Ultimate Acceleration to Calculate Atomic Spin

Huaiyang Cui
Department of Physics, Beihang University, Beijing, 102206, China
Email: hycui@buaa.edu.cn
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Abstract: In analogy with the ultimate speed c, there is an ultimate acceleration β, nobody's acceleration can exceed this limit β, for electrons and quarks, β=2.327421e+29(m/s^2). Because this ultimate acceleration is a large number, any effect connecting to β will become easy to test. In this paper, an approach is put forward to connect the ultimate acceleration with quantum theory, consequently, the matter wave can bear the spin concept. This paper also carefully explains how the matter wave to display its spin effect in Stern-Gerlach experiments. It is completely a new aspect to quantum mechanics for the relativistic matter wave to contain spin.

1. Introduction

This year is 99th anniversary of the initiative of de Broglie's matter wave. In 1922, the Louis de Broglie considered blackbody radiation as a gas of light quanta [4], he tried to reconcile the concept of light quanta with the phenomena of interference and diffraction. In 1923 and 1924, the concept that matter behaves like a wave was proposed by Louis de Broglie [5,6]. It is also referred to as the de Broglie hypothesis, matter waves are referred to as de Broglie waves.

In analogy with the ultimate speed c, there is an ultimate acceleration β, nobody's acceleration can exceed this limit β, for electrons and quarks, β=2.327421e+29(m/s2). Because this ultimate acceleration is a large number, any effect connecting to β will become easy to test. In this paper, an approach is put forward to connect the ultimate acceleration with quantum theory, consequently, the matter wave can bear the spin concept. This paper also carefully explains how the matter wave to display its spin effect in Stern-Gerlach experiments. It is completely a new aspect to quantum mechanics for the relativistic matter wave to contain spin.

2. How to connect the ultimate acceleration with quantum theory

In the relativity, the speed of light c is an ultimate speed, nobody's speed can exceed this limit c. The relativistic velocity u of a particle in the coordinate system ($x_1,x_2,x_3,x_4=ict$) satisfies

$$u_1^2+u_2^2+u_3^2+u_4^2=-c^2 \quad . \tag{1}$$

No matter what particles (electrons, molecules, neutrons, quarks), their 4-vector velocities all have the same magnitude: $|u|=ic$. All particles gain **equality** because of the same magnitude of the 4-velocity u. The acceleration a of a particle is given by

$$a_1^2+a_2^2+a_3^2=a^2; \quad (a_4=0; \quad \because x_4=ict) \tag{2}$$

Assume that particles have an ultimate acceleration β as limit, no particle can exceed this acceleration limit β. Subtracting the both sides of the above equation by β^2, we have

$$a_1^2 + a_2^2 + a_3^2 - \beta^2 = a^2 - \beta^2; \qquad a_4 = 0 \tag{3}$$

It can be rewritten as

$$[a_1{}^2 + a_2{}^2 + a_3{}^2 + 0 + (i\beta)^2]\frac{1}{1 - a^2/\beta^2} = -\beta^2 \tag{4}$$

Now, the particle subjects to an acceleration whose five components are specified by

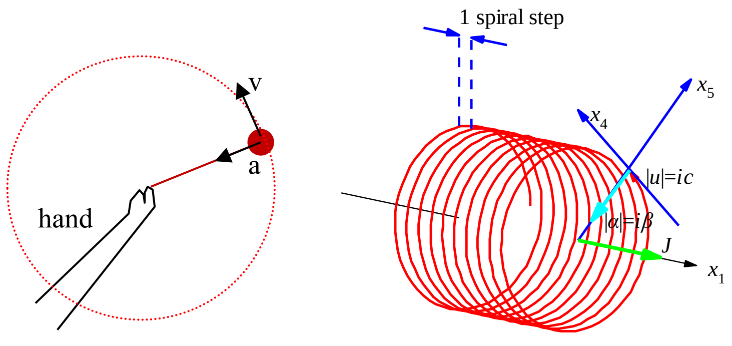
$$\begin{aligned} &\alpha_1 = \frac{a_1}{\sqrt{1 - a^2/\beta^2}}; \quad \alpha_2 = \frac{a_2}{\sqrt{1 - a^2/\beta^2}}; \\ &\alpha_3 = \frac{a_3}{\sqrt{1 - a^2/\beta^2}}; \quad \alpha_4 = 0; \quad \alpha_5 = \frac{i\beta}{\sqrt{1 - a^2/\beta^2}}; \end{aligned} \tag{5}$$

