A New Fastest Method to Find Perfect Square Roots, Non Perfect Square Roots & Even Higher Roots Like Cube Roots 4^{th}, 5^{th}, 6^{th}, 7^{th} …. n^{th} Roots.

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Introduction-

Perfect square root of any √ X can be describe as x is the square root of y and it is represented as $x^2 = y$. For example square root if 3 is 9, $3^2 = 9$.

A non-perfect square root of √ X is that every Integer that is not the result of squaring an integer with itself.

E.g, 5, 7, 10, 21

There are various method of finding square roots like Initial estimate, Babylonian method, Bakhshali method, Digit-by-digit calculation, Newton's method etc. Many of those method are about guessing method with trial and error.

Some of them take longer time to compute with vast series of calculations.

I introduce a new method which is simple, yet fastest method to calculate with a 100% accuracy to find perfect square roots and with better approximation in case of non prefect square roots of any given √X. Also using this method one can easily find higher roots like cube roots, 4^{th}, 5^{th}, 6^{th}, 7^{th} …. n^{th} roots.

Formula -

Those below are the simple formula which we will use to find 'Key Integer' to find m which is required for calculating c.

$$\frac{x}{72} = m$$

1.1) Repeated Subtraction of Series –

To find the key integer ‘c' we will use the below infinite **repeated subtraction series** where we will subtract every difference of series using the following **Sequence Series** -

Repeated Subtraction Series

$$m_1 - 1 = m_2$$
$$m_2 - 2 = m_3$$
$$m_3 - 3 = m_4$$

Img 1.1

$$Eg.$$
$$m_1 = 5$$
$$5 - 1 = 4$$
$$4 - 2 = 2$$

Img 1.2

1) While calculating, note that each difference of the series can be strictly 0 or any positive integers. E.g as shown on the above image 1.2 all the difference are positive i.e. 5-1 = **4**, 4 - 2 = **2**

2 – 3 = -1 (like this series must not have negative difference)

Note - Always use below any one of the two methods to find Key Integer ‘c’

1.2) First Method of Finding Key Integer ‘c’ - Repeated Subtraction Series.

$$m_1 - 1 = m_2$$
$$m_2 - 2 = m$$
$$m_3 - 3 = m$$

$$Eg.$$
$$m_1 = 5$$
$$5 - 1 = 4$$
$$4 - 2 = 2$$

Img 1.3 img 1.4

1) As we find m from the formula X / 72= m where X is 360 / 72 = 5

Take m as m_1 = 5

Substituting in the ' Repeated Subtraction Series' as shown below.

5 -1 = 4

4 - 2 = 2

2 – 3 = -1 (continuing from above when we subtract 2 by 3 we get -1).

But as stated above at explanation 1.1, we cannot have negative difference.

This way one can go on calculating until the series reaches the point where we start getting negative difference as result. Once we reach at this point of the series where we start getting negative results, we must stop the calculation. Therefore above shown example series ends at 4 - 2 = 2.

3) The Key Integer 'c' we be the last negative integer of the following **Sequence of Series.**

E.g, here we got -2 as **Key Integer.**

4) We must always convert the negative key integer to positive. E.g, **Key Integer -2** will be 2 as positive integer. Conversion using simple equation -a × -1 = c, here as per the example a = -2

Therefore, -2 × -1 = 2

At above example, we got the last series as 4 -2 = 2

Now, notice the last subtracting sequence term that is -2

So, we must consider the negative integer -2 as positive integer. So, we get **Key Integer 'c'.** Therefore**, c = 2** as positive integer.

1.3) Second Method of Finding Key Integer 'c' – Successive Series.

Successive Series –

0 -- 1 has zero difference

1 --- 3 has only as difference of one i.e 2

3 ---- 6 has a difference of two i.e. 4 and 5

6 ---- 10 has a difference of three i.e. 7 to 9

10 ---- 15 has a difference of four i.e. 11 to 14

15 ---- 21 has a difference of five i.e. 16 to 20

21 ---- 28 has a difference of six i.e. 22 to 27

28 ---- 36 has a difference of seven i.e. 28 to 35

36 ---- 45 has a difference of eight i.e. 37 to 44

45 ---- 55 has a difference of nine i.e. 46 to 54

55 ---- 66 has a difference of ten i.e. 56 to 65

66 ---- 78 has a difference of eleven i.e. 67 to 77

78 ---- 91 has a difference of twelve i.e. 79 to 90

91 ---- 105 has a difference of thirteen i.e. 92 to 104

105 ---- 120 has a difference of fourteen i.e. 106 to 119

120 ---- 136 has a difference of fifteen i.e. 121 to 135

136 ---- 153 has a difference of sixteen i.e. 137 to 152

This series is a form of successive series where sequence of numbers on the right hand side that is, 3, 6, 10, 15, 21….
Is the result of addition of sequence of positive integers such as 1, 2, 3, 4, 5, 6, 7, 8….
E.g, 1 + **2** = 3, 3 + **3** = 6, 6 + 4 = 10 ….

