Fine-structure constant from the Archimedes constant

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Abstract

A new interpretation and a very accurate value of the fine-structure constant has been discovered in terms of the Archimedes constant. The equation is simple,elegant and symmetrical in a great physical meaning. This accurate expression is the most impressive since it is simple and contains just a few prime numbers and the Archimedes constant.

Keywords

Fine-structure constant , Dimensionless physical constants , Madelung constant , Archimedes constant

1. Introduction

Archimedes constant π is a mathematical constant that appears in many types in all fields of mathematics and physics. It is also found in various types from other disciplines,such as Cosmology,numbers,Statistics,fractals, thermodynamics, engineering,and electromagnetism. Archimedes constant π appears in the cosmological constant, Heisenberg's uncertainty principle,Einstein's field equation of general relativity,Coulomb's law for the electric force in vacuum,Magnetic permeability of free space,Period of a simple pendulum with small amplitude,Kepler's third law of planetary motion,the buckling formula,etc.

Golden ratio φ is an omnipresent number in nature,found in the architecture of living creatures as well as human buildings,music,finance,medicine,philosophy,and of course in physics and mathematics including quantum computation. It is the most irrational number known and a number-theoretical chameleon with a self-similarity property. The golden ratio can be found in nearly all domains of Science,appearing when self-organization processes are at play and/or expressing minimum energy configurations. Several non-exhaustive examples are given in biology (natural and artificial phyllotaxis,genetic code and DNA),physics (hydrogen bonds,chaos,superconductivity), astrophysics (pulsating stars,black holes),chemistry (quasicrystals,protein AB models),and technology (tribology, resistors,quantum computing,quantum phase transitions,photonics). The fifth power of the golden mean appears in Phase transition of the hard hexagon lattice gas model,Phase transition of the hard square lattice gas model,One-dimensional hard-boson model,Baryonic matter relation according to the E-infinity theory,Maximum quantum probability of two particles,Maximum of matter energy density,Reciprocity relation between matter and dark matter,Superconductivity phase transition,etc.

Euler's number e is an important mathematical constant,which is the base of the natural logarithm. All five of these numbers play important and repetitive roles in mathematics and these five constants appear in a formulation of Euler's identity. Euler's number has many practical uses,especially in higher level mathematics such as calculus,differential equations,trigonometry,complex analysis,statistics,etc. Euler's number frequently appears in problems related to growth or decay,where the rate of change is determined by the present value of the number being measured. One example is in biology,where bacterial populations are expected to double at reliable intervals. Another case is radiometric dating,where the number of radioactive atoms is expected to decline over the fixed half-life of the element being measured. From Euler's identity the following relation of the mathematical constant e can emerge $e=i^{-2i/\pi}$.

Gelfond's constant,in mathematics,is the number e^{π},e raised to the power π. Like e and π,this constant is a transcendental number. It was named after the Soviet mathematician Aleksandr Gelfond. Gelfond's constant were singled out in Hilbert's 7th problem as an example of numbers whose excess was an open problem. This was first established by Gelfond and may now be considered as an application of the Gelfond–Schneider theorem,noting that $e^{\pi}=(e^{i\pi})^{-i}=(-1)^{-i}=i^{-2i}$.

Euler's constant is a mathematical constant usually denoted by the lowercase Greek letter gamma (γ). The number γ has not been proved algebraic or transcendental. In fact,it is not even known whether γ is irrational. The numerical value of Euler's constant is $\gamma=0,57721566490153286...$

The imaginary unit i is a solution to the quadratic equation $x^2+1=0$. Although there is no real number with this property,it can be used to extend the real numbers to what are called complex numbers,using addition and multiplication. Despite their misleading name,imaginary numbers are not only real but also very useful,with application in electricity,signal processing and many other applications. They are widely used in electronics,for the representation of alternating currents and in waves.

Euler's identity is considered to be an exemplar of mathematical beauty as it shows a profound connection between the most fundamental numbers in mathematics:

$$e^{i\pi}+1=0$$

The expression who connects the six basic mathematical constants,the number 0,the number 1,the golden ratio φ,the Archimedes constant π,the Euler's number e and the imaginary unit i is:

$$e^{\frac{i\pi}{1+\varphi}}+e^{\frac{-i\pi}{1+\varphi}}+e^{\frac{i\pi}{\varphi}}+e^{\frac{-i\pi}{\varphi}}=0$$

In [7] we presented exact and approximate expressions between the Archimedes constant π,the golden ratio φ,the Euler's number e and the imaginary number i.