where α_5 is the newly defined acceleration in five dimensional space-time ($x_1, x_2, x_3, x_4 = ict, x_5$). Thus, we have

$$\alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \alpha_4^2 + \alpha_5^2 = -\beta^2; \quad \alpha_4 = 0 \tag{6}$$

It means that the magnitude of the newly defined acceleration α for every particle takes the same value: $|\alpha| = i\beta$ (constant imaginary number), all particle accelerations gain **equality** for the sake of the same magnitude.

How to resolve the velocity u and acceleration α into x, y, and z components? In realistic world, a hand can rotate a ball moving around a circular path at constant speed v with constant centripetal acceleration a, as shown in Fig.1(a).



(a) (b)

Fig.1 (a) A hand rotates a ball moving around a circular path at constant speed v with constant centripetal acceleration a. (b) The particle moves along the x1 axis with the constant speed |u|=ic in the u direction and constant centripetal force in the x5 axis at the radius iR (imaginary number).

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<Clet2020 Script>// Clet is a C compiler[26]
double D[100],S[2000];int i,j,R,X,N;
int main(){R=50;X=50;N=600;D[0]=-50;D[1]=0;D[2]=0;D[3]=X;D[4]=0;D[5]=0;D[6]=-50;D[7]=R;D[8]=0;
D[9]=600;D[10]=10;D[11]=R;D[12]=0;D[13]=3645;
Lattice(SPIRAL,D,S);SetViewAngle(0,80,-50);DrawFrame(FRAME_NULL,1,0xffffff);
Draw("LINE,0,2,XYZ,0","-150,0,0,-50,0,0");Draw("ARROW,0,2,XYZ,10","50,0,0,150,0,0");
SetPen(2,0xff0000);Plot("POLYLINE,0,600,XYZ",S[9]);i=9+3*N-6;Draw("ARROW,0,2,XYZ,10",S[i]);
TextHang(S[i],S[i+1],S[i+2],"    #if|u|=ic#t");TextHang(150,0,0,"    #ifx#sd1#t");SetPen(2,0x005fff);
Draw("LINE,1,2,XYZ,8","-50,0,50,-50,0,100");Draw("LINE,1,2,XYZ,8","-40,0,50,-40,0,100");
Draw("ARROW,0,2,XYZ,10","-80,0,100,-50,0,100");Draw("ARROW,0,2,XYZ,10","-10,0,100,-40,0,100");
TextHang(-50,0,110,"1 spiral step");i=9+3*N;S[i]=50;S[i+1]=10;S[i+2]=10;
Draw("ARROW,0,2,XYZ,10","50,0,0,50,80,80");TextHang(50,80,80," #ifx#sd5#t");
Draw("ARROW,0,2,XYZ,10","50,72,0,50,0,72");TextHang(50,0,72,"    #ifx#sd4#t");
SetPen(3,0x00ffff);Draw("ARROW,0,2,XYZ,15",S[i-3]);TextHang(S[i],S[i+1],S[i+2],"    #if|α|=iβ#t");
SetPen(3,0x00ff00);Draw("ARROW,0,2,XYZ,15","50,0,0,120,0,0");TextHang(110,5,5," #ifJ#t");
TextHang(-60,0,-80," right hand chirality");}#v07=?>A
```

In analogy with the ball in a circular path, consider a particle in one dimensional motion along the x_1 axis at the speed v, in the Fig.1(b) it moves with the constant speed $|u|=ic$ almost along the x_4 axis and slightly along the x_1 axis, and the constant centripetal acceleration $|\alpha|=i\beta$ in the x_5 axis at the constant radius iR (imaginary number); the coordinate system ($x_1,x_4=ict,x_5=iR$) establishes a cylinder coordinate system in which this particle moves spirally at the speed v along the x_1 axis. According to usual centripetal acceleration formula $a=v^2/r$, the acceleration in the x_4-x_5 plane is given by

