How To use Starobinsky inflationary potential plus argument from Alder, Bazin, and Schiffer as radial acceleration to obtain first order approximation as to where/when Cosmological constant may form

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Abstract
Using the Klauder enhanced quantization as a way to specify the cosmological constant as a baseline for the mass of a graviton, we eventually come up and then we will go to the . Starobinsky potential as a replacement for the term N used in Eq. (3) and Eq,. (4). From there we will read in a way to describe conditions allowing for where the cosmological constant may be set. The idea also is to describe a regime of space-time where the initial perturbation/ start to inflation actually occurred, as is alluded to in the final part of the document.

1. Input from General Relativity First integral. From [1], [2], [3],[4]

We use the Padmanabhan 1st integral [1], [2] of the form , with the third entry of Eq. (1) having a Ricci scalar defined via [9] and usually the curvature $\aleph$ set as extremely small, with the general relativity[3], [4]

$$\begin{aligned} &S_1 = \frac{1}{2\kappa}\int\sqrt{-g}\cdot d^4x\cdot\left(\Re - 2\Lambda\right) \\ &\& -g = -\det g_{uv} \\ &\& \Re = 6\cdot\left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{\aleph}{a^2}\right) \end{aligned} \tag{1}$$

Also, the variation of $\delta g_{tt} \approx a_{\min}^2\phi$ as given by [5,6] will have an inflaton, ϕ given by[7] [6][9] Leading to [7,8] to the inflaton which is combined into other procedures for a solution to the cosmological constant problem. Here, $a_{\min}$ is a minimum value of the scale factor [8,9]

1a. Next for the idea from Klauder

We are going to go to page 78 by Klauder [4] of what he calls on page 78 a restricted Quantum action principle which he writes as: S_2 where we write a 1-1 equivalence as in [1] , which is also seen in [3]

$$S_2 = \int_0^T dt\cdot\left[p(t)\dot{q}(t) - H_N\left(p(t),q(t)\right)\right] \approx S_1 = \frac{1}{2\kappa}\int\sqrt{-g}\cdot d^4x\cdot\left(\Re - 2\Lambda\right) \tag{2}$$

Our assumption is that Λ is a constant, hence we assume then the following approximation , from[3]

$$
\begin{aligned}
&\frac{p_0{}^2}{2} = \frac{p_0{}^2(N)}{2} + N; \quad for \quad 0 < N \le \infty, and \quad q = q_0 \pm p_0 t \\
&V_N(x) = 0; \quad for \quad 0 < x < 1 \\
&V_N(x) = N; \quad otherwise \\
&H_N\left(p(t), q(t)\right) = \frac{p_0^2}{2} + \frac{\left(\hbar \cdot \pi\right)^2}{2} + N; \quad for \quad 0 < N \le \infty
\end{aligned}
\tag{3}
$$

Our innovation is to then equate $q = q_0 \pm p_0 t \sim \phi$ and to assume small time step values. Then

$$
\Lambda \approx \frac{-\left[\dfrac{V_0}{3\gamma - 1} + 2N + \dfrac{\gamma \cdot \left(3\gamma - 1\right)}{8\pi G \cdot \tilde{t}^{\,2}}\right]}{\dfrac{1}{\kappa}\int \sqrt{-g} \cdot d^3 x} + \left.\left(6 \cdot \left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right)\right)\right|_{t = \tilde{t}} \tag{4}
$$

These are terms within the bubble of space-time given in [1] using the same inflaton potential. The scale factor is presumed here to obey the value of the scale factor in [9]

2. Modification of Eq. (4) if the term of N in Eq. (3) if N replaced by the Starobinsky potential

In order to do this , we will replace the term, N, as appearing in Eq. (3) and Eq. (4) by the Starobinsky early inflation potential

From [10] pp 152 to153 where we assume q ~ 1

$$
N \approx V\left(\phi\right) = V_0 \cdot \left(1 - \exp\left[-q \cdot \phi / m_P\right]\right)^2 \tag{5}
$$

Also, making use of [7] and

$$
\left(\frac{\ddot{a}}{a}\right) = \dot{H} + H^2 \tag{6}
$$

And

$$
\begin{aligned}
\left(\frac{\ddot{a}}{a}\right) = \dot{H} + H^2 = \frac{-t \cdot \gamma \cdot \left(3\gamma - 1\right)}{m_P G} \cdot \sqrt{\frac{1}{8\pi}} \\
+ \frac{8\pi G}{3} \cdot \left[V_0 \cdot \left(1 - \exp\left[-q \cdot \phi / m_P\right]\right)^2\right]
\end{aligned}
\tag{7}
$$

