ON TRYING TO MODIFY EINSTEIN FIELD EQUATIONS HYPOTHESIS

TOMASZ KOBIERZYCKI
KOBIERZYCKITOMASZ@GMAIL.COM
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Abstract. At quantum scale particles can be understood as complex vector fields that from is derived Einstein Field Equations. Space-time geometry is defined by those complex vectors fields. I present in general and explain all mathematical model that is need to make this idea be consistent with itself. Idea uses symmetry $SU(n)$ that for our space-time is just $SU(4)$ that is four dimensional.

Contents

1. Complex space-time and energy tensor

Conflict between General Relativity and quantum physics is one of most important unsolved problems in theoretical physics. In this hypothesis I will try to show how to quantize it by starting with simple assumption, I start from complex space-time [1] [2] [3] [4] tensor that I will denote $(\psi\psi^*)^\alpha_\rho$ that is mixed tensor field that takes two vectors of complex space-time , one normal complex vector and second it's complex conjugate and creates out of it a mixed tensor. First I need to impose that field is symmetric tensor field:

$$(\psi\psi^*)^\alpha_\rho = (\psi\psi^*)^\rho_\alpha \tag{1.1}$$

From fact it's a complex field that uses $SU(n)$ matrix as rotation times it's complex conjugate I can rotate that field by this matrix and still get a valid solutions, where it's a mixed tensor field of space-time:

$$U^\alpha_\kappa \left(U^\dagger\right)^\phi_\rho (\psi\psi^*)^\kappa_\phi = (\psi\psi^*)^\alpha_\rho \tag{1.2}$$

Energy tensor has to obey few simple rules, first it has to be symmetric with respect to indexes:

$$T^\alpha_{\lambda\mu\nu} = T^\alpha_{\lambda\nu\mu} = T^\lambda_{\alpha\nu\mu} = T^\lambda_{\alpha\mu\nu} \tag{1.3}$$

$$T^\gamma_{\rho\mu\nu} = T^\gamma_{\rho\nu\mu} = T^\rho_{\gamma\nu\mu} = T^\rho_{\gamma\mu\nu} \tag{1.4}$$

Then contraction of that tensor will give energy-stress tensor:

$$T^\alpha_{\alpha\mu\nu} = T^\rho_{\rho\mu\nu} = T_{\mu\nu} \tag{1.5}$$

From it follows that there is conservation of energy of that that tensor:

$$g^{\mu\phi}g^{\nu\varphi}T^\alpha_{\alpha\phi\varphi} = T^{\mu\nu} \tag{1.6}$$

$$g^{\mu\phi}g^{\nu\varphi}T^\rho_{\rho\phi\varphi} = T^{\mu\nu} \tag{1.7}$$

$$\nabla_\nu g^{\mu\phi}g^{\nu\varphi}T^\alpha_{\alpha\phi\varphi} = \nabla_\nu T^{\mu\nu} = 0 \tag{1.8}$$

$$\nabla_\nu g^{\mu\phi}g^{\nu\varphi}T^\rho_{\rho\phi\varphi} = \nabla_\nu T^{\mu\nu} = 0 \tag{1.9}$$

Contraction of that tensor field gives probability scalar function:

$$\psi^2(\mathbf{x}) = (\psi\psi^*)^\alpha_\alpha \tag{1.10}$$

$$\rho^2(\mathbf{x}) = \int (\psi\psi^*)^\alpha_\alpha \, d^{n-1}x \tag{1.11}$$

That represents probability of being at position $\left(x^1, .., x^{n-1}\right)$ in some time x^0. Where this tensor can be split into two complex vector fields:

$$(\psi\psi^*)^\alpha_\rho = \psi^\alpha\psi^*_\rho \tag{1.12}$$

2. Curvature of complex space-time

I can extend this idea to curved scape-time, first I write relation between curvature tensor [5] [6] [7] [8] and Einstein field equations, I multiply each term in Einstein field equation by wave function squared that is first step:

$$R_{\mu\nu}\psi^2\left(\mathbf{x}\right)-\frac{1}{2}R\psi^2\left(\mathbf{x}\right)g_{\mu\nu}=\kappa T_{\mu\nu}\psi^2\left(\mathbf{x}\right) \tag{2.1}$$