$$a=\frac{v^2}{r} \quad \Rightarrow \quad i\beta=\frac{|u|^2}{iR}=-\frac{c^2}{iR}=i\frac{c^2}{R} \quad . \tag{7}$$

Therefore, the track of the particle in the cylinder coordinate system ($x_1,x_4=ict,x_5=iR$) forms a shape, called as **acceleration-roll**. The faster the particle moves along the x_1 axis, the longer the spiral step is.

As like a steel spring with elastic wave, the track in the acceleration-roll in Fig.1(b) can be described by a wave function whose phase changes 2π for one spiral step. Apparently, this wave is the de Broglie's matter wave for electrons, protons or quarks, etc.

Theorem: the acceleration-roll bears the matter wave.

Proof: The wave function phase changes 2π for one spiral circumference $2\pi(iR)$, then a small displacement of the particle on the spiral track is $|u|d\tau=icd\tau$ in the 4-vector u direction, thus this wave phase along the spiral track is evaluated by

$$phase=\int_0^\tau \frac{2\pi}{2\pi(iR)}icd\tau=\int_0^\tau \frac{c}{R}d\tau \quad . \tag{8}$$

Substituting the radius R into it, the wave function ψ is given by

$$\psi=\exp(-i\cdot phase)=\exp(-i\int_0^\tau \frac{c}{R}d\tau)=\exp(-i\frac{\beta}{c}\int_0^\tau d\tau) \quad . \tag{9}$$

In the theory of relativity, we known that the integral along $d\tau$ needs to transform into realistic line integral, that is

$$\begin{aligned} d\tau &=-c^2\frac{d\tau}{-c^2}=(u_1^2+u_2^2+u_3^2+u_4^2)\frac{d\tau}{-c^2} \\ &=(u_1dx_1+u_2dx_2+u_3dx_3+u_4dx_4)\frac{1}{-c^2} \end{aligned} \quad . \tag{10}$$

Therefore, the wave function ψ is evaluated by

$$\begin{aligned} \psi &=\exp(-i\frac{\beta}{c}\int_0^\tau d\tau) \\ &=\exp(i\frac{\beta}{c^3}\int_0^x (u_1dx_1+u_2dx_2+u_3dx_3+u_4dx_4)) \end{aligned} \quad . \tag{11}$$

This wave function may have different explanations, depending on the particle under investigation. If the β is replaced by the Planck constant, the wave function of electrons is given by

$$
\begin{aligned}
&assume: \quad \beta = \frac{mc^3}{\hbar} \\
&\psi = \exp\left(\frac{i}{\hbar}\int_0^x (mu_1 dx_1 + mu_2 dx_2 + mu_3 dx_3 + mu_4 dx_4)\right)
\end{aligned}
\qquad (12)
$$

where mu_4dx_4=-Edt, it strongly suggests that the wave function is just the de Broglie's matter wave [4,5,6]. Proof is done.

In Fig.1(b), the acceleration-roll of particle moves with two distinctions: right-hand chirality and left-hand chirality. The direction of the angular momentum J would be slightly different from the x_1 due to spiral precession. It is easy to calculate the ultimate acceleration β , the radius R and the angular momentum J in the plane x_4-x_5 for a spiraling electron as

$$
\begin{aligned}
&\beta = \frac{c^3 m}{\hbar} = 2.327421\text{e+29 (M/s}^2) \\
&R = \frac{c^2}{\beta} = 3.861593\text{e-13 (M)} \\
&J = \pm m\,|u|\,iR = \mp\hbar
\end{aligned}
\qquad (13)
$$