As show above, one can continue create this series as record and use it whenever required to calculate Key Integer 'c'. Series can go to infinity . This second method is very useful when we need to find larger Key Integer 'c' of any larger √ X.
Example - Let suppose we want to find key integer of √ 2209
Using

$$\frac{x}{72} = m$$

√ 2209 / 72 = 30.68056
Ignoring the decimals, we got m = 30
To find key integer from m = 30 , check the above 'Pattern Series' at 1.3
30 appears between this part of above series at 28 --30-- 36 and has a difference of seven i.e. 28 to 35
n.
Therefore, we must take Key Integer 'c'

c = 7

1.4) Calculation of Steps For Higher Roots –

Note – The below conditions must be only followed in case while finding Key Integer ‘c' of any higher roots like cube roots, 4^{th} , 5^{th} , 6^{th}, 7^{th} roots…..

For Cube root – up to 2 steps.
For 4^{th} root – up to 3 steps.
For 5^{th} root – up to 4 steps
Fir 6^{th} root – up to 5 steps
For 7^{th} roots up to 6 steps (and so on increase one step for every nth roots)

As higher roots increases, the steps of calculation increase by one as shown at the above. Use the applicable steps depending on which roots you want to find.
Example- we want to find **Key Integer ‘ c’** of cube root 3 √1331.
Check the required steps of calculation for cube root at above calculation steps explanation 1.4
Therefore, for cube root, we must calculate up till 2 steps of calculation.

As usual first find m using formula.
√ 1331 / 72 = 18.486...
Ignoring the decimals, we got m = 18

Following 1^{st} step -
To find key integer from m = 18 , check the above ‘Pattern Series’ at 1.3
18 somewhere appears between this part of above series 15 --18-- 21 and has a difference of **five** i.e. from 16 to 20

Therefore, we must take Key Integer ‘c'
c = 5
Following 2^{st} step -
Carried forward c = 5 from 1^{st} step and substitute it with m
To find key integer from m = 5 , check the above ‘Pattern Series’ at 1.3
5 somewhere appears between this part of above series 3 ---- 6 and has a difference of **two** i.e. 4 and 5

Therefore, we must take Key Integer ‘c'
c = 2

This way one can easily find the Key Integer ‘c' using the above explained method.

1.5 Multiplying c by Constant 6 -

This step is very important and is calculated after we get Key Integer value ‘c'
E.g, at above explanation of calculation of steps at 1.4 we get c = 2

$2 \times 6 = 12$
Now we get c = 12
Very Important to Note - that in some case of finding solutions we may get c itself as final answer. We can check c by dividing it with associated √ X . If it is divisible, and we get square roots solutions as required, then c is the final answer, if it doesn't yields solution then proceed to below rules to find final answer c (for better understanding check various examples solutions shown at this paper).

1.6) Rules of Finding ‘c In case of Square Roots -

As we find the final answer ‘c as square root through the formula **r/6= s** we must consider the following rule that is required to get the perfect square root with 100% accurately and in case of non perfect square root we can get to some degree of accuracy.

First Rule - If the √ X is any odd integer and is not divisible by 2 or 3, then the final answer will be either adding c by 5 i.e. c + 5 adding c by 7 i.e. c + 7

Second Rule - If the√ X is any even integer and is divisible by 2 or 3, then the final answer will be by either adding c by 3 i.e. ‘c - 3’ or adding c by 6 i.e. ‘c - 6’.

Third Rule- If the √ X is any even integer which is divisible by 2, then the final answer will be either adding c by 4 i.e. ‘c + 4 or adding c by 8 i.e. ‘c + 8'.

Fourth Rule - If the √ X is any odd integer and is divisible by 3, then the final answer c will be subtracting c by 3 i.e. ‘c + 3'.

To find square root of any given √ X integer we must follow all the above explained formula, series and rules to get the precise results. No matter how larger digit the √ X integers would be, using the method explained in this paper, one can get the answer very quickly and precisely.

Below Are Some of The Examples Solutions of Finding Square Roots -

First Example –

To find perfect square root of $\sqrt{196} = s$, where s is the square root.

1st Step - Using the below formula by inputting 196 to get m.

$$\frac{x}{72} = m$$

196/ 72 = 2.7222 …..
By ignoring the decimals, we get m = 2
We will take m value as m_1 to input it in the below repeated subtraction series.
$m_1 = 2$

2nd Step – (Finding The Key Integer 'c' Using First Method-Repeated Subtraction Series. ….. from 1.2

Note that just for explanation purpose, i have shown both example of finding Key Integer c. One can use any of one of the two method for finding Key Integer c).

$$m_1 - 1 = m_2$$
$$m_2 - 2 = m_3$$
$$m_3 - 3 = m_4$$

By substituting the $m_1 = 2$ in the repeated subtraction series we get,

Where $m_1 = 2$
2 -1 = 1

As per the 'condition of series' we cannot further calculate 1- 2 = -1 since it gives negative value.
Therefore we stop the calculation at 2 – 1 = 1
Last negative series of sequence is -1 and by converting it as positive integer, -1 × -1 = 1
we got **Key Integer 'c' as** 1
Therefore, c =1

Explanation of Using Second Method to Finding Key Integer 'c' …… from 1.3

To find key integer c of √ 196
using

$$\frac{x}{72} = m$$

√ 196 / 72 = 2. 72222…
Ignoring the decimals, we got m = 2
To find key integer from m = 2 , check the above 'Successive Series'.
2 appears between this part of above successive series somewhere between 1 --- 3 and this series has a difference of one.
Therefore, we must take **Key Integer 'c'**
c = 1

2th Step – Multiplying c by constant 6.

