# Demystifying the Mystery of Quantum Superposition 

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A magician, seems to be analogous to quantum mechanics, throws the spectators into a bewildering surprise by exhibiting a magic trick similar to the quantum superposition. The trick appears to be strange, weird and counter-intuitive like the quantum superposition, as long as the underlying secret behind its working is unknown. In the present article, the mystery of quantum superposition is demystified at a single-quantum level. Also, the counterfactual reality and the causality in Young's double-slit and Wheeler's delayed-choice experiments are pointed out, respectively.

## I. INTRODUCTION

As long as the applications of quantum mechanics depend on the statistical averages, the mainstream Copenhagen interpretation [1-5] works excellently. However, the statistical averages must be derived from the behavior of individual quanta. Therefore, describing the quantum mechanics at a single-quantum level is important for the further progress in fundamental physics. In this regard, the "wave-particle non-dualistic interpretation of quantum mechanics at a single-quantum level" is proposed [6-21], which yields the Copenhagen interpretation statistically - Born's rule is derived as a limiting case of the relative frequency of detection in the former case, whereas, the same is accepted as one of the postulates in the later case.

This article is organized as follows: In Section-II, the mystery of quantum superposition [22] is briefed. In Section-III, a solution, along with a possible experimental test, is provided - resolving the mystery in the contexts of Young's double-slit and Mach-Zehnder interferometer experiments. Also, the counterfactual reality and the causality in Young's double-slit and Wheeler's delayed choice [23-29] experiments are pointed out, respectively. Section-IV contains the conclusions.

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## II. MYSTERY OF THE QUANTUM SUPERPOSITION

The mystery of quantum superposition is elucidated in Ref. [22] and the same is summarized in the present section:

Consider the Mach-Zehnder interferometer in FIG. 1(a). An emitted free-particle from the single-particle source (SPS) is described by the quantum state vector, $|\psi\rangle$, and is subjected to a $50: 50$ beam splitter, $\mathrm{BS}_{1}$, whose vector space is spanned by the transmitted eigenstate, $\left|\psi_{T}\right\rangle$, and the reflected eigenstate, $\left|\psi_{R}\right\rangle$, of the operator, $\widehat{\mathrm{BS}_{1}}$ :

$$
\begin{equation*}
\widehat{\mathrm{BS}_{1}} \left\lvert\, \psi>=\frac{1}{\sqrt{2}}\left(\left|\psi_{T}>+\right| \psi_{R}>\right)\right. \tag{1}
\end{equation*}
$$

Both these eigenstates encounter the vector space of another beam splitter, $\mathrm{BS}_{2}$, similar to $\mathrm{BS}_{1}$ :

$$
\begin{equation*}
\frac{1}{\sqrt{2}} \widehat{\mathrm{BS}_{2}}\left|\psi_{T}>=-\frac{1}{2}\left(\left|\psi_{1}>-\right| \psi_{2}>\right) ; \quad \frac{1}{\sqrt{2}} \widehat{\mathrm{BS}_{2}}\right| \psi_{R}>=\frac{1}{2}\left(\left|\psi_{1}>+\right| \psi_{2}>\right) \tag{2}
\end{equation*}
$$

where, $\mid \psi_{1}>$ and $\mid \psi_{2}>$ are the eigenstates of $\widehat{\mathrm{BS}_{2}}$ - which can be felt by the particle detectors, $\mathrm{D}_{1}$ and $\mathrm{D}_{2}$, respectively:

$$
\begin{equation*}
\widehat{\mathrm{BS}_{2} \mathrm{BS}_{1}}\left|\psi>=\frac{1}{\sqrt{2}}\left(\widehat{\mathrm{BS}_{2}}\left|\psi_{T}>+\widehat{\mathrm{BS}_{2}}\right| \psi_{R}>\right)=\left|\psi_{2}>=\right| \psi>,\right. \tag{3}
\end{equation*}
$$

where, $<\psi_{R}\left|\psi_{T}>=<\psi_{2}\right| \psi_{1}>=0$. Therefore, the particle entering $\mathrm{BS}_{1}$ comes out of $\mathrm{BS}_{2}$ to be detected only by $\mathrm{D}_{2}$, hence not by $\mathrm{D}_{1}$. It has two paths to take from $\mathrm{BS}_{1}$ to $\mathrm{BS}_{2}$, while being described by the superposition state in Eq. (1). Not having unambiguous answers to all the possible logical questions, enumerated below using the labels of figures, constitute the mystery of quantum superposition:

- FIG. 1(b). Does the particle take Path-1 or Path-2?
- FIG. 1(c). Does the particle take both paths at the same time?
- FIG. 1(d). Does the particle take neither paths, though emerges out of $\mathrm{BS}_{2}$ ?

