows them as members of
distinct subsets, $S_m$ and $S_w$, both members of $M$. It does not exclude either as a member of $M$.

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