Therefore, 1× 6 = 6
Now we get c = 6
Checking whether c is final answer by dividing 196 by 6 . We found it is not divisible.
Therefore, we will proceed to below 3rd step of checking **Rules of Finding 'c.**

3th Step – Checking the **Rules of Finding 'c'** to find precise perfect square root of c = 6

√X = 196 is an even integer divisible by 2. Therefore it follows the third rule of finding 'c.
Third rule states - If the √ X is any even integer which is divisible by 2, then the final answer will be either adding c by 4 i.e. 'c + 4 or adding s by 8 i.e. 'c + 8'

Checking both options by substituting value of c at all options.
6 + 4 = 10
6 + 4 = 14
Since 196 is divisible by 14
Therefore, our required answer is 6 + 4 = 14
So, √196 = 14

Second Example –

To find perfect square root of √3249

1st Step - Using the formula by substituting 3249 at X to get m.

$$\frac{x}{72} = m$$

3249 / 72 = 45.125 …..
By ignoring the decimals, we get m = 45
We will take m value as m_1 to input it in the below repeated subtraction series.
m_1 = 45

2nd Step – Finding the Key Integer 'c' Using repeated subtraction series. ……..from 1.2
By substituting the m_1 = 45 in the series we get,

Where m_1 = 45
45 - 1 = 1
44 - 2 = 42
42 - 3 = 39
39 - 4 = 35
35 - 5 = 30
30 - 6 = 24
24 - 7 = 17
17 - 8 = 9
9 - 9 = 0

As per the ‘condition of series’ we cannot further subtract 0 – 10 = -10 since it gives negative value.
Therefore we stop the calculation at 9 – 9 = 0
Now, the last negative series of sequence at the above series is -9 and by converting it to positive integer, -9 × -1 = 9
we get **Key Integer ‘c’ as 9**
Therefore, c = 9

Explanation of Using Second Method to Finding Key Integer ‘c’ …… from 1.3

3249 / 72 = 45.125 …..
By ignoring the decimals, we get m = 45

To find key integer from m = 45 , check the above ‘Successive Series’.
45 appears at above successive series that is at 45 ---- 55 and has a difference of nine.

Therefore, we must take **Key Integer ‘c'**
c = 9

2th Step – Multiplying c by constant 6.

Therefore, 9 × 6 = 54
Now we get c = 54
Checking whether c is final answer by dividing 3249 by 54 . We found it is not divisible.
Therefore, we will proceed to below 3rd step of checking **Rules of Finding ‘c.**

3th Step – Checking the **Rules of Finding ‘c’** to find precise perfect square root of c = 54

√X = √3249 is divisible integer divisible by 3. Therefore it follows the fourth rule of finding c.

Fourth Rule - If the √ X is any odd integer and is divisible by 3, then the final answer c will be adding s by 3 i.e. ‘c + 3'.

54 + 3 = 57

Since 3249 is divisible by 57

Therefore, √3249 = 57

Third Example –

To find perfect square root of √529

1st Step - Using the formula by substituting 529 at x to get m.

529/ 72 = 7.347…
Ignore the decimals, we get m = 7
We will take m value as m_1 and substitute it in the below repeated subtraction series.
m_1 = 7

2nd Step – Finding the **Key Integer 'c' Using repeated subtraction series.**
By substituting m_1 =7 in the series we get,

Where m_1 = 7
7 -1 = 6
6 -2 = 4
4 -3 = 1

As per the 'condition of series' we cannot further subtract 1 – 4 = -3 since it gives negative value.
Therefore we stop the calculation at 4 – 3 = 1
Now, the last negative series of sequence at the above series is -3 and by taking it as positive integer, we get **Key Integer 'c' as 3.**
9 × -1 = 9
Therefore, c = 3

2th Step – Multiplying c by constant 6.

Therefore, 3 × 6 = 18
Now we get c = 18
Checking whether c is final answer by dividing 529 by 18 . We found it is not divisible.

Therefore, we will proceed to below 3rd step of checking **Rules of Finding 'c.**

3th Step – Checking the **Rules of Finding 'c'** to find precise perfect square root of c = 18

√X = √529 is an integer not divisible by 2 or 3. Therefore it follows the first rule of finding c.
First Rule - If the √ X is any odd integer and is not divisible by 2 or 3, then the final answer will be either adding c by 5 i.e. c + 5 adding c by 7 i.e. c + 7

18 + 5 = 23

18 + 7 = 25

Since 529 is divisible by 23 therefore, our answer is at 18 + 5 = 23

Therefore, √3249 = 23

Fourth Example (Non Perfect Square Root) –

Note – in the case of finding non perfect square root of any integer √X we can get only the better approximation with decimals.
To find non perfect square root of √1817
1st Step - Using the formula by substituting 1817 to X to get m.

$$\frac{x}{72} = m$$

1817 / 72 = 25.26111
Ignore the decimals, we get m = 25
We will take m value as m_1 to input it in the below repeated subtraction series.
m_1 = 25

2nd Step – Finding the **Key Integer 'c' Using repeated subtraction series.**
By inputting the m_1 =25 in the series we get,

Where m_1 = 25
25 -1 = 24
24 -2 = 22
22 -3 = 20
20 -4 = 16
16 -5 = 11
11 -6 = 5

As per the 'condition of series' we cannot further subtract 5 - 7 = -2 since it will give negative value as -2
Therefore we stop the calculation at 11 – 6 = 5
Now, the last negative series of sequence at the above series is -6 and by taking it as positive integer, we get **Key Integer 'c' as 6**
Therefore, c = 6

2th Step – Multiplying c by constant 6.