Above questions are analyzed in the following in order to figure out the reason underlying the mystery:

FIG. 1(b). Does the particle take Path-1 or Path-2?

From Eq. (3) and FIG. 1(a), a single-particle detected by $\mathrm{D}_{2}$ can be inferred to have transited through either Path-1 or Path-2. Assume that it takes Path-2 (Path-1) - which


FIG. 1. Schematic Diagram for a Particle in the Mach-Zehnder Interferometer: SPS is a single-particle source, $B S_{1}$ and $B S_{2}$ are $50: 50$ beam splitters, $M_{1}$ and $M_{2}$ are $100 \%$ reflecting mirrors, $\mathrm{D}_{1}$ and $\mathrm{D}_{2}$ are single-particle detectors and $\mathrm{B}_{T}$ and $\mathrm{B}_{R}$ stand for "Blockers" in Path-1 and Path-2, respectively. A particle is emitted by SPS and is detected by either $\mathrm{D}_{1}$ or $\mathrm{D}_{2}$. The energy eigenstate, $|\psi\rangle$, encounters the vector space of $\mathrm{BS}_{1}$ spanned by the reflected and transmitted eigenstates, $\left|\psi_{T}\right\rangle$ and $\left|\psi_{R}\right\rangle$, respectively, and hence, $|\psi\rangle=(1 / \sqrt{2})\left(\left|\psi_{T}\right\rangle+\left|\psi_{R}\right\rangle\right)$. The vector space of $\mathrm{BS}_{2}$ is spanned by $\mid \psi_{1}>$ and $\mid \psi_{2}>$ and its action is similar to $\mathrm{BS}_{1}$.
may be verified by introducing a blocker, $\mathrm{B}_{\mathrm{T}}\left(\mathrm{B}_{\mathrm{R}}\right)$, in Path-1 (Path-2) as shown in FIG. $1(\mathrm{~b})$. Let $\widehat{\mathrm{B}}_{i}$ be the operator representing the blocker $\mathrm{B}_{i}$ :

$$
\begin{align*}
\widehat{\mathrm{B}}_{i} \widehat{\mathrm{BS}_{1}} \mid \psi> & \left.=\frac{1}{\sqrt{2}} \widehat{\mathrm{~B}}_{i}\left(\left|\psi_{T}>+\right| \psi_{R}>\right)=\frac{1}{\sqrt{2}} \right\rvert\, \psi_{\bar{i}}>, \\
\Longrightarrow \widehat{\mathrm{BS}}_{2} \widehat{\mathrm{~B}}_{i} \widehat{\mathrm{BS}_{1}} \mid \psi> & =\frac{1}{\sqrt{2}} \widehat{\mathrm{BS}_{2}} \left\lvert\, \psi_{\bar{i}}>=\frac{1}{2}\left[(-1)^{n(\bar{i})}\left|\psi_{1}>+\right| \psi_{2}>\right]\right., \tag{4}
\end{align*}
$$

where, $i=T, R: T=\bar{R} ; \bar{T}=R ; n(T)=1$ and $n(R)=2$. From the above equation, the given below are clear:

1. The output from $\mathrm{BS}_{2}$ is down by $50 \%$ as it should be due to the presence of the blocker.
2. Both $\mathrm{D}_{1}$ and $\mathrm{D}_{2}$ register $25 \%$ each, contradicting the expectation of $0 \%$ and $50 \%$, respectively.

Therefore, inferring the particle to have transited through either Path-1 or Path-2 has be incorrect!