```
<Clet2020 Script>// Clet is a C compiler[26]
double beta,R,J,m,D[10];char str[200];
int main(){m=ME;beta=SPEEDC*SPEEDC*SPEEDC*m/PLANCKBAR;
R=SPEEDC*SPEEDC/beta;J=PLANCKBAR;Format(str,"beta=%e, #nR=%e, #nJ=%e",beta,R,J);
TextAt(50,50,str);ClipJob(APPEND,str);}#v07=#t
```

Considering another explanation to ψ for planets in the solar system, no Planck constant can be involved. But, in a many-body system with the total mass M, the data-analysis [28] tells us that the ultimate acceleration can be rewritten in terms of **Planck-constant-like constant** h as

$$
\begin{aligned}
&assume: \quad \beta = \frac{c^3}{hM} \\
&\psi = \exp\left(\frac{i}{hM}\int_0^x (u_1 dx_1 + u_2 dx_2 + u_3 dx_3 + u_4 dx_4)\right)
\end{aligned}
\qquad (14)
$$

The constant h will be determined by experimental observations. This paper will show that this wave function is applicable to several many-body systems in the solar system, the wave function is called as the **acceleration-roll wave**.

Tip: actually, ones cannot get to see the acceleration-roll of particle in the relativistic space-time ($x_1,x_2,x_3,x_4=ict$) ; only get to see it in the cylinder coordinate system ($x_1,x_4=ict,x_5=iR$).

3. How to determine the ultimate acceleration

In the Bohr's orbit model for planets or satellites, as shown in Fig.2, the circular quantization condition is given in terms of relativistic matter wave in gravity by

$$\left.\begin{aligned}&\frac{\beta}{c^3}\oint_L v_l\,dl = 2\pi n\\ &v_l=\sqrt{\frac{GM}{r}}\end{aligned}\right\} \Rightarrow \sqrt{r}=\frac{c^3}{\beta\sqrt{GM}}n;\quad n=0,1,2,\ldots \tag{15}$$

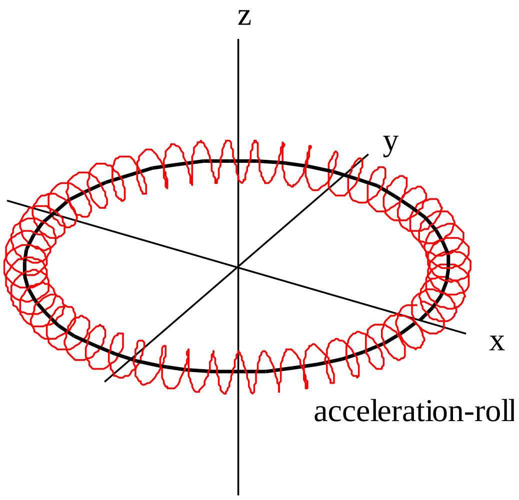

Fig.2 A planet 2D orbit around the sun, an acceleration-roll winding around the planet.

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<Clet2020 Script>// Clet is a C compiler[26]
int i,j,k; double r,rot,x,y,z,D[20],F[20],S[200];
int main(){SetViewAngle("temp0,theta60,phi-30");
DrawFrame(FRAME_LINE,1,0xafffaf);r=80;Spiral(); TextHang(r,-r,0,"acceleration-roll");
r=110;TextHang(r,0,0,"x");TextHang(0,r,0,"y");TextHang(0,0,r,"z");}
Spiral(){r=80;j=10;rot=j/r;k=2*PI/rot+1;
for(i=0;i<k;i+=1) {D[0]=x;D[1]=y;D[2]=z;D[6]=x;D[7]=y;D[8]=r;
x=r*cos(rot*i);y=r*sin(rot*i);z=0;if(i==0) continue;
SetPen(2,0x00);F[0]=D[0];F[1]=D[1];F[2]=x;F[3]=y;Draw("LINE,0,2,XY,",F);SetPen(1,0xff0000);
D[3]=x;D[4]=y;D[5]=z; D[9]=40;D[10]=10;D[11]=8;D[12]=0;D[13]=360;
Lattice(SPIRAL,D,S);Plot("POLYLINE,0,40,XYZ",S[9]);}
}#v07=?>A
```
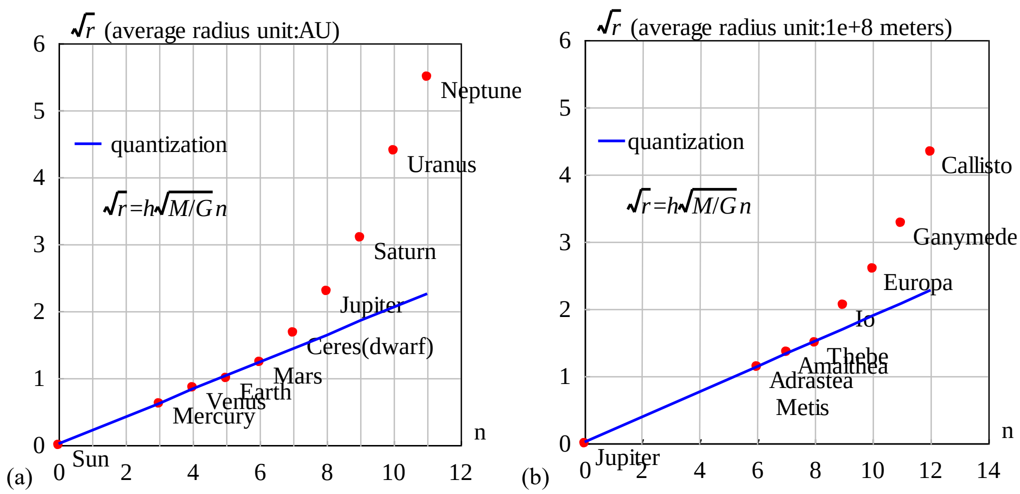