Where we have used

$$
\left(\frac{\dot{a}}{a}\right)^2 = H^2 \equiv \frac{8\pi G}{3} \cdot \left[V_0 \cdot \left(1 - \exp\left[-q \cdot \phi / m_P\right]\right)^2\right] \tag{8}
$$

And so we obtain if we have a scale factor behaving as

$$
\begin{aligned}
\Lambda &\approx \frac{-\left[\frac{V_0}{3\gamma-1}+2N+\frac{\gamma\cdot(3\gamma-1)}{8\pi G\cdot\tilde{t}^2}\right]}{\frac{1}{\kappa}\int\sqrt{-g}\cdot d^3x}+\left(6\cdot\left(\frac{\ddot{a}}{a}+\left(\frac{\dot{a}}{a}\right)^2\right)\right)\Bigg|_{t=\tilde{t}} \\
&\approx \frac{-\left[\frac{V_0}{3\gamma-1}+2V_0\cdot\left(1-\exp\left[-q\cdot\phi/m_P\right]\right)^2+\frac{\gamma\cdot(3\gamma-1)}{8\pi G\cdot\tilde{t}^2}\right]}{\frac{1}{\kappa}\int\sqrt{-g}\cdot d^3x} \\
&+6\cdot\frac{-t\cdot\gamma\cdot(3\gamma-1)}{m_P G}\cdot\sqrt{\frac{1}{8\pi}} \\
&+\frac{48\pi G}{3}\cdot\left[V_0\cdot\left(1-\exp\left[-q\cdot\phi/m_P\right]\right)^2\right]
\end{aligned}
\tag{9}
$$

The time interval we are specifying in Eq,. (9) is of about Planck time

We can write an expression for V_0 from [10], page 153 taking the form of

$$
V_0^{1/4}=\frac{.022m_P}{\sqrt{qN_{e-folds}}} \tag{10}
$$

This value will be important when we make a linkage to Gravitons in the end of this analysis

To complete our analysis, we will go to [11], and an interesting piece of analysis, about the cosmological constant. Before doing that we will say a bit about evaluation of a term in one of the denominators of Eq. (9)

3. Addressing the term of $\frac{1}{\kappa}\int\sqrt{-g}\cdot d^3x$ in Eq. (9)

Here – g as in $\frac{1}{\kappa}\int\sqrt{-g}\cdot d^3x$

We will be assuming, also that as from [7] that

$$
\begin{aligned}
&a(t)=a_{initial}t^{\nu} \\
&\Rightarrow \phi=\ln\left(\sqrt{\frac{8\pi G V_0}{\nu\cdot(3\nu-1)}}\cdot t\right)^{\sqrt{\frac{\nu}{16\pi G}}} \\
&\Rightarrow \dot{\phi}=\sqrt{\frac{\nu}{4\pi G}}\cdot t^{-1} \\
&\Rightarrow \frac{H^2}{\dot{\phi}}\approx\sqrt{\frac{4\pi G}{\nu}}\cdot t\ \ \cdot T^4\cdot\frac{(1.66)^2\cdot g_*}{m_P^2}\approx 10^{-5}
\end{aligned}
\tag{11}
$$

The numbers here are extremely small, hence we will be considering a restricted version of g in

$\frac{1}{\kappa}\int\sqrt{-g}\cdot d^3x$ with regards to [11]

From [11], page 399, in what is called considering the dynamics of a non vanishing cosmological constant, namely if φ is the classical mechanics potential for how a particle travels along a geodestics of a metric so considered

$$\begin{aligned} g &\to -g_{00} = -1 - 2\varphi / c^2 \\ \varphi &= -\frac{\kappa M}{r} - \frac{1}{6}\cdot\Lambda\cdot c^2 r^2 \end{aligned} \tag{12}$$

The claim from page 399 of [11] is that then there would be a net acceleration of a test particle we could write as

$$a_{radial} = \frac{\Lambda}{3}\cdot c^2 r \tag{13}$$

Our supposition that such a small initial acceleration would be pertinent to the production / acceleration of Gravitons from the initial part / place of expansion of the universe , i.e. what would be the starting gun for inflation

If this were the initial nudge of space-time this would be commensurate with the following interpretation of $\frac{1}{\kappa}\int\sqrt{-g}\cdot d^3x$ with regards to [11] will to a certain degree presuppose not paying attention to the following initial line metric

$$\begin{aligned} &dS_{Schwartchild}\Big|_{Spatial-part-line-element} \simeq \\ &dl^2\Big|_{M\to 0} = \left(1-\Lambda r^2/3\right)^{-1} dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\vartheta^2 \end{aligned} \tag{14}$$

I am pre supposing that in the initial stages of space time that the term g appearing in $\frac{1}{\kappa}\int\sqrt{-g}\cdot d^3x$ with regards to [11] would be treatable as being a constant, so as to place the initial configuration of the cosmological "constant ' parameter as a straight out inter-play between a positive set of terms, and a negative set of terms, so as to nearly cancel, which is such that if the initial acceleration as to gravitons would be, at its initial inception very small

However, once perturbed, i.e a barrier of sorts breached , inflation would commence as a violent universe creating phenomenon.