## FIG. 1(c). Does the particle take both paths?

From Eq. (3) and FIG. 1(a), a single particle detected by $\mathrm{D}_{2}$ can counter-intuitively be inferred to have taken both the paths simultaneously. This can be tested by inserting the particle detectors, $D_{1}$ in Path- 1 and $D_{2}$ in Path-2, as shown in FIG. 1(c), where, $\left|\psi_{T}\right\rangle=$ $\mid \psi_{1}>$ and $\left|\psi_{R}\right\rangle=\left|\psi_{2}\right\rangle$ :

$$
\widehat{\mathrm{BS}_{1}} \left\lvert\, \psi>=\frac{1}{\sqrt{2}}\left(\left|\psi_{\mathrm{R}}>+\right| \psi_{\mathrm{T}}>\right) \xrightarrow[\text { State Vector }]{\text { Reduction of }}\left\{\begin{array}{l}
\left.\frac{1}{\sqrt{2}} \right\rvert\, \psi_{\mathrm{T}}>\text { only } \mathrm{D}_{1} \text { clicks }  \tag{5}\\
\left.\frac{1}{\sqrt{2}} \right\rvert\, \psi_{\mathrm{R}}>\text { only } \mathrm{D}_{2} \text { clicks }
\end{array}\right.\right.
$$

Always either $D_{1}$ or $D_{2}$ click, but not both at the same time, clarifying the fact that the whole particle is present in one of the paths only. Therefore, inferring the particle as going through both the paths simultaneously cannot be the truth.

## FIG. 1(d). Does the particle take neither paths, though emerges out of $\mathrm{BS}_{2}$ ?

What if the particle takes neither paths, but still manages to exit out of $\mathrm{BS}_{2}$ ? This can easily be checked by placing the blockers in both paths as in FIG. 1(d), which is indeed equivalent to FIG. 1(c):

$$
\begin{equation*}
\left.\widehat{\mathrm{BS}_{2}}\left(\widehat{\mathrm{~B}_{R}} \widehat{\mathrm{~B}_{T}} \widehat{\mathrm{BS}_{1}} \mid \psi>\right)=\widehat{\mathrm{BS}_{2}}(0)=0 ; \text { here, }\left[\widehat{\mathrm{B}_{T}}, \widehat{\mathrm{~B}_{R}}\right]=0\right), \tag{6}
\end{equation*}
$$

not resulting any click in both the detectors. Therefore, inferring the particle to have taken neither paths must not be the reality.

From the above analysis, the overall conclusion drawn is that, the particle going through the Mach-Zehnder interferometer in FIG. 1(a) does not take Path-1, Path-2, both paths simultaneously or neither paths between the beam splitters, but still magically emerges out of $\mathrm{BS}_{2}$ towards $\mathrm{D}_{2}$. Surely, the particle is doing something unknown or unknowable, known as being in the state of quantum superposition (Eq. (3)).

## III. DEMYSTIFYING THE MYSTERY OF QUANTUM SUPERPOSITION

In the "wave-particle non-dualistic interpretation of quantum mechanics at a singlequantum level" [6-21], the physical reality of Schrödinger's wave function is shown to be an