The solar system, Jupiter's satellites, Saturn's satellites, Uranus' satellites, Neptune's satellites as five different many-body systems are investigated with the Bohr's orbit model. After fitting observational data as shown in Fig.3, their ultimate accelerations are obtained in Table 1. The predicted quantization-blue-lines in Fig.3(a), Fig.3(b), Fig.3(c), Fig.3(d) and Fig.3(e) agree well with experimental observations for those *inner constituent planets or satellites*.



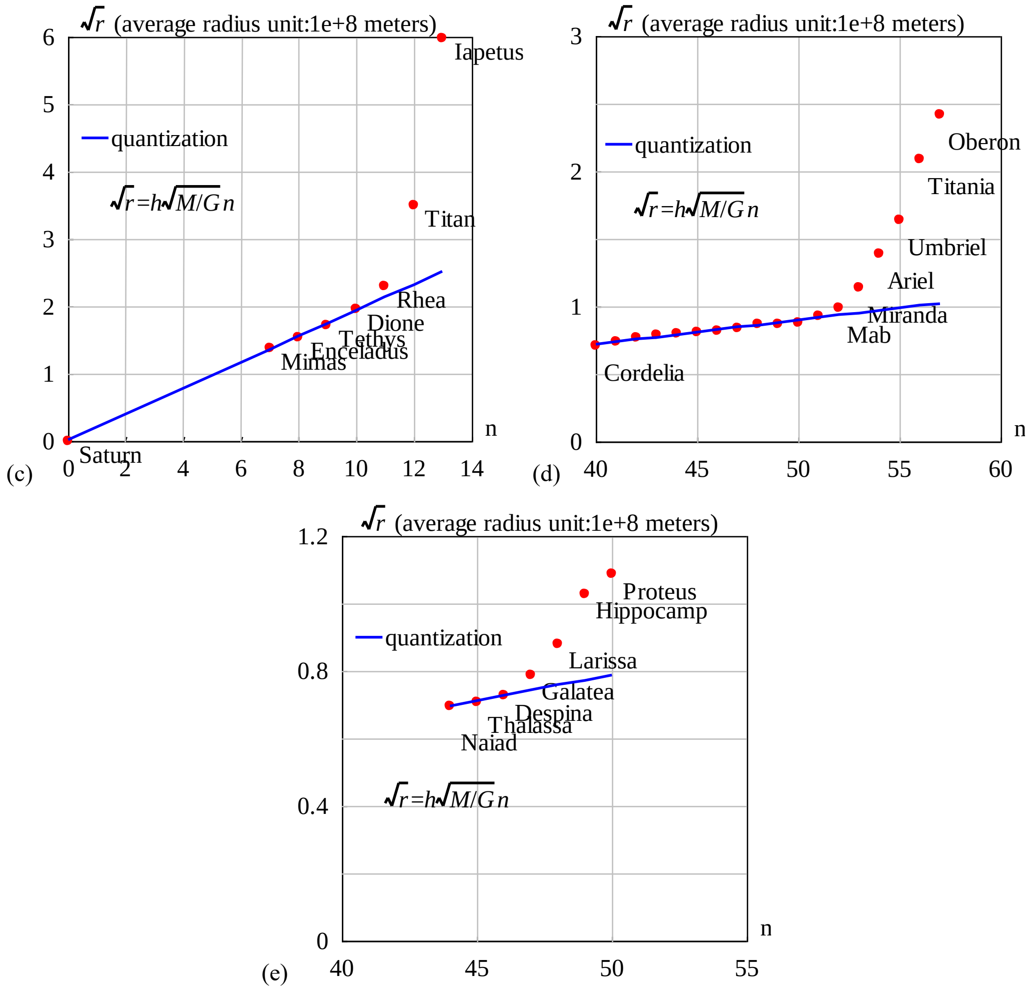


Fig.3 The orbital radii are quantized for inner constituents. (a) the solar system with h=4.574635e-16 ($m^2s^{-1}kg^{-1}$). The relative error is less than 3.9%. (b) the Jupiter system with h=3.531903e-16 ($m^2s^{-1}kg^{-1}$). Metis and Adrastea are assigned the same quantum number for their almost same radius. The relative error is less than 1.9%. (c) the Saturn system with h=6.610920e-16 ($m^2s^{-1}kg^{-1}$). The relative error is less than 1.1%. (d) the Uranus system with h=1.567124e-16 ($m^2s^{-1}kg^{-1}$). n=0 is assigned to the Uranus. The relative error is less than 2.5%. (e) the Neptune system with h=1.277170e-16 ($m^2s^{-1}kg^{-1}$). n=0 is assigned to the Neptune. The relative error is less than 0.17%.

Table 1 Planck-constant-like constant h, N is constituent particle number with smaller inclination.

system	N	M/M_{earth}	β (m/s^2)	h ($m^2s^{-1}kg^{-1}$)	Prediction