This term still lacks dependence on mixed tensor field, but I can expand it in two possible ways, first way to expand it is to expand wave function into wave field:

$$R_{\mu\nu}\left(\psi\psi^*\right)^{\lambda}_{\rho}-\frac{1}{2}R\left(\psi\psi^*\right)^{\alpha}_{\rho}g_{\mu\nu}=\kappa T_{\mu\nu}\left(\psi\psi^*\right)^{\lambda}_{\rho} \tag{2.2}$$

Then I expand curvature tensor and energy tensor and contract it with wave field to make indexes match middle term:

$$R^{\alpha}_{\lambda\mu\nu}\left(\psi\psi^*\right)^{\lambda}_{\rho}-\frac{1}{2}R\left(\psi\psi^*\right)^{\alpha}_{\rho}g_{\mu\nu}=\kappa T^{\alpha}_{\lambda\mu\nu}\left(\psi\psi^*\right)^{\lambda}_{\rho} \tag{2.3}$$

This equation is field equation I need, it comes from fact that it can contract and by contraction I will arrive at Einstein Field equations multiply by wave function squared:

$$R^{\alpha}_{\alpha\mu\nu}\left(\psi\psi^*\right)^{\alpha}_{\alpha}-\frac{1}{2}R\left(\psi\psi^*\right)^{\alpha}_{\alpha}g_{\mu\nu}=\kappa T^{\alpha}_{\alpha\mu\nu}\left(\psi\psi^*\right)^{\alpha}_{\alpha} \tag{2.4}$$

$$R_{\mu\nu}\psi^2\left(\mathbf{x}\right)-\frac{1}{2}R\psi^2\left(\mathbf{x}\right)g_{\mu\nu}=\kappa T_{\mu\nu}\psi^2\left(\mathbf{x}\right) \tag{2.5}$$

This equation can be done opposite way where I contract curvature tensor with not contravariant indexes but covariant:

$$R^{\lambda}_{\rho\mu\nu}\left(\psi\psi^*\right)^{\alpha}_{\lambda}-\frac{1}{2}R\left(\psi\psi^*\right)^{\alpha}_{\rho}g_{\mu\nu}=\kappa T^{\lambda}_{\rho\mu\nu}\left(\psi\psi^*\right)^{\alpha}_{\lambda} \tag{2.6}$$

And contraction gives again a wave function squared times Einstein Field Equations:

$$R^{\rho}_{\rho\mu\nu}\left(\psi\psi^*\right)^{\rho}_{\rho}-\frac{1}{2}R\left(\psi\psi^*\right)^{\rho}_{\rho}g_{\mu\nu}=\kappa T^{\rho}_{\rho\mu\nu}\left(\psi\psi^*\right)^{\rho}_{\rho} \tag{2.7}$$

$$R_{\mu\nu}\psi^2\left(\mathbf{x}\right)-\frac{1}{2}R\psi^2\left(\mathbf{x}\right)g_{\mu\nu}=\kappa T_{\mu\nu}\psi^2\left(\mathbf{x}\right) \tag{2.8}$$

This field equations before contraction and after contraction has to be both fulfilled to make this idea work.

3. Meaning of field equations

In this hypothesis there are two main equations first one is general field equation that can be written in two ways:

$$R^{\alpha}_{\lambda\mu\nu}\left(\psi\psi^{*}\right)^{\lambda}_{\rho}-\frac{1}{2}R\left(\psi\psi^{*}\right)^{\alpha}_{\rho}g_{\mu\nu}=\kappa T^{\alpha}_{\lambda\mu\nu}\left(\psi\psi^{*}\right)^{\lambda}_{\rho} \tag{3.1}$$

$$R^{\lambda}_{\rho\mu\nu}\left(\psi\psi^{*}\right)^{\alpha}_{\lambda}-\frac{1}{2}R\left(\psi\psi^{*}\right)^{\alpha}_{\rho}g_{\mu\nu}=\kappa T^{\lambda}_{\rho\mu\nu}\left(\psi\psi^{*}\right)^{\alpha}_{\lambda} \tag{3.2}$$

This equation relates curvature to complex field and energy. But It can be reduced by contraction of indexes to Einstein Field Equations with a scalar function multiply:

$$R_{\mu\nu}\psi^{2}\left(\mathbf{x}\right)-\frac{1}{2}R\psi^{2}\left(\mathbf{x}\right)g_{\mu\nu}=\kappa T_{\mu\nu}\psi^{2}\left(\mathbf{x}\right) \tag{3.3}$$