FIG. 2. Schematic diagrams of Young's Double-Slit and Mach-Zehnder Interferometer
Experiments: SPS - Single-Particle Source, $\mid \psi>$ - energy eigenstate, $D_{1}$ and $D_{2}$ - single particle detectors and P - Particle. (a) NS - Narrow Slit, DSA - Double-Slit Assembly, DS - Detection Screen, $\left|\psi_{s}\right\rangle=\left|\psi_{1}\right\rangle+\left|\psi_{2}\right\rangle,\left\langle\psi_{s}\right|$ - induced dual in DS, IPI $=<\psi_{s} \mid \psi_{s}>$ - Inner-Product Interaction, IRSM - Instantaneous Resonant Spatial Mode, SWF - Schrödinger's Wave Function and IRSM $=$ SWF. The particle at SPS, $|\psi\rangle,\left|\psi_{s}\right\rangle$ and the IPI - all appear at the same instant. A classical path of least action is traced in the IRSM by the eigenvalues of a particular position eigenstate, evolving according to the Heisenberg equations of motion, where the moving particle resides. The points $\mathrm{O}^{\prime}$ and $\mathbf{r}_{p}\left(t_{a}\right)$ lie in the central bright fringe and some other bright fringe, respectively. (b) $\mathrm{BS}_{1}$ and $\mathrm{BS}_{2}-50: 50$ Beam Splitters, $\mathrm{M}_{1}$ and $\mathrm{M}_{2}-100 \%$ reflecting Mirrors. $\mid \psi_{\mathrm{T}}>$ and $\mid \psi_{\mathrm{R}}>$ are transmitted and reflected eigenstates, respectively. $\left|\psi_{1}\right\rangle$ and $\left|\psi_{2}\right\rangle$ are the eigenstates spanning the vector space of $\mathrm{BS}_{2}$ and detected by $\mathrm{D}_{1}$ and $\mathrm{D}_{2}$, respectively. Another set of $D_{1}$ and $D_{2}$ can be used as both particle detectors and blockers at the same time on path- 1 and path-2, respectively. The $\left\langle\psi_{T} \mid \psi_{T}\right\rangle,\left\langle\psi_{R} \mid \psi_{R}\right\rangle,\left\langle\psi_{1} \mid \psi_{1}\right\rangle$ and $\left\langle\psi_{2} \mid \psi_{2}\right\rangle$ are IPIs occuring at their respective detectors.
"instantaneous resonant spatial mode (IRSM)", in which a particle moves akin to the test particle in curved spacetime of general theory of relativity. A classical path of least action is traced in the IRSM by the eigenvalues of a particular position eigenstate, evolving according to the Heisenberg equations of motion, where the moving particle resides.

Consider Young's double-slit experiment in FIG. 2(a). A free-particle, carrying the energy eigenvalue, is emitted by the single-particle source (SPS) and is described by the energy eigenstate, $|\psi\rangle$. The moment it comes out of SPS, its Schrödinger wave function, $<\mathbf{r}|\psi\rangle$, resonantly and instantaneously appears as a superposition of all eigenstates of the position operator $\hat{\mathbf{r}}$ :

$$
\begin{equation*}
\left|\psi>=\iiint_{\mathbb{R}^{3}} d^{3} \mathbf{r}\right| \mathbf{r}><\mathbf{r} \mid \psi> \tag{7}
\end{equation*}
$$

where, $\hat{\mathbf{r}}|\mathbf{r}>=\mathbf{r}| \mathbf{r}>$; the set of position eigenvalues, $\left\{\mathbf{r} \mid \mathbf{r} \in \mathbb{R}^{3}\right\}$, spans the 3-dimensional Euclidean space. Along with the wave function, the superposition state, $\left|\psi_{s}\right\rangle=\left|\psi_{1}\right\rangle$ $+\mid \psi_{2}>$ (a projection due to the double-slit assembly, DSA) and the inner-product interaction (IPI), $\left\langle\psi_{s} \mid \psi_{s}\right\rangle$, on the AB surface of detection screen (DS) also appear at the same time. There always exists in Eq. (7), a particular position eigenstate, say $\left|\mathbf{r}_{p}><\mathbf{r}_{p}\right| \psi>$, having the same phase as the global-phase of $\mid \psi>$ and the particle enters that particular eigenstate at SPS; the subscript $p$ in $\mathbf{r}_{p}$ indicates "particle". Later, depending on the same global phase, it enters either $\mid \psi_{1}>$ or $\mid \psi_{2}>$ and hits the region of IPI, say at $\mathbf{r}_{p}\left(t_{a}\right)$; here, $t_{a}$ is the "arrival time" of the particle.

The DS can be associated with an operator $\hat{D}=\left|\psi_{D}\right\rangle<\psi_{s} \mid$ for the scattering of $\left|\psi_{s}\right\rangle$ into $\mid \psi_{D}>$ :

$$
\begin{equation*}
\hat{D}\left|\psi_{s}>=\left|\psi_{D}><\psi_{s}\right| \psi_{s}>\right. \tag{8}
\end{equation*}
$$

which contains the final boundary condition to be imposed to $\mid \psi_{s}>$ in accordance with the quantum formalism. The particle interacts at $\mathbf{r}_{p}\left(t_{a}\right)$ in the region of $\left\langle\psi_{s} \mid \psi_{s}\right\rangle$ - a set of real numbers serving as the ground for the detection of eigenvalues of the Observables:

$$
\begin{equation*}
<\psi_{s}\left|\psi_{s}>=\iiint d^{3} \mathbf{r}<\psi_{s}\right| \mathbf{r}><\mathbf{r} \mid \psi_{s}>\xrightarrow[\text { at DS }]{\text { Detection }}\left|<\mathbf{r}_{p}\left(t_{a}\right)\right| \psi_{s}>\left.\right|^{2}, \tag{9}
\end{equation*}
$$

where, only $\left|\mathbf{r}_{p}\left(t_{a}\right)><\mathbf{r}_{p}\left(t_{a}\right)\right| \psi_{s}>$ contains the particle, hence contributes to the above detection process. All other remaining position eigenstates contribute nothing, since they are empty. Since $<\psi_{2} \mid \psi_{1}>\neq 0$, the above equation represents the landing of particle at some bright fringe of the interference pattern.

A property of any eigenvalue equation is that if the eigenvalue changes suddenly, then the eigenstate also has to do the same. Therefore, the moment the particle is detected at $\mathbf{r}_{p}\left(t_{a}\right)$, its energy eigenvalue and the $\mid \psi>$ change instantaneously - which is analogous to the wave function collapse advocated in the Copenhagen Interpretation [1-5].

The Schrödinger wave function, being an IRSM, itself induces all possible interactions, because, it is a delocalized entity existing everywhere at once, but the particle itself participates in one particular interaction, because, it is a localized entity. That particular interaction is determined by the global phase of the wave function - these are in agreement with "Bohr's set of all possible experimental outcomes" [30, 31].

If the which-path detectors, $\mathrm{D}_{1}$ and $\mathrm{D}_{2}$, are ON , then $\mid \psi_{s}>$ gets scattered into, say $\mid \psi_{s}^{\prime}>$ :

$$
\begin{equation*}
\left|\psi_{s}>=\left|\psi_{1}>+\right| \psi_{2}>\xrightarrow[\text { are ON }]{\mathrm{D}_{1} \text { and } \mathrm{D}_{2}}\right| \psi_{s}^{\prime}>=\left|\psi_{1}^{\prime}>+\right| \psi_{2}^{\prime}>. \tag{10}
\end{equation*}
$$

The $\mid \psi_{s}^{\prime}>$ interacts with its excited dual, $<\psi_{s}^{\prime} \mid$, at DS according to the IPI:

$$
\begin{equation*}
<\psi_{s}^{\prime} \psi_{s}^{\prime}>=\left|<\psi_{1}^{\prime}\right| \psi_{1}^{\prime}>\left.\right|^{2}+\left|<\psi_{2}^{\prime}\right| \psi_{2}^{\prime}>\left.\right|^{2} . \tag{11}
\end{equation*}
$$

Notice that $<\psi_{1}^{\prime} \mid \psi_{2}^{\prime}>=0$, because, $\mathrm{D}_{1}\left(\mathrm{D}_{2}\right)$ can detect the particle only when it passes through the slit-1 (slit-2). The particle at SPS, $|\psi\rangle,\left|\psi_{s}^{\prime}\right\rangle$, and the IPI - all appear at the same moment. Once the particle transits from $\mid \psi_{s}>$ to $\mid \psi_{s}^{\prime}>$ at DSA, then the former can be treated as a disappeared state and the later as an appeared state, because, they are connected in series unlike $\mid \psi_{1}^{\prime}>$ and $\mid \psi_{2}^{\prime}>\left(\right.$ or $\left.\left|\psi_{1}>\&\right| \psi_{2}\right\rangle$ ). While $\mathrm{D}_{1}$ and $\mathrm{D}_{2}$ are switched on, the particle enters either $\mid \psi_{1}^{\prime}>$ or $\mid \psi_{2}^{\prime}>$ depending on the global phase of $\mid \psi>$ and gets detected by DS in any one of the clump-patterns, either $\left|<\psi_{1}^{\prime}\right| \psi_{1}^{\prime}>\left.\right|^{2}$ or $\left|<\psi_{2}^{\prime}\right| \psi_{2}^{\prime}>\left.\right|^{2}$, respectively.