Solar planets	9	333000	2.961520e+10	4.574635e-16	Fig.3(a)
Jupiter' satellites	7	318	4.016793e+13	3.531903e-16	Fig.3(b)
Saturn's satellites	7	95	7.183397e+13	6.610920e-16	Fig.3(c)
Uranus' satellites	18	14.5	1.985382e+15	1.567124e-16	Fig.3(d)
Neptune 's satellites	7	17	2.077868e+15	1.277170e-16	Fig.3(e)

Besides every β, our interest shifts to the constant h in Table 1, which is defined as

$$h = \frac{c^3}{M\beta} \quad \Rightarrow \quad \sqrt{r} = h\sqrt{\frac{M}{G}}n \quad . \tag{16}$$

In a many-body system with the total mass M, a constituent particle has the mass m and moves at the speed v, it is easy to find that the wavelength of de Broglie's matter wave should be modified for planets and satellites as

$$\lambda_{de\_Broglie} = \frac{2\pi\hbar}{mv} \quad \Rightarrow \text{modify} \Rightarrow \quad \lambda = \frac{2\pi hM}{v} \quad . \tag{17}$$

where h is a **Planck-constant-like constant**. Usually the total mass M is approximately equal to the central-star's mass. It is found that this modified matter wave works for quantizing orbits correctly in Fig.3 [28,29]. The key point is that the various systems have almost same Planck-constant-like constant h in Table 1 with a mean value of 3.51e-16 $m^2s^{-1}kg^{-1}$, at least have the same magnitude! The acceleration-roll wave is a generalized matter wave as a planetary scale wave.