Therefore, 6× 6 = 36
Now we get c = 36

Checking whether c is final answer by dividing 1817 by 36 . We found it is not divisible. Therefore, we will proceed to below 3rd step of checking **Rules of Finding 'c.**

3th Step – Checking the **Rules of Finding 'c'** to find precise perfect square root of c = 36

√X = 1817 is an odd integer not divisible by 3. Therefore it follows the first rule of finding c.
First Rule - If the √ X is any odd integer and is not divisible by 2 or 3, then the final answer will be either adding c by 5 i.e. c + 5 adding c by 7 i.e. c + 7

Checking both options by substituting value of c at all options.
36 + 5 = 41
36 + 7 = 43
The close approximation of 1817 is 43
Therefore, our required answer is at 36 + 7 = 43

So, √1817 = 43

Fifth Example –

To find perfect square root of √16

1st Step - Using the formula by inputting 16 to get m.

$$\frac{x}{72} = m$$

8 / 72 = 0.2222222..
Ignore the decimals, we get m = 0
We will take m value as m_1 to substitute it in the below repeated subtraction series as $m_1 = 0$

2nd Step – Finding the **Key Integer 'c' Using repeated subtraction series.**
By inputting the $m_1 = 0$ in the series we get,

Where $m_1 = 0$
0 -1 = -1

As per the 'condition of series' at the first step itself we got 0 -1 = -1 and since it gives negative value as -1 therefore, this step of series is negligible.
Therefore we stop the calculation at first step and the **Key Integer 'c'** will be 0
Therefore, c = 0

3th Step – Multiplying c by constant 6.

Therefore, 0 × 6 = 0
Now we get c = 0

Checking whether c is final answer by dividing 16 by 0 . We found no solution. Therefore, we will proceed to below 4^{rd} step of checking **Rules of Finding 'c.**

4^{th} Step – Checking the **Rules of Finding 'c'** to find precise perfect square root.

√X = 16 is an even integer divisible by 2. Therefore it follows the third rule of finding c.

Third Rule- If the √ X is any even integer which is divisible by 2, then the final answer will be either adding c by 4 i.e. 'c + 4 or adding c by 8 i.e. 'c + 8'

Checking both options by substituting value of c at all options.

0 + 4 = 4

0 + 7 = 7

Therefore, our required answer is at 0 + 4 = 4

So, √16 = 4

Finding The Cube Roots & Higher n^{th} Roots Like 4^{th}, 5^{th} 6^{th} Roots of Given √X And so on..

In the case of finding higher roots we need to take slightly different approach such as using rules explained below at 2.1, steps of 'Repeated Subtraction Series' or 'Successive Series' to get the final result.

2. 1) Rule of finding 'c' (to find final answer) -

First rule - If the √X is any odd integer and is not divisible by 2 or 3, then we must do subtraction c by 1, 5 or 7 i.e. 'c - 1', 'c - 5', 'c - 7'.

Second Rule - If the √X is any even integer and is divisible by 2 or 3, then we must do subtraction c by 3, 6 or 9 i.e. 'c - 3', 'c - 6', 'c - 9'.

Third Rule - If the √ X is any even integer which is divisible by 2, then we must do subtraction c by 2, 4, 8 or 10 i.e. 'c - 2', 'c - 4', 'c - 8', 'c - 10'.

Fourth Rule - If the √ X is any odd integer and is divisible by 3, then we must take do as subtraction c by 3, 6 or 9 i.e. ‘c - 3’, ‘c - 6’, ‘c - 9’.

Important Note- In case of finding higher roots like cube roots, 4^{th} , 5^{th} , 6^{th} roots….. Rules of addition or subtraction of c might get increase to c – 12, c – 14 etc…but I am not sure though. One can calculate higher roots and check whether such problem appears and where we require such solutions like c – 12, c – 15, c – 16 …..c - n or we don’t require at any of the above at their associated stated rules.

2.3) Steps of Repeated Subtraction Series For Each Higher Roots -

Cube root – up till 2 steps.
4^{th} root – up till 3 steps.
5^{th} root – up till 4 steps
6^{th} root – up till 5 steps
7^{th} roots up till 6 steps and so on …..
As higher roots increases, the steps of calculating repeated subtraction series increase by one as shown at the above.

Below Are The Example of Solutions –

To find the result always follow the below three steps –

1^{st} Step - Using the formula √X / 72 = m, to get m.

2^{nd} Step – Finding the **Key Integer ‘c’** Using repeated subtraction series or progressive series by following first, second, third …. Step of calculation depending on which nth root we are finding.
3^{rd} Step – Following and check the Rule to find c as the final result.

First Example-

Let suppose we want to find cube root of

$$\sqrt[3]{1331}$$

1^{st} Step - Using the below formula by substituting 1331 at X to get m.

$$\frac{x}{72} = m$$

1331/ 72 = 18.486111

Ignore the decimals, we get m = 18
We will take m value as m_1 to input it in the below repeated subtraction series.
m_1 = 18

2^{nd} Step – Finding the **Key Integer 'c' Using repeated subtraction series.**
By inputting the m_1 = 18 in the series we get,
In this case of finding cube root we must to calculate **repeated subtraction series.**
up to two steps.