In physics,a dimensionless physical constant is a physical constant that is dimensionless,a pure number having no units attached and having a numerical value that is independent of whatever system of units may be used. The term fundamental physical constant is used to refer to some universal dimensionless constants. A long-sought goal of theoretical physics is to find first principles from which all of the fundamental dimensionless constants can be calculated and compared to the measured values. In the 1.920s and 1.930s,Arthur Eddington embarked upon extensive mathematical investigation into the relations between the fundamental quantities in basic physical theories,later used as part of his effort to construct an overarching theory unifying quantum mechanics and cosmological physics. The mathematician Simon Plouffe has made an extensive search of computer databases of mathematical formulas,seeking formulas for the mass ratios of the fundamental particles. An empirical relation between the masses of the electron,muon and tau has been discovered by physicist Yoshio Koide,but this formula remains unexplained.

Dimensionless physical constants cannot be derived and have to be measured. Developments in physics may lead to either a reduction or an extension of their number:discovery of new particles,or new relationships between physical phenomena,would introduce new constants,while the development of a more fundamental theory might allow the derivation of several constants from a more fundamental constant. A long-sought goal of theoretical physics is to find first principles from which all of the fundamental dimensionless constants can be calculated and compared to the measured values. The laws of physics have a set of fundamental constants,and it is generally admitted that only dimensionless combinations of constants have physical significance. These combinations include the electromagnetic and gravitational fine structure,along with the ratios of elementary particles masses. Cosmological measurements clearly depend on the values of these constants in the past and can therefore give information on their time dependence if the effects of time-varying constants can be separated from the effects of cosmological parameters.

The fine-structure constant is one of the most fundamental constants of physics. It describes the strength of the force of electromagnetism between elementary particles in what is known as the standard model of particle physics. In particular, the fine-structure constant sets the strength of electromagnetic interaction between light (photons) and charged elementary particles such as electrons and muons. The quantity α was introduced into physics by A. Sommerfeld in 1.916 and in the past has often been referred to as the Sommerfeld fine-structure constant. In order to explain the observed splitting or fine structure of the energy levels of the hydrogen atom, Sommerfeld extended the Bohr theory to include elliptical orbits and the relativistic dependence of mass on velocity.

One of the most important numbers in physics is the fine-structure constant α which defines the strength of the electro-magnetic field. It is a dimensionless number independent of how we define our units of mass,length,time or electric charge. A change in these units of measurement leaves the dimensionless constant unchanged. The number can be seen as the chance that an electron emits or absorbs a photon. It’s a pure number that shapes the universe to an astonishing degree. Paul Dirac considered the origin of the number «the most fundamental unsolved problem of physics». The constant is everywhere because it characterizes the strength of the electromagnetic force affecting charged particles such as electrons and protons. Many eminent physicists and philosophers of science have pondered why α itself has the value that it does,because the value shows up in so many important scenarios and aspects of physics. Nobody has come up with any ideas that are even remotely convincing. A similar situation occurs with the proton-electron mass ratio μ,not because of its ubiquity,but rather how chemistry can be based on two key electrically charged particles of opposite electric charge that are opposite but of seemingly identical magnitude while their masses have a ratio that is quite large yet finite. These two questions have a huge bearing on why physics and chemistry behave the way they do. The product of the two quantities appears,at least at first glance,not to be so important.

The fine-structure constant α is defined as:

$$\alpha = \frac{q_e^2}{4\pi\varepsilon_0 \hbar c}$$

The 2.018 CODATA recommended value of the fine-structure constant is α=0,0072973525693(11) with standard uncertainty $0{,}0000000011\times10^{-3}$ and relative standard uncertainty $1{,}5\times10^{-10}$. Also the fine-structure constant is universal scaling factor:

$$\alpha = \frac{2\pi r_e}{\lambda_e} = \frac{\lambda_e}{2\pi\alpha_0} = \frac{r_e}{l_{pl}}\frac{m_e}{m_{pl}} = \sqrt{\frac{r_e}{\alpha_0}}$$

There is a dream,which,albeit more often not confessed,occupies the most secret aspirations of theoreticians and is that of reducing the various constants of Physics to simple formula involving integers and transcendent numbers. The fine-structure constant plays an important role in modern physics. Yet it continues to be a mystery as to exactly what it represents and why it has the mystical value it has. The purpose of many sciences is to find the most accurate mathematical formula that can be found in the experimental value of fine-structure constant. Attempts to find a mathematical basis for this dimensionless constant have continued up to the present time. However,no numerological explanation has ever been accepted by the physics community.