Here, it is worth pointing out the counterfactual reality [32-35] in Young's double-slit experiment: Notice an inference from the above analysis that if an event of the particle passing through the slit-1 does happen, then simultaneously, a similar event does not happen at the slit-2 and visa versa. But, the event that does not happen can have an unavoidable physical effect resulting in interference pattern at the DS as given in Eq. (9). On the other hand, any attempt to confirm the inference by direct observation using $D_{1}$ and $D_{2}$ completely changes the interference pattern to the clump-patterns as in Eq. (11).

All the above analysis done for Young's double-slit experiment goes through as it is to the case of Mach-Zehnder interferometer, because, a particular choice of paths, $\mathrm{O}\left(\right.$ slit-1) $\mathrm{O}^{\prime}$ and O (slit-2) $\mathrm{O}^{\prime}$, in FIG. 2(a) and the configuration of paths, (SPS)(path-1) $\mathrm{D}_{2}$ and (SPS)(path2) $\mathrm{D}_{2}$, in FIG. 2(b) are identical to each other.

Consider FIG. 2(b) ( = FIG. 1): The actual solution to the mystery of quantum superposition mentioned in Section-II is, "Depending on the global-phase of $\mid \psi>$ (Eq. (1)),


FIG. 3. Modified Mach-Zehnder Interferometer Experiment: SPS is a single-particle source and P is a particle. BS and IBS are $50: 50$ beam splitter and inverse beam splitter, $\mathrm{M}_{1}$ and $M_{2}$ are $100 \%$ reflecting mirrors, and $D_{1}$ and $D_{2}$ are single-photon detectors, respectively. A singleparticle energy eigenstate $\mid \psi>$ encountering BS gets partially transmitted and partially reflected as $\mid \psi_{T}>$ and $\left|\psi_{R}\right\rangle$ along Path $_{1}$ and Path $_{2}$, respectively. The path difference, Path ${ }_{2}-$ Path $_{1}$, is chosen to yield the constructive and the destructive interferences, CI and DI, towards the detectors $\mathrm{D}_{1}$ and $\mathrm{D}_{2}$, where, the inner-product interactions $<\psi_{1}\left|\psi_{1}\right\rangle$ and $<\psi_{2}\left|\psi_{2}\right\rangle$ happen, respectively. The moment P appears at SPS, $\left.|\psi\rangle,\left|\psi_{T}\right\rangle,\left|\psi_{R},\right| \psi_{1}\right\rangle,\left\langle\psi_{1} \mid \psi_{1}\right\rangle \neq 0$ and $\left\langle\psi_{2} \mid \psi_{2}\right\rangle=0$ - all appear at the same moment. Since $<\psi_{2}\left|\psi_{2}\right\rangle=0, \mathrm{P}$ always lands in $\left\langle\psi_{1} \mid \psi_{1}\right\rangle$ at $\mathrm{D}_{1}$ with the arrival time $T_{1}$ or $T_{2}$, depending on whether it takes path- 1 or path- 2 , respectively.
the particle enters either $\left|\psi_{T}\right\rangle$ or $\left|\psi_{R}\right\rangle$, hence takes either path- 1 or path- 2 , respectively, and always gets detected by $\mathrm{D}_{2}$ (Eq. (3))" (which is also the same as the counterfactual reality mentioned in Young's double-slit experiment). This becomes evident when $\mathrm{D}_{1}$ and $\mathrm{D}_{2}$ are placed on path- 1 and path-2, where, the IPIs, $<\psi_{T} \mid \psi_{T}>$ and $<\psi_{R}\left|\psi_{R}\right\rangle$, occur, respectively. If the particle is present in path-1, then it hits $\mathrm{D}_{1}$ in $\left\langle\psi_{T} \mid \psi_{T}\right\rangle$ and at the same time, $<\psi_{R} \mid \psi_{R}>$ in $\mathrm{D}_{2}$ receives no hit. Therefore, always one of the detectors clicks, but not both at the same time, proving the fact that the particle takes only one path at a time.