First Step of Calculation (Repeated Subtraction Series). -

Where m_1 = 18
18 -1 = 17
17 -2 = 15
15 -3 = 12
12 - 4 = 8
8 - 5 = 3

As per the 'condition of series' we cannot further 3 – 6 = -3 since it gives negative value.
Therefore we stop the calculation at 8 – 5 = 3
Now, the last negative series of sequence at the above series is - 5 and by taking it as positive integer, -5× -5 = 5
we get **Key Integer 'c' as 5**

Second Step of Calculation (Repeated Subtraction Series). -

To calculate second step, carry forward the **Key Integer c = 5**
from the above first step series. Again here m_1 = 5
Calculating again the below **repeated subtraction series.**

5 -1 = 4

4 -2 = 2

As per the ‘condition of series’ we cannot further 2 -3 = -1
since it gives negative value.
Therefore we stop the calculation at 4 -2 = 2

Now, the last final negative series of sequence at the above series is -2 and by taking it as positive integer, we get **Key Integer ‘c’ as 2, c = 2**

Explanation of Using Second Method to Finding Key Integer ‘c’ …… from 1.3

(Note that just for explanation purpose, i have shown both example of finding Key Integer c. One can use any of one, out of the two method for finding Key Integer c).

As per the condition stated at ‘ **1.4 - Steps For Each Higher Roots’**
we must calculate up to two steps for cube root finding.

1st Step of Calculation -

1331/ 72 = 18.486111

Ignore the decimals, we get m = 18
To find key integer from m = 18 , check the above ‘Progressive Series’.
18 appears at above pattern series that is 15 --18-- 21 and has a difference of five.

Therefore, we must take **Key Integer ‘c'**
c = 5

2nd Step of Calculation -
Carried forward c = 5 from ‘1st step of calculation’.
To find key integer from 5, check the above ‘Successive Series’.
5 appears at above successive series somewhere between 3 ---- 6 and has a difference of two.

Therefore, we must take **Key Integer ‘c'**
c = 2

3th Step – Multiplying c by constant 6.

Therefore, $2 \times 6 = 12$
Now we get c = 12
Checking whether c is final answer by dividing 1331 by 12 . We found it is not divisible.
Therefore, we will proceed to below 4rd step of checking **Rules of Finding 'c.**

4.th Step – Checking the **Rules of Finding 'c'** to find precise perfect square root.
√X = 1331 is an odd integer not divisible by 2 or 3. Therefore it follows the first rule of finding c.

First rule - If the √X is any odd integer and is not divisible by 2 or 3, then we must do subtraction c by 1, 5 or 7 i.e. 'c - 1', 'c - 5', 'c - 7'.

Checking by substituting value of c at all options.
12 – 1 = 11 .
12 - 5 = 7
12 - 7 = 5

Therefore, our required answer is at 12 - 1 = 11
So,

$\sqrt[3]{1331}$
= 11

Second Example-

To find cube root
$\sqrt[3]{5832}$

1st Step - Using the below formula by substituting 5832 at X to get m.

$$\frac{x}{72} = m$$

5832/ 72 = 81

We got m = 81

We will take m value and substitute it to m_1 in the below repeated subtraction series.

m_1 = 81

2nd Step – Finding the **Key Integer 'c' Using repeated subtraction series.**

m_1 = 81

In this case of finding cube root we must calculate the same **repeated subtraction series.**

up to two steps as shown below.

First Step of Calculation (Repeated Subtraction Series). -

Where m_1 = 81

81 - 1 = 80

80 - 2 = 78

78 - 3 = 75

75 - 4 = 71

71 - 5 = 66

66 - 6 = 60

60 – 7 = 53

53 – 8 = 45

45 – 9 = 36

36 – 10 = 26

26 – 11 = 15

15 – 12 = 3

As per the 'condition of series' we cannot further calculate 3 – 13 = -10 since it gives negative value.

Therefore we stop the calculation at 15 – 12 = 3

Now, the last negative series of sequence at the above series is - 12 and by taking it as positive integer, -12× -1 = 12

we got **Key Integer 'c' as 12**

Second Step of Calculation (Repeated Subtraction Series). -

To calculate second step, carry forward the **Key Integer c = 12**

from the above first step series. Again here m_1 = 12

Calculating again the below **repeated subtraction series.**

12 -1 = 11

11- 2 = 9

9 - 3 = 6
6 – 4 = 2

As per the 'condition of series' we cannot further calculate 2 -3 = -1
since it gives negative value.
Therefore we stop the calculation at 6 - 4 = 2

Now, the last final negative series of sequence at the above series is - 4 and by taking it as positive integer, we get **Key Integer 'c' as 4, c = 4**

Explanation of Using Second Method to Finding Key Integer 'c' …… from 1.3

As per the condition stated at ' **1.4 - Steps For Each Higher Roots'**
we must calculate up to two steps for cube root finding.
1st step of calculation -

5832 / 72 = 81 ....Ignore the decimals, we got m = 81

To find key integer from 81 , check the above 'Successive Series'.
81 appears at above successive series somewhere between i.e. 78 --81-- 91 and has a difference of twelve.

Therefore, we must take **Key Integer 'c'**
c = 12

2nd Step of Calculation -
Carried forward c = 12 from '1st step of calculation'.
To find key integer from 12 m, check the above 'Successive Series'.
12 appears at above successive series somewhere between 10 --12---- 15 and has a difference of four.