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The 2.018 CODATA recommended value of α is:

$$\alpha=0.0072973525693(11)$$

With standard uncertainty $0{,}0000000011\times10^{-3}$ and relative standard uncertainty $1{,}5\times10^{-10}$. For reasons of convenience,historically the value of the reciprocal of the fine-structure constant is often specified. The 2.018 CODATA recommended value is given by:

$$\alpha^{-1}=137{,}035999084(21)$$

With standard uncertainty $0{,}000000021\times10^{-3}$ and relative standard uncertainty $1{,}5\times10^{-10}$. There is general agreement for the value of α,as measured by these different methods. The preferred methods in 2.019 are measurements of electron anomalous magnetic moments and of photon recoil in atom interferometry. The most precise value of α obtained experimentally (as of 2.012) is based on a measurement of g using a one-electron so-called "quantum cyclotron" apparatus,together with a calculation via the theory of QED that involved 12.672 tenth-order Feynman diagrams:

$$\alpha^{-1}=137{,}035999174(35)$$

This measurement of α has a relative standard uncertainty of $2{,}5\times10^{-10}$. This value and uncertainty are about the same as the latest experimental results. Further refinement of this work were published by the end of 2.020,giving the value:

$$\alpha^{-1}=137{,}035999206(11)$$

with a relative accuracy of 81 parts per trillion.

2. The search for mathematical expression for the fine-structure constant

The mystery about the fine-structure constant is actually a double mystery. The first mystery – the origin of its numerical value – has been recognized and discussed for decades. The second mystery – the range of its domain – is generally unrecognized.

— M. H. MacGregor (2.007). The Power of Alpha.

When I die my first question to the Devil will be: What is the meaning of the fine structure constant?

— Wolfgang Pauli

“God is a pure mathematician!' declared British astronomer Sir James Jeans. The physical Universe does seem to be organized around elegant mathematical relationships. And one number above all others has exercised an enduring fascination for physicists: 137,0359991.... It is known as the fine-structure constant and is denoted by the Greek letter alpha (α).”

— Paul Davies

“While twentieth-century physicists were not able to identify any convincing mathematical constants underlying the fine structure, partly because such thinking has normally not been encouraged, a revolutionary suggestion was recently made by the Czech physicist Raji Heyrovska, who deduced that the fine structure constant, ...really is defined by the [golden] ratio ....”

— Carl Johan Calleman, The Purposeful Universe: How Quantum Theory and Mayan Cosmology Explain the Origin and Evolution of Life

The fine-structure constant plays an important role in modern physics. Yet it continues to be a mystery as to exactly what it represents and why it has the mystical value it has. The elementary charge of electron e was proposed by Stoney in 1.894 and discovered by Thomson in 1.896,then Planck introduced the energy quanta $h\cdot v$ in 1.901 and explained it as photon $E=h\cdot v$ by Einstein in 1.905. Planck first noticed in 1.905 that e^2/c and h have the same dimension. In 1.909,Einstein found that there are two fundamental velocities in physics: c and e^2/h requiring explanation. He said, “It seems to me that we can conclude from $h=e^2/c$ that the same modification of theory that contains the elementary quantum e as a consequence,will also contain as a consequence the quantum structure of radiation.” Albert Einstein was the first to use a mathematical formula for the fine-structure constant α in 1.909. This expression is:

$$\alpha = \frac{7\pi}{3.000}$$

with numerical value α=0,00733038286 with an error accuracy of 0,45%. Later many scientists used other mathematical formulas for fine-structure constant but they are not at all accurate. These are Jeans 1.913,Lewis Adams 1.914,Lunn in 1.922,Peirles in 1.928 and others. Arthur Eddington was the first to focus on its inverse value and suggested that it should be an integer,that the theoretical value is $\alpha^{-1}=136$. In his original document 1.929 he applied the value:

$$\alpha^{-1}=16+1/2\times16\times(16-1)=136$$

However,the experiments themselves consistently showed that $\alpha^{-1}\approx137$. This forced him to look for an error in his original theory. He soon came to the conclusion that:

$$\alpha^{-1}=137$$

He thus argued that the extra unit was a consequence of the initial exclusion of every elementary particle pair in the universe. In the document of 1.929,Eddington considered that fine-structure constant relates in a simple way to the cosmological constants,as given by the expression:

$$\alpha = \frac{2\pi mcR_E}{h\sqrt{N}}$$

where N the cosmic number,the number of electrons and protons in the closed universe. Eddington always kept the name and the symbol α:

$$\alpha = \frac{hc}{2\pi q_e^2}$$

The first to find an exact formula for the fine-structure constant α was the Swiss mathematician Armand Wyler in 1.969. Based on the arguments concerning the congruent group,the group consists of simple Lorentz transformations such as the space-time dimensions that leave the Maxwell equations unchanged. The first form of the Wyler constant type is:

$$\alpha_w = \left(\frac{9}{16\pi^3}\right)\left(\frac{\pi}{5!}\right)^{\frac{1}{4}}$$

With numerical value α_w=0,00729735252... At the time it was proposed,they agreed with the experiment to be within 1.5 ppm for the value α^{-1}.

We proposed in [8] the exact formula for the fine-structure constant α in terms of the golden angle,the relativity factor and the fifth power of the golden mean:

$$\alpha^{-1}=360\cdot\varphi^{-2}-2\cdot\varphi^{-3}+(3\cdot\varphi)^{-5} \tag{1}$$

with numerical value:

$$\alpha^{-1}=137{,}035999164.....$$

Another beautiful forms of the equations are:

$$\frac{1}{\alpha} = \frac{360}{\varphi^2} - \frac{2}{\varphi^3} + \frac{1}{3^5\varphi^5} \tag{2}$$

$$\frac{1}{\alpha} = \frac{360}{\varphi^2} - \frac{2}{\varphi^3} + \frac{3^{-5}}{\varphi^5} \tag{3}$$

Other equivalent expressions for the fine-structure constant are:

$$\alpha^{-1}=(362-3^{-4})\cdot\varphi^{-2}-(1-3^{-5})\cdot\varphi^{-1} \tag{4}$$

$$\alpha^{-1}=(362-3^{-4})+(3^{-4}+2\cdot3^{-5}-364)\cdot\varphi^{-1} \tag{5}$$

$$\alpha^{-1}=1-2\cdot\varphi^{-1}+360\cdot\varphi^{-2}-\varphi^{-3}+(3\cdot\varphi)^{-5} \tag{6}$$

4. Fine-structure constant from the Archimedes constant

We proposed in [9] the simple and absolutely accurate expression for the fine-structure constant in terms of the Archimedes constant π:

$$\alpha^{-1} = \frac{2.706}{43}\pi\ln 2 \tag{7}$$

with absolutely accurate numerical value:

$$\alpha^{-1}=137{,}035999078...$$

$$\alpha=0{,}00729735256959...$$

The equivalent expression for the fine-structure constant is:

$$\alpha^{-1}=2\cdot3\cdot11\cdot41\cdot43^{-1}\cdot\pi\cdot\ln2 \tag{8}$$

Other equivalent expression for the fine-structure constant are:

$$\alpha^{-1}=[(14^3-38)\cdot43^{-1}]\cdot\ln2\cdot\pi \tag{9}$$

$$\frac{2}{43}\times3\times11\times41\,\pi\log(2)=\frac{2706}{43}\,\pi\log(a)\log_a(2) \tag{10}$$

$$\frac{2}{43}\times3\times11\times41\,\pi\log(2)=\frac{5412}{43}\,\pi\coth^{-1}(3) \tag{11}$$

$$\frac{2}{43}\times3\times11\times41\,\pi\log(2)=\frac{5412}{43}\,\pi\tanh^{-1}\left(\frac{1}{3}\right) \tag{12}$$

$$\frac{2}{43}\times3\times11\times41\,\pi\log(2)=\frac{5412}{43}\,i\,\pi\cot^{-1}(3\,i) \tag{13}$$

$$\frac{2}{43}\times3\times11\times41\,\pi\log(2)=-\frac{2706}{43}\,\pi\,S_{0,1}(-1) \tag{14}$$

$$\frac{2}{43}\times3\times11\times41\,\pi\log(2)=\frac{5412}{43}\,i\,\pi\tan^{-1}\left(-\frac{i}{3}\right) \tag{15}$$

$$\frac{2}{43}\times3\times11\times41\,\pi\log(2)=\frac{2706\,\pi}{43}\int_1^2\frac{1}{t}\,dt \tag{16}$$

$$\frac{2}{43}\times3\times11\times41\,\pi\log(2)=-\frac{1353\,i}{43}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\quad\text{for } -1<\gamma<0 \tag{17}$$