Replacing $\mathrm{D}_{1}$ and $\mathrm{D}_{2}$ by $\mathrm{B}_{T}$ and $\mathrm{B}_{R}$, respectively, are one and the same with respect to the output at $\mathrm{BS}_{2}$, i.e. FIG. $1(\mathrm{c}) \sim$ FIG. $1(\mathrm{~d})$. If any one of the detectors is placed on any one of the paths, then the output from $\mathrm{BS}_{2}$ is detected by both $\mathrm{D}_{1}$ and $\mathrm{D}_{2}$ (Eq. (4))

- this must be the case, because, the context of $\mid \psi_{R}>$ alone encountering $\mathrm{BS}_{2}$, while $\mathrm{B}_{T}$ is placed on path-1, is the same as that of $\mid \psi>$ encountering $\mathrm{BS}_{1}$. In other words, placing $\mathrm{B}_{T}$ on path-1 is equal to replacing the question, "Does the particle that emerges out of $\mathrm{BS}_{1}$ take path-1 or path-2?" by "Does the particle that emerges out of $\mathrm{BS}_{2}$ get detected by $\mathrm{D}_{1}$ or $D_{2}$ ?" Importantly, in this case, the coincidence detection between $D_{1}$ and $D_{2}$ is zero, confirming the fact that a single particle does enter $\mathrm{BS}_{2}$. Therefore, the conclusion drawn in FIG. 1(b) that the particle does not take path-2 is not supported by the very experiments, which characterizes the single-particle sources [36-39].

Nevertheless, the above solution may merely appear as a theoretical inference, though, it successfully resolves the mystery of quantum superposition. But, notice that, it is falsifiable using a modified Mach-Zehnder interferometer with unequal path lengths, as shown in FIG. 3. Notice that, a particular choice of paths - $\mathrm{O}(\operatorname{slit}-1) \mathbf{r}_{p}\left(t_{a}\right)$ and $\mathrm{O}\left(\operatorname{slit-2)} \mathbf{r}_{p}\left(t_{a}\right)\right.$ - in FIG. 2(a) and the configuration of paths - (SPS)(path-1) $D_{1}$ and (SPS)(path-2) $D_{1}$ - in FIG. 3 are identical to each other. Let $\mathrm{T}_{1}$ and $\mathrm{T}_{2}$ be the arrival times of the particle along Path ${ }_{1}$ and $\mathrm{Path}_{2}$, respectively. If the difference, $T_{2}-T_{1}$, is sufficiently large and is also greater than all the possible experimental errors involving in the determination of the initial time of emission and the final time of detection of the particle, then half of the total number of particles detected by $D_{1}$ will have arrival time $T_{1}$ and the remaining half will have $T_{2}$. Therefore, in this particular experiment, the path taken by the particle can be known by merely measuring $\mathrm{T}_{1}$ and $\mathrm{T}_{2}$, but a negative result invalidates the wave-particle non-dualistic interpretation itself.

It is worth pointing out the causality in Wheeler's delayed-choice experiment [23-29] using FIG. 2(b). While the particle, P , is moving along path- 1 , the $\mathrm{BS}_{2}$ can be removed and reinserted randomly again and again, but it continues its journey along path-1. If it manages to pass through $\mathrm{BS}_{2}$, then it is detected by $D_{2}$ or otherwise, by $\mathrm{D}_{1}$. On the other hand, if it takes path-2, then it gets detected always by $\mathrm{D}_{2}$, irrespective of transiting through $\mathrm{BS}_{2}$ or not. Notice that, its motion is completely causal and hence, the retrocausal effects are absent in Wheeler's delayed-choice experiment.

## IV. CONCLUSIONS

Using the "wave-particle non-dualistic interpretation of quantum mechanics", the mystery of quantum superposition is demystified at a single-quantum level. Similar to the relation between an eigenstate and its eigenvalue via an eigenvalue equation, the wave and particle natures are related in non-duality. The global-phase of the eigenstate decides the particular path to be taken by the particle, which automatically provides a solution to the mystery this also expalins the counterfactual reality in Young's double-slit and Mach-Zehnder interferometer experiments. A modified Mach-Zehnder interferometer experiment is proposed to verify the correcness of the solution. Finally, the existence of causality in Wheeler's delayed-choice experiment is pointed out in the context of Mach-Zehnder interferometer experiment.
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