Therefore, we must take **Key Integer 'c'**
c = 4
Next,

3th Step – Multiplying c by constant 6.

Therefore, 4 × 6 = 24
Now we get c = 24

Checking whether c is final answer by dividing 5832 by 24 . We found it is divisible but it is not our final square roots answer.

Therefore, we will proceed to below 4rd step of checking **Rules of Finding 'c.**

4th Step – Checking the **Rules of Finding 'c'** to find precise perfect square root of c = 24

√X = 5832 is an even integer divisible both by 2 and 3 . Therefore it follows the second rule of finding c.

Second Rule - If the √X is any even integer and is divisible by 2 or 3, then we must do subtraction c by 3, 6 or 9 i.e. 'c - 3', 'c - 6', 'c - 9'.

Checking by substituting value of c at all options.

24 – 3 = 21

24 - 6 = **18**

24 – 9 = 15

Therefore, our required answer is at 24 - 6 = 18

So,

$$\sqrt[3]{5832}$$

= 18

Third Example-

To find fourth root of

$$\sqrt[4]{10000}$$

1st Step - Using the below formula by inputting 10000 to get m.

$$\frac{x}{72} = m$$

10000/ 72 = 138.88889

Ignoring decimals we got,

m =138
We will take m value as m_1 to substitute it in the below repeated subtraction series.
m_1 = 138

<u>2nd Step –</u> Finding the **Key Integer 'c' Using repeated subtraction series.**

As per the condition stated at ' Steps of Repeated Subtraction Series'. In this case of finding fourth root we must calculate the same repeated subtraction series up to three steps as shown below.
inputting the m_1 = 138 in the series.
<u>First Step of Calculation (Repeated Subtraction Series). -</u>

Where m_1 = 138
138 - 1 = 137
137 - 2 = 135
135 - 3 = 132
132 - 4 = 128
128 - 5 = 123
123 - 6 = 117
117 – 7 = 110
110 – 8 = 102
102 – 9 = 93
93 – 10 = 83
83 – 11 = 72
72 – 12 = 60
60 – 13 = 47
47 – 14 = 33
33 – 15 = 18
18 – 16 = 2

As per the 'rule of series' we cannot further calculate 2 – 17 = -15 since it gives negative value.
Therefore we stop the calculation at 18 – 16 = 2

Now, the last negative series of sequence at the above series is - 16 and by taking it as positive integer, $-16 \times -1 = 16$
we get **Key Integer 'c' as 16**

Second Step of Calculation (Repeated Subtraction Series). -

To calculate second step, carry forward the **Key Integer c = 16**
from the above first step series. Again here $m_1 = 16$
Calculating again the below **repeated subtraction series.**

16 -1 = 15
15 - 2 = 13
13 - 3 = 10
10 – 4 = 6
6 – 5 = 1

As per the 'condition of series' we cannot further subtract 1 - 6 = -5
since it gives negative value.
Therefore we stop the calculation at 6 - 5 = 1
Now, the last negative series of sequence at the above series is - 5 and by taking it as positive integer, $-5 \times -1 = 5$
we get **Key Integer 'c' = 5**

Third Step of Calculation (Repeated Subtraction Series). -

To calculate third step, carry forward the **Key Integer c = 5**
from the above first step series. Here $m_1 = 5$
Calculating again the below **repeated subtraction series.**

5 -1 = 4
4 - 2 = 2

As per the 'condition of series' we cannot further subtract 2 - 3 = -1 since it gives negative value.
Therefore we stop the calculation at 4 - 2 = 2
Now, the last final negative series of sequence at the above series is - 2 and by taking it as positive integer, $-2 \times -1 = 2$
We get **Key Integer 'c' as 2, c = 2**

<u>Explanation by Using Second Method to Find Key Integer ‘c’</u> **…… from 1.3**
As per the condition stated at ‘ **1.4 - Steps For Each Higher Roots’**
we must calculate up to three steps for fourth root finding.

1^{st} step of calculation -

Finding m using formula,

10000 / 72 = 81.88...

Ignore the decimals, we got m = 138

To find key integer from 138, check the above ‘Pattern Series’.

138 appears at above pattern series somewhere between i.e. 136 -138----- 153 and has a difference of sixteen.
Therefore, we must take **Key Integer ‘c'**
c = 16
2^{nd} Step of Calculation -
Carried forward c = 16 from ‘1^{st} step of calculation’.
To find key integer from 16, check the above ‘Pattern Series’.
12 appears at above pattern series somewhere between 15 --16---- 21 and has a difference of five.

Therefore, we must take **Key Integer ‘c'**
c = 5
Next,

3^{rd} Step of Calculation -
Carried forward c = 5 from ‘2^{nd} step of calculation’.
To find key integer from 5, check the above ‘Pattern Series’.
12 appears at above pattern series somewhere between 3 --5---- 6 and has a difference of two.

Therefore, we must take **Key Integer ‘c'**
c = 2
Next,
<u>3^{th} Step –</u> Multiplying c by constant 6.

Therefore, 2 × 6 = 12

Now we get c = 12

4th Step – Checking the **Rules of Finding 'c'** to find precise perfect square root of c = 12

√X = 10000 is an even integer divisible by 2. Therefore it follows the third rule of finding c.

Third Rule - If the √ X is any even integer which is divisible by 2, then we must do subtraction c by 2, 4, 8 or 10 i.e. 'c - 2', 'c - 4', 'c - 8', 'c - 10'.