That scalar function represents probability, it means that contraction of field equation leads to some possible geometry of space-time with probability at each point of that space-time. But probability for given moment of time can't change that can be written as integral over whole space being equal to one [9] :

$$\int_{\text{Space}}\psi^{2}\left(\mathbf{x}\right)d^{n-1}x=1 \tag{3.4}$$

So eat each point of time there is probability of let's say particle being at position $\left(x^{1},...,x^{n-1}\right)$ but this function also depends on time, so each change of this function and how it's evolving is there. When there is measurement done this function collapses to just being at one location, and there classical field equation solution is measured. It means that there is no longer a probability at each point of space-time, now there is just a classical gravitational field. Before measurement there is still a gravity field but it's not localized in space. After measurement field equation solutions change to being well defined field solutions. It means that gravity field before measurement represents not one field equation solutions but for each point of space-time a new equation solution where each solution will say that particle is at position of that point. So there is infinite number of fields equation solutions each representing a possible position of particle. After measurement it collapses to just one position and one field equation solution.

4. Wave function as manifold- quantum gravity principle

Meaning of field equation is just one side of coin, field equations states that all parts of Einstein Field Equation is derive from complex field that turns into real field. It means that both distance and time is measured in terms of wave function it means that each wave function can be threat like a manifold a pseudo-manifold that lives in Minkowski space-time. But each wave function has it's own way of measuring time and distance that is relative to itself. To make it consistent with quantum physics there is need to use Planck units as base units. All particles will move along shortest possible space-time path:

$$\psi^2(\mathbf{x})\left(\frac{d^2x^\mu(\mathbf{x})}{ds^2}+\Gamma^\mu_{\alpha\beta}\frac{dx^\alpha(\mathbf{x})}{ds}\frac{dx^\beta(\mathbf{x})}{ds}\right)=0 \tag{4.1}$$

At from it follows quantum gravity principle:
At all possible points of space-time wave function that is spread in all of points in space, will follow shortest possible space-time path for each point of space.
This principle says that for each point of space I have a geodesic equation to solve. So I need to solve infinite number of geodesic equations for each point of space for given instant of time. Space-time interval defines distance between two events. Space-time interval for all possible points of space-time can be written:

$$ds^2(\mathbf{x})=\psi^2(\mathbf{x})\,g_{\mu\nu}dx^\mu(\mathbf{x})\,dx^\nu(\mathbf{x}) \tag{4.2}$$

Where for each point of space there is probability function of particle being in that point and there is need to solve interval for all possible space points. So there is infinite number of space-time intervals for each point of space, when there is measurement done it changes to just one space-time interval with probability equal to wave function squared:

$$\psi^2(\mathbf{x})\to\delta(\mathbf{x}) \tag{4.3}$$

$$ds^2(\mathbf{x})=\delta(\mathbf{x})\,g_{\mu\nu}(\mathbf{x})\,dx^\mu(\mathbf{x})\,dx^\nu(\mathbf{x}) \tag{4.4}$$

Where $\delta(\mathbf{x})$ is function [10] that has zero everywhere and one at point of measurement in space for each instant of time. So space-time interval changes from being a field that has a particle at each possible position in space so infinite many possible space-time intervals to just have one possible position in space that gives one space-time interval. Same can be apply to geodesic equation:

$$\delta(\mathbf{x})\left(\frac{d^2x^\mu(\mathbf{x})}{ds^2}+\Gamma^\mu_{\alpha\beta}\frac{dx^\alpha(\mathbf{x})}{ds}\frac{dx^\beta(\mathbf{x})}{ds}\right)=0 \tag{4.5}$$

5. Many particle systems

Many particle systems are key to understand more complex objects than just point particles. It's very easy to extend all field equations for many particles i just add dependence on more coordinates sets where each set represents one particle:

$$\begin{aligned} &R^{\alpha}_{\lambda\mu\nu}\left(\mathbf{x_1},...,\mathbf{x_n}\right)\left(\psi\psi^*\right)^{\lambda}_{\rho}\left(\mathbf{x_1},...,\mathbf{x_n}\right) \\ -&\frac{1}{2}R\left(\mathbf{x_1},...,\mathbf{x_n}\right)\left(\psi\psi^*\right)^{\alpha}_{\rho}\left(\mathbf{x_1},...,\mathbf{x_n}\right)g_{\mu\nu}\left(\mathbf{x_1},...,\mathbf{x_n}\right) \\ &=\kappa T^{\alpha}_{\lambda\mu\nu}\left(\mathbf{x_1},...,\mathbf{x_n}\right)\left(\psi\psi^*\right)^{\lambda}_{\rho}\left(\mathbf{x_1},...,\mathbf{x_n}\right) \end{aligned} \tag{5.1}$$