In Fig.3(a), the blue straight line expresses the linear regression relation among the Sun, Mercury, Venus, Earth and Mars, their quantization parameters are hM=9.098031e+14(m^2/s). The ultimate acceleration is fitted out to be β=2.961520e+10 (m/s^2). Where, n=3,4,5,.. were assigned to solar planets, the sun was assigned a quantum number n=0 because the sun is in the **central state**.

4. Influence of the ultimate distance in all directions

Position, velocity and acceleration are three basic concepts in particle physics, correspondingly, we have ultimate distance, ultimate velocity, and ultimate acceleration, respectively. Consider there exists an **ultimate distance** D which is automatically recognized as the diameter of our universe: among the D rang nobody can escape. The ultimate distance provides us a useful insight into cosmic microwave background, the Hubble law and dark matter.

Consider a star that have distance r to the sun (to us), we establish a frame of reference with the origin at the sun, as shown in Fig.4 in the Cartesian coordinates (x,y,z), the Pythagorean theorem tells us

$$x^2 + y^2 + z^2 = r^2 \tag{18}$$

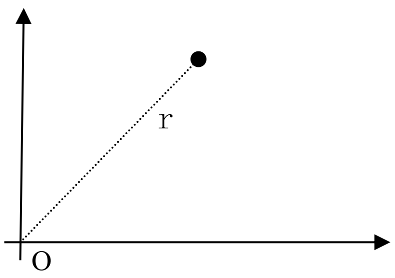


Fig.4 A star in the solar reference frame.

Because the distance is a very large quantity for the star, we worry about non-Euclidian effect that may involve within, we modify it by Taylor expansion with the first order small quantity

$$x^2 + y^2 + z^2 = r^2 + kr \tag{19}$$

where the kr term represents the non-Euclidian effect. Suppose there is the ultimate distance D in the universe, then we have

$$
\begin{aligned}
&x^2+y^2+z^2-D^2=-D^2+r^2+kr\\
&or\\
&x^2+y^2+z^2-D^2=-D^2(1-\frac{kr}{D^2}-\frac{r^2}{D^2})
\end{aligned}
\tag{20}
$$

It can be rewritten as

$$
\begin{aligned}
&\left(\frac{x}{\sqrt{1-kr/D^2-r^2/D^2}}\right)^2+\left(\frac{y}{\sqrt{1-kr/D^2-r^2/D^2}}\right)^2\\
&+\left(\frac{z}{\sqrt{1-kr/D^2-r^2/D^2}}\right)^2+\left(\frac{iD}{\sqrt{1-kr/D^2-r^2/D^2}}\right)^2=-D^2
\end{aligned}
\tag{21}
$$

Then, new coordinates can be established, are specified by

$$
\begin{aligned}
&x'=\frac{x}{\sqrt{1-kr/D^2-r^2/D^2}} \quad y'=\frac{y}{\sqrt{1-kr/D^2-r^2/D^2}}\\
&z'=\frac{z}{\sqrt{1-kr/D^2-r^2/D^2}} \quad d'=\frac{iD}{\sqrt{1-kr/D^2-r^2/D^2}}
\end{aligned}
\tag{22}
$$

In the new coordinates (*x',y',z',d'*), all stars to the origin are the same for their same distance:

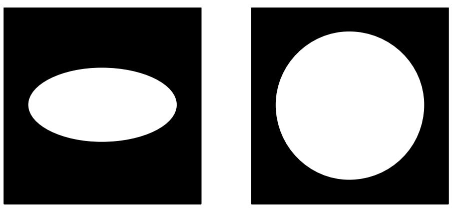
$$
|x'^2+y'^2+z'^2+d'^2|=iD \tag{23}
$$

The magnitude of the distance is a constant! All positions gain **equality** for the sake of the same distance *D*. The new coordinates (*x',y',z',d'*) is named as the **position equality space** in the followings.