Checking by substituting value of c at all options.
12 – 2 = 10
12 - 4 = 8
12 – 8 = 4
12 – 10 = 2

Therefore, our required answer is at 12 – 2 = 10

So,

$\sqrt[4]{10000}$
= 10

Fifth Example-

To find cube root of

$\sqrt[3]{4913}$

1st Step - Using the below formula by inputting 4913 to get m.

$$\frac{x}{72} = m$$

4913 / 72 = 67.236111

Ignoring decimals we get whole number as

$m = 67$
We will take m value as m_1 to input it in the below repeated subtraction series.
$m_1 = 67$

<u>2nd Step –</u> Finding the **Key Integer 'c' Using repeated subtraction series.**

As per condition stated at 2.3 - 'Steps of Repeated Subtraction Series', in this case of finding third root we must calculate the same **repeated subtraction series.**
up to two steps as shown below.
Substituting the $m_1 = 67$ in the series.
<u>First Step of Calculation (Repeated Subtraction Series). -</u>

Where $m_1 = 68$
68 - 1 = 67
67 - 2 = 65
65 - 3 = 62
62 - 4 = 58
58 - 5 = 53
53 - 6 = 47
47 – 7 = 40
40 – 8 = 32
32 – 9 = 23
23 – 10 = 13
13 – 11 = 2

As per the 'condition of series explained at 1.1' we cannot further calculate 2 – 12 = -10 since it gives negative value.
Therefore we stop the calculation at 13 – 11 = 2
Now, the last negative series of sequence at the above series is - 11 and by taking it as positive integer, -11 × -1 = 11
we get **Key Integer 'c' as 11**

<u>Second Step of Calculation (Repeated Subtraction Series). -</u>

To calculate second step, carry forward the **Key Integer c = 11**
from the above first step series. Again here $m_1 = 11$

Calculating again the below **repeated subtraction series.**

11 -1 = 10
10 - 2 = 8
8 - 3 = 5
5 – 4 = 3

As per the ‘condition of series explained at 1.1’ we cannot further subtract 3 – 5 = - 2 since it gives negative value.
Therefore we stop the calculation at 5 - 4 = 3
Now, the last negative series of sequence at the above series is – 4 and by taking it as positive integer, -4 × -1 = 4
we got **Key Integer ‘c’ = 4**

3th Step – Multiplying c by constant 6.

Therefore, 4 × 6 = 24
Now we get c = 24

4th Step – Checking the **Rules of Finding ‘c’** to find precise perfect square root of c = 24

√X = 4913 is an odd integer not divisible by 2 or 3 Therefore it follows the first rule of finding c.
First rule - If the √X is any odd integer and is not divisible by 2 or 3, then we must do subtraction c by 1, 5 or 7 i.e. ‘c - 1’, ‘c - 5’, ‘c - 7’.

Checking by substituting value of c at all options.
24 – 1 = 28
24 - 5 = 19
24 – 7 = 17

Therefore, our required answer is at 24 – 7 = 17
So,

$$\sqrt[3]{4913}$$

= 17

Fourth Example -

To find fourth root of

$\sqrt[4]{14641}$

1st Step - Using the below formula by substituting 14641 at X to get m.

$$\frac{x}{72} = m$$

14641 / 72 = 203.34722

Ignoring decimals we got, m = 203

We will take m value as m_1 to substitute it in the below repeated subtraction series.
m_1 = 203

2nd Step – Finding the **Key Integer 'c' Using repeated subtraction series.**

As per the condition stated at ' 2.3 - Steps of Repeated Subtraction Series', so in this case of finding fourth root we must calculate the same **repeated subtraction series** up to three steps as shown below.
inputting the m_1 = 203 in the series.
First Step of Calculation (Repeated Subtraction Series). -

Where m_1 = 203
203 - 1 = 202
202 - 2 = 200
200 - 3 = 197
187 - 4 = 193
193 - 5 = 188
187 - 6 = 182

182 – 7 = 175
175 – 8 = 167
167 – 9 = 158
158 – 10 = 148
148 – 11 = 137
137 – 12 = 125
125 – 13 = 112
112 – 14 = 98
97 – 15 = 83
83 – 16 = 67
67 – 17 = 50
50 – 18 = 32
32 – 19 = 13

As per the ‘condition of series’ we cannot further calculate 13 – 20 = -7 since it gives negative value.
Therefore we stop the calculation at 32 – 19 = 13
Now, the last negative series of sequence at the above series is - 19 and by taking it as positive integer, -19 × -1 = 19
we get **Key Integer ‘c’ as 19**

<u>Second Step of Calculation (Repeated Subtraction Series). -</u>

To calculate second step, carry forward the **Key Integer c = 19**
from the above first step series. Again here m_1 = 19
Calculating again the below **repeated subtraction series.**

19 -1 = 18
18 - 2 = 16
16 - 3 = 13
13 – 4 = 9
9 – 5 = 4

As per the ‘condition of series’ we cannot further subtract 4 - 6 = - 2
since it gives negative value.
Therefore we stop the calculation at 8 - 5 = 4
Now, the last negative series of sequence at the above series is - 5 and by taking it as positive integer, -5 × -1 = 5
we get **Key Integer ‘c’ = 5**

Third Step of Calculation (Repeated Subtraction Series). -

To calculate third step, carry forward the **Key Integer c = 5** from the above first step series. Here $m_1 = 5$
Calculating again the below **repeated subtraction series.**

5 -1 = 4
4 - 2 = 2

As per the 'condition of series' we cannot further subtract 2 - 3 = -1 since it gives negative value.
Therefore we stop the calculation at 4 - 2 = 2
Now, the last final negative series of sequence at the above series is - 2 and by taking it as positive integer, -2 × -1 = 2
We get **Key Integer 'c' as 2, c = 2**

3^{th} Step – Multiplying c by constant 6.