For the record, the term r in Eq. (13) would be assumed to be very small, of the magnitude of Plancks length, whereas the driving force to expansion would be doable as linked to Eq. (10)

Note plank mass would be presumed to be about 2.1764 times 10^-5 grams whereas a common mass of the graviton , if it has mass would be about 10^-65 grams, whereas, if we have by [12][13] compared with Eq. (10) the mass- energy equivalent of 10^62 gravitons, which may be in the situation specified as to the cosmological constant

4. Conclusion: Do we have a 1-1 correspondence via this "cosmological constant" argument in magnitude with the mass of a massive graviton ?

Klauder's program[4] is to embed via Eq.(6) as a quantum mechanical well for a Pre Planckian-system for inflaton physics as given by Eq. (3). as given in Klauder's treatment of the action integral as of page 87 of [4] where Klauder talks of the weak correspondence principle, where an enhanced classical Hamiltonian, is given 1-1 correspondence with quantum effects, in a non-vanishing fashion. If so, by Novello [3,12]

$$m_g = \frac{\hbar \cdot \sqrt{\Lambda}}{c} \tag{15}$$

This formula needs to be confirmed via likely CMB radiation as could be ascertained by interaction of the CMB spectra , and gravitational waves. However, if Eq. (15) were true, it may mean that the initial regime of cosmological constant creation may be able to give a description as to what exactly initiated the transition to inflation to begin with. And that is an open question.

The author has presupposed in all of this using the following Uncertainty principle [13] [14] [15] [16]

$$\begin{aligned}
&\Delta t \geq \frac{\hbar}{\Delta E} + \gamma t_P^2 \frac{\Delta E}{\hbar} \Rightarrow (\Delta E)^2 - \frac{\hbar \Delta t}{\gamma t_P^2} (\Delta E)^1 + \frac{\hbar^2}{\gamma t_P^2} = 0 \\
&\Rightarrow \Delta E = \frac{\hbar \Delta t}{2\gamma t_P^2} \cdot \left(1 + \sqrt{1 - \frac{4\hbar^2}{\gamma t_P^2 \cdot \left(\frac{\hbar \Delta t}{2\gamma t_P^2} \right)^2}} \right) = \frac{\hbar \Delta t}{2\gamma t_P^2} \cdot \left(1 \pm \sqrt{1 - \frac{16\hbar^2 \gamma t_P^2}{(\hbar \Delta t)^2}} \right)
\end{aligned} \tag{16}$$

$$\begin{aligned}
&\Delta E \approx \frac{\hbar \Delta t}{2\gamma t_P^2} \cdot \left(1 \pm \left(1 - \frac{8\hbar^2 \gamma t_P^2}{(\hbar \Delta t)^2} \right) \right) \\
&\Rightarrow \Delta E \approx either\, \frac{\hbar \Delta t}{2\gamma t_P^2} \cdot \frac{8\hbar^2 \gamma t_P^2}{(\hbar \Delta t)^2}, or\, \frac{\hbar \Delta t}{2\gamma t_P^2} \cdot \left(2 - \frac{8\hbar^2 \gamma t_P^2}{(\hbar \Delta t)^2} \right)
\end{aligned} \tag{17}$$

$$\Delta E \approx \frac{\hbar \Delta t}{2\gamma t_P^2} \cdot \frac{8\hbar^2 \gamma t_P^2}{(\hbar \Delta t)^2} \equiv \frac{4\hbar}{\Delta t} \tag{18}$$

Our supposition is that the initial radial distance regime of creation of space time allowing for a cosmological constant in the beginning of the universe is commensurate with a time interval Δt . Again, if say 10^62 gravitons may be produced as of the time when a cosmological constant is produced, and if some of the gravitons are created by the decay of mini black holes, this says a lot about initial conditions which may be verifiable later as to suitable data sets in gravitational wave astronomy

In particular, Eq. (10) is directly from Starobinsky as reported in [10] and it would be of stunning import if the following is confirmed

$$V_0^{1/4} \approx 5.4\times10^{16} GeV \Big/ \sqrt{qN_{efolds}} \qquad (10)$$

Also see this [17], [18], [19], [20] [21][22]

The term q is of about 1 in magnitude, where as N_{efolds} is the number of evolds in inflation. Tying this in, say if a graviton were of 10^-65 grams would be huge importance as far as GW astronomy is concerned

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