Notice that a star moving at the classical position (*x,y,z*) will never be able escape from us in the position equality space (*x',y',z',d'*), simply because

$$
\begin{aligned}
&x'=\frac{x}{\sqrt{1-kr/D^2-r^2/D^2}} \quad\Rightarrow\quad |x'|<D\\
&y'=\frac{y}{\sqrt{1-kr/D^2-r^2/D^2}} \quad\Rightarrow\quad |y'|<D\\
&z'=\frac{z}{\sqrt{1-kr/D^2-r^2/D^2}} \quad\Rightarrow\quad |z'|<D
\end{aligned}
\tag{24}
$$

In other words, all distance stars are confined in an equivalent cavity with the diameter *D*. Our universe is a blackbody cavity in terms of the position equality space, in which all electromagnetic radiations warm up our universe, as shown in Fig.5.



classical cavity　　cavity confined by r≤D

Fig.5　A classical cavity, and an equivalent cavity confined by r<D in the position equality space (x',y',z',d').

```
<Clet2020 Script>//[26]
int i,j,k,nP[10];
double r,x,y,z,dP[10],D[10];
```

```
int main(){DrawFrame(FRAME_NULL,1,0xafffaf); r=40;x=-50;D[0]=x-r;D[1]=-r;D[2]=x+r;D[3]=r;
SetPen(1,0x000000); Draw("RECT,1,2,XY,10",D); x=50;r=40;D[0]=x-r;D[1]=-r;D[2]=x+r;D[3]=r;
Draw("RECT,1,2,XY,10",D); r=30;x=-50;D[0]=x-r;D[1]=-r/2;D[2]=x+r;D[3]=r/2;
Draw("ELLIPSE,3,2,XY,0xffffff",D); r=30;x=50;D[0]=x-r;D[1]=-r;D[2]=x+r;D[3]=r;
Draw("ELLIPSE,3,2,XY,0xffffff",D);
TextHang(-x-x,-x,0,"classical cavity");TextHang(0,-x,0,"cavity confined by r<D");
}#v07=?>A
```

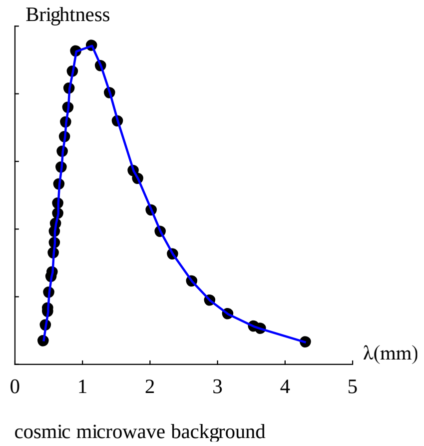


Fig.6 The measurement of the cosmic microwave background.

```
<Clet2020 Script>//[26]
int i,j,k,nP[10];char str[100]; double
D[74]={0.44,6.53,0.48,11.04,0.51,15.09,0.51,16.17,0.53,21.00,0.56,25.51,0.57,26.91,0.59,32.50,0.60,35.46,0.61,38.72,0.62,41.0
6,0.65,44.32,0.66,47.28,0.68,52.88,0.70,57.85,0.72,62.36,0.76,66.87,0.77,71.07,0.80,75.43,0.83,81.18,0.87,86.31,0.93,92.07,1.1
6,93.78,1.29,87.87,1.42,79.94,1.54,71.54,1.78,56.92,1.84,54.43,2.04,45.26,2.18,38.72,2.36,32.19,2.64,24.11,2.90,18.66,3.17,14.
62,3.56,10.73,3.65,10.11,4.33,6.22,};
int main(){ SetAxis(X_AXIS,0,0,5,"λ(mm);0;1;