Therefore, 2 × 6 = 12
Now we get c = 12

4^{th} Step – Checking the **Rules of Finding 'c'** to find precise perfect square root of c = 12

√X = 14641 is an odd integer not divisible by 2 or 3 Therefore it follows the first rule of finding c.
First rule - If the √X is any odd integer and is not divisible by 2 or 3, then we must do subtraction c by 1, 5 or 7 i.e. 'c - 1', 'c - 5', 'c - 7'.

Checking by substituting value of c at all options.
12 – 1 = 11
12 - 5 = 7
12 – 7 = 5

Therefore, our required answer is at 12 – 1 = 11
So,

$\sqrt[4]{14641}$

= 11

Finding p and q the multiples of 'n', using the explain method and formulas -

(Note – this method may also work for p × q = n up till p and q has a gap of eight. But I am not sure whether the solution will be correct or not, but it works perfectly for p × q = n, where p and q has a gap of 2, as explained below).

The formula method explained in this paper also works for any given p × q = n
Where, p and q integers has a gap of two.

E.g, Twin Prime Numbers are the set of two integers that have exactly one composite number between them, therefore having gap of two between two primes.

p	q (twin primes has gap of two).	p	q (two integers having gap of two).
5	7	10	12
11	13	24	25

Using the explained formulas we can also find n of those multiplied p × q = n
E.g, finding solution of 65 where its multiplied p and q has a gap of 8.
In this paper we will consider only about twin primes.

First Example –

To find p and q of given n = 143

1st Step - Using the formula by inputting 143 to get m.

$$\frac{x}{72} = m$$

143 / 72 = 1.9861111

Ignore the decimals, we get m = 1
We will take m value as m_1 to input it in the below repeated subtraction series.
$m_1 = 1$

<u>2^{nd} Step –</u> Finding the **Key Integer 'c' Using repeated subtraction series.**
By substituting the $m_1 = 0$ in the series we get,

Where $m_1 = 1$
1 -1 = 0

As per the 'condition of series' we cannot further 0 – 2 = -2 since it gives negative value.
Therefore we stop the calculation at 1 – 1 = 0
Now, the last negative series of sequence at the above series is – 1 and by taking it as positive integer, we get **Key Integer 'c' as 1**

<u>3^{th} Step –</u> Multiplying c by constant 6.

Therefore, $1 \times 6 = 6$
Now we get c = 6

<u>4^{th} Step –</u> Checking the **Rules of Finding 'c'** to find precise perfect square root of c = 6

$\sqrt{X} = 143$ is an odd integer not divisible by 2 or 3 therefore it follows the first rule of finding c.

Note – In the case of finding solutions to such problems always take stated rules at – '1.2) Rules of finding 'c'' for square roots.

So in this case we will take below rule -

First rule - If the $\sqrt{X}$ is any odd integer and is not divisible by 2 or 3, then we must add c by 1, 5 or 7 i.e. 'c - 1', 'c - 5', 'c - 7'.

Checking by substituting value of c at all options.
6 + 1 = 7

6 + 5 = 11
6 + 7 = 13
Therefore, our required answer is at 6 + 5 = 11
6 + 7 = 13

So,
143 is a multiple of 11× 13

Second Example -

To find p and q of given n = 323

<u>1st Step -</u> Using the formula by inputting 323 to get m.

323 / 72 = 4.486…
Ignore the decimals, we get m = 4
We will take m value as m_1 to substitute it in the below repeated subtraction series.
m_1 = 1

<u>2nd Step –</u> Finding the **Key Integer ‘c’ Using repeated subtraction series.**
By substituting the m_1 = 4 in the series we get,

Where m_1 = 4
4 -1 = 3
3 - 2 = 1

As per the ‘condition of series’ we cannot further 1 – 3 = - 2 since it gives negative value.
Therefore we stop the calculation at 3 – 2 = 1
Now, the last negative series of sequence at the above series is – 2 and by taking it as positive integer, we get **Key Integer ‘c’ c= 2**

<u>3th Step –</u> Multiplying c by constant 6.

Therefore, $2 \times 6 = 12$
Now we get c = 12

4th Step – Checking the **Rules of Finding 'c'** to find precise perfect square root of c = 12

√X = 323 is an odd integer not divisible by 2 or 3 therefore it follows the first rule of finding c.

Note – in the case of finding solutions to such problems always take - 1.2 rules of finding 'c' for square roots

So in this case we will take below rule -

First rule - If the √X is any odd integer and is not divisible by 2 or 3, then we must do subtraction c by 1, 5 or 7 i.e. 'c - 1', 'c - 5', 'c - 7'.

Checking by substituting value of c at all options.
12 + 1 = 13
12 + 5 = 17
12 + 7 = 19
Therefore, our required answer is at 12 + 5 = 17
12 + 7 = 19

So,
323 is a multiple of 17 and 19

Reference –

https://en.m.wikipedia.org/wiki/Methods_of_computing_square_roots