This accurate expression is the most impressive since it is simple and contains just a few prime numbers and the Archimedes constant. These prime numbers can be possibly connected to finite groups (Group of Lie type).
The series representations for the fine-structure constant are:

$$\begin{aligned}\frac{2}{43}\times3\times11\times41\log(2)\,\pi=&\frac{5412}{43}\,i\,\pi^2\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor+\\&\frac{2706}{43}\,\pi\log(x)-\frac{2706}{43}\,\pi\sum_{k=1}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}}{k}\quad\text{for } x<0\end{aligned} \tag{18}$$

$$\begin{aligned}\frac{2}{43}\times3\times11\times41\log(2)\,\pi=&\frac{2706}{43}\,\pi\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\frac{2706}{43}\,\pi\log(z_0)+\\&\frac{2706}{43}\,\pi\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log(z_0)-\frac{2706}{43}\,\pi\sum_{k=1}^{\infty}\frac{(-1)^k\,(2-z_0)^k\,z_0^{-k}}{k}\end{aligned} \tag{19}$$

$$\begin{aligned}\frac{2}{43}\times3\times11\times41\log(2)\,\pi=&\frac{5412}{43}\,i\,\pi^2\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\\&\frac{2706}{43}\,\pi\log(z_0)-\frac{2706}{43}\,\pi\sum_{k=1}^{\infty}\frac{(-1)^k\,(2-z_0)^k\,z_0^{-k}}{k}\end{aligned} \tag{20}$$

We pose:

$$\alpha_b = 2.706/43 = 2 \cdot 3 \cdot 11 \cdot 41 \cdot 43^{-1}$$

This combination of prime numbers we will call the coefficient for the fine-structure constant. So the expression for the fine-structure constant can written as:

$$\alpha^{-1} = -\alpha_b \cdot \pi \cdot \ln 2 \tag{21}$$

The repeating decimal of the coefficient for the fine-structure constant α_b is:

$$62{,}\overline{930232558139534883720} \text{ (period 21)}$$

The continued fraction for the α_b is:

$$62 + \cfrac{1}{1 + \cfrac{1}{13 + \frac{1}{3}}}$$

The egyptian fraction expansion for the α_b is:

$$62 + \frac{1}{2} + \frac{1}{3} + \frac{1}{11} + \frac{1}{167} + \frac{1}{473946}$$

The repeating decimal of the α_b^{-1} is:

$$0{,}\overline{01589061345} \text{ (period 10)}$$

So from (21) the expression for the fine-structure constant with the Archimedes constant is:

$$\alpha^{-1} = -\alpha_b \cdot \pi \cdot \ln 2$$

$$\alpha \cdot \pi \cdot \ln 2 = -0{,}\overline{01589061345} \tag{22}$$

5. Continued fraction for the fine-structure constant

The pattern of the continued fraction for the fine-structure constant is:

[137; 27, 1, 3, 1, 1, 16, 1, 4, 45, 12, 2, 4, 1, 6, 4, 2, 1, 155, 2, 7, 26, 1, 2, 1,
2, 10, 2, 3, 11, 2, 1, 2, 1, 4, 2, 1, 2, 7, 7, 1, 1, 1, 1, 2, 1, 5, 1, 1, 1, 1, 5, 1, 3,
1, 1, 1, 1, 1, 1, 5, 2, 9, 1, 1, 3, 2, 1, 27, 1, 11, 1, 1, 1, 11, 2, 5, 1, 3, 41, 1, 4,
1, 13, 1, 1, 38, 1, 4, 2, 1, 2, 6, 1, 2, 18, 33, 1, 1, 4, 1, 9611, 1, 6, 5, 5, 1, 5,
1, 1, 2, 2, 1, 2, 1, 2, 2, 1, 2, 12, 2, 1, 3, 8, 1, 1, 5, 2, 8, 2, 8, 111, 12, 3, 3, 2,
1, 1, 18, 1, 1, 1, 1, 15, 1, 6, 3, 7, 1, 1, 1, 1, 2, 9, 1, 2, 1, 2, 2, 1, 1, 1, 1, 1, 3,
1, 9, 2, 2, 1, 12, 16, 1, 4, 1, 1, 3, 3, 54, 1, 1, 1, 2, 1, 2, 2, 3, 1, 9, 1, 2, 4, 1,
1, 2, 1, 6, 4, 8, 42, 1, 1, 9, 12, 1, 7, 9, 1, 1, 1, 3, 3, 3, 16, 2, 1, 2, 12, 1, 2, 2,
1, 3, 1, 5, 7, 2, 1, 3, 3, 1, 2, 1, 3, 2, 2, 6, 1, 6, 1, 2, 1, 1, 9, 1, 2, 2, 106, 1, 1,
1, 1, 8, 1, 1, 1, 27, 2, 55, 2, 1, 6, 2, 1, 3, 1, 9, 1, 3, 2, 1, 6, 4, 1, 13, 2, 2, 1,
2, 6, 1, 2, 1, 1, 3, 6, 1, 2, 189, 3, 8, 3, 4, 4, 1, 3, 5, 1, 1, 6, 565, 1, 1, 2, 7, 2,
1, 1, 4, 1, 2, 2, 3, 1, 3, 96, 9, 2, 1, 2, 67, 1, 1, 1, 1, 8, 3, 1, 4, 1, 2, 26, 1, 2,
1, 3, 5, 13, 11, 3, 1, 1, 10, 1, 1, 2, 1, 2, 1, 2, 3, 1, 2, 1, 1, 1, 1, 2, 1, 1, 1, 1,
11, 5, 9, 14, 1, 1, 4, 2, 1, 5, 1, 9, 2, 77, 5, 1, 1, 1, 5, 4, 1, 1, 3, 1, 10, 2, 1, 5,
10, 4, 1, 6, 1, 2, 1, 2, 1, 2, 16, 708, 1, 4, 3, 1, 1, 14, 1, 3, 1, 1, 2, 38, 46, 1,
1, 4, 1, 1, 2, 3, 2, 2, 15, 1, 3, 1, 1, 20, 1, 14, 6, 2, 7, 2, 1, 2, 14, 4, 3, 1, 1, …]

The continued fraction for the fine-structure constant is:

$$137+\cfrac{1}{27+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{16+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{45+\cfrac{1}{12+\cfrac{1}{2+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{6+\cfrac{1}{4+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{\cdots}}}}}}}}}}}}}}}}}}$$

6. Fine-structure constant from the Madelung constant

The Madelung constant is used in determining the electrostatic potential of a single ion in a crystal by approximating the ions by point charges. It is named after Erwin Madelung,a German physicist. Because the anions and cations in an ionic solid attract each other by virtue of their opposing charges,separating the ions requires a certain amount of energy. This energy must be given to the system in order to break the anion–cation bonds. The energy required to break these bonds for one mole of an ionic solid under standard conditions is the lattice energy. The Madelung constant is also a useful quantity in describing the lattice energy of organic salts. Izgorodina and coworkers have described a generalized method (called the EUGEN method) of calculating the Madelung constant for any crystal structure.

The quantities obtained [4] from cubic,hexagonal,etc.lattice sums, evaluated at s=1,are called Madelung constants. For cubic lattice sums:

$$b_n(2s) \equiv \sum_{k_1,\ldots,k_n=-\infty}^{\infty}{}' \frac{(-1)^{k_1+\ldots+k_n}}{\left(k_1^2+\ldots+k_n^2\right)^s},$$

The Madelung constants are expressible in closed form for even indices n,a few examples of which are summarized in the following table, where beta(n) is the Dirichlet beta function and eta(n) is the Dirichlet eta function. To obtain the closed form for b2(s),break up the double sum into pieces that do not include i=j=0:

$$b_2(2s) = 4\left[\sum_{i,j=1}^{\infty} \frac{(-1)^{i+j}}{\left(i^2+j^2\right)^s} + \sum_{i=1}^{\infty} \frac{(-1)^i}{i^{2s}}\right].$$