$$\begin{aligned} &R^{\lambda}_{\rho\mu\nu}\left(\mathbf{x_1},...,\mathbf{x_n}\right)\left(\psi\psi^*\right)^{\alpha}_{\lambda}\left(\mathbf{x_1},...,\mathbf{x_n}\right) \\ -&\frac{1}{2}R\left(\mathbf{x_1},...,\mathbf{x_n}\right)\left(\psi\psi^*\right)^{\alpha}_{\rho}\left(\mathbf{x_1},...,\mathbf{x_n}\right)g_{\mu\nu}\left(\mathbf{x_1},...,\mathbf{x_n}\right) \\ &=\kappa T^{\lambda}_{\rho\mu\nu}\left(\mathbf{x_1},...,\mathbf{x_n}\right)\left(\psi\psi^*\right)^{\alpha}_{\lambda}\left(\mathbf{x_1},...,\mathbf{x_n}\right) \end{aligned} \tag{5.2}$$

$$\begin{aligned} &R_{\mu\nu}\left(\mathbf{x_1},...,\mathbf{x_n}\right)\psi^2\left(\mathbf{x_1},...,\mathbf{x_n}\right) \\ -&\frac{1}{2}R\left(\mathbf{x_1},...,\mathbf{x_n}\right)\psi^2\left(\mathbf{x_1},...,\mathbf{x_n}\right)g_{\mu\nu}\left(\mathbf{x_1},...,\mathbf{x_n}\right) \\ &=\kappa T_{\mu\nu}\left(\mathbf{x_1},...,\mathbf{x_n}\right)\psi^2\left(\mathbf{x_1},...,\mathbf{x_n}\right) \end{aligned} \tag{5.3}$$

Now when measurement is done I just use many delta functions for each particle for space-time interval and geodesic:

$$\delta\left(\mathbf{x_1}\right)...\delta\left(\mathbf{x_n}\right)\left(\frac{d^2x^{\mu}\left(\mathbf{x_1},...,\mathbf{x_n}\right)}{ds^2}+\Gamma^{\mu}_{\alpha\beta}\frac{dx^{\alpha}\left(\mathbf{x_1},...,\mathbf{x_n}\right)}{ds}\frac{dx^{\beta}\left(\mathbf{x_1},...,\mathbf{x_n}\right)}{ds}\right)=0 \tag{5.4}$$

$$ds^2\left(\mathbf{x}\right)=\delta\left(\mathbf{x_1}\right)...\delta\left(\mathbf{x_n}\right)g_{\mu\nu}\left(\mathbf{x_1},...,\mathbf{x_n}\right)dx^{\mu}\left(\mathbf{x_1},...,\mathbf{x_n}\right)dx^{\nu}\left(\mathbf{x_1},...,\mathbf{x_n}\right) \tag{5.5}$$

And before measurement I will have wave field that is contraction of complex mixed tensor field:

$$\psi^2\left(\mathbf{x_1},...,\mathbf{x_n}\right)\left(\frac{d^2x^{\mu}\left(\mathbf{x_1},...,\mathbf{x_n}\right)}{ds^2}+\Gamma^{\mu}_{\alpha\beta}\frac{dx^{\alpha}\left(\mathbf{x_1},...,\mathbf{x_n}\right)}{ds}\frac{dx^{\beta}\left(\mathbf{x_1},...,\mathbf{x_n}\right)}{ds}\right)=0 \tag{5.6}$$

$$ds^2\left(\mathbf{x}\right)=\psi^2\left(\mathbf{x_1},...,\mathbf{x_n}\right)g_{\mu\nu}\left(\mathbf{x_1},...,\mathbf{x_n}\right)dx^{\mu}\left(\mathbf{x_1},...,\mathbf{x_n}\right)dx^{\nu}\left(\mathbf{x_1},...,\mathbf{x_n}\right) \tag{5.7}$$

From fact that I do expand number of coordinates there is expansion of dimension of all fields, so if field it self has n dimension for k objects i will get dimensions $N = nk$.

References

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