The second of these sums can be done analytically as:

$$\sum_{i=1}^{\infty} \frac{(-1)^i}{i^{2s}} = -4^s\left(4^s-2\right)\zeta(2s),$$

which in the case s=1 reduces to:

$$\sum_{i=1}^{\infty} \frac{(-1)^i}{i^2} = -\tfrac{1}{12}\pi^2.$$

The first sum is more difficult, but in the case s=1 can be written:

$$\sum_{i,j=1}^{\infty} \frac{(-1)^{i+j}}{i^2+j^2} = \frac{1}{12}\pi(\pi - 3\ln 2).$$

Combining these then gives the original sum as:

$$b_2(2) = -\pi \ln 2.$$

So from (7) the equivalent expressions for the fine-structure constant with the madelung constant $b_2(2)$ are:

$$\alpha^{-1} = -\frac{2.706}{43} b_2(2) \tag{23}$$

$$\alpha^{-1} = -2 \cdot 3 \cdot 11 \cdot 41 \cdot 43^{-1} \cdot b_2(2) \tag{24}$$

$$\alpha^{-1} = -[(14^3-38) \cdot 43^{-1}] \cdot b_2(2) \tag{25}$$

Also the expression for the fine-structure constant can written as:

$$\alpha^{-1} = -\alpha_b \cdot b_2(2)$$

So from the expression for the fine-structure constant with the madelung constant $b_2(2)$ is:

$$\alpha \cdot b_2(2) = -0,\overline{01589061345} \tag{26}$$

6. Conclusions

We presented new exact formula for the fine-structure constant in terms of the Archimedes constant:

$$\alpha^{-1} = -2 \cdot 3 \cdot 11 \cdot 41 \cdot 43^{-1} \cdot \pi \cdot \ln 2$$

This accurate expression is the most impressive since it is simple and contains just a few prime numbers and the Archimedes constant. These prime numbers can be possibly connected to finite groups. This exact formula should be studied further.

References

[1] www.en.wikipedia.org/
[2] www.wolframalpha.com/
[3] www.math.stackexchange.com/
[4] www.mathworld.wolfram.com/
[5] www.numberempire.com/
[6] http://physics.nist.gov/cuu/Constants/
[7] Stergios Pellis Unification Archimedes Constant π,Golden Ratio φ,Euler's Number e and Imaginary Number i http://dx.doi.org/10.2139/ssrn.3975869, 2.021
[8] Stergios Pellis Exact formula for the fine-structure constant α in terms of the golden ratio φ http://dx.doi.org/10.2139/ssrn.4160769, 2.021,
[9] Stergios Pellis Exact expressions of the fine-structure constant http://dx.doi.org/10.2139/ssrn.3975861, 2.021
[10] Stergios Pellis Exact mathematical expressions of the proton to electron mass ratio http://dx.doi.org/10.2139/ssrn.3967998, 2.021
[11] Stergios Pellis Exact mathematical Formula that connect 6 dimensionless physical constants http://dx.doi.org/10.2139/ssrn.3963427, 2.021

[12] Stergios Pellis Unity formula that connect the fine structure constant and the proton to electron mass ratio http://dx.doi.org/10.2139/ssrn.3963425, 2.021
[13] Stergios Pellis Theoretical value for the strong coupling constant
http://dx.doi.org/10.2139/ssrn.3998175, 2022
[14] Stergios Pellis Unity formulas for the coupling constants and the dimensionless physical constants
http://dx.doi.org/10.2139/ssrn.4003636, 2022
[15] Stergios Pellis Fine-Structure Constant from the Madelung Constant
http://dx.doi.org/10.2139/ssrn.4174644, 2.022
[16] Stergios Pellis Dimensionless Unification of the Fundamental Interactions
http://dx.doi.org/10.2139/ssrn.4201780, 2.022
[17] Stergios Pellis Unification of the fundamental interactions
DOI: 10.13140/RG.2.2.12296.70405, 2.022
[18] Stergios Pellis Unification of the Fundamental Forces
DOI: 10.13140/RG.2.2.33651.60967, 2.022
[19] Stergios Pellis Fine-Structure Constant from the Golden Angle,the Relativity Factor and the Fifth Power of the Golden Mean
DOI: 10.13140/RG.2.2.33651.60967, 2.022
[20] Stergios Pellis Unification of atomic physics and cosmology
DOI: 10.13140/RG.2.2.11493.88804, 2.022
[21] Stergios Pellis Dimensionless solution for the cosmological constant
http://dx.doi.org/10.2139/ssrn.4219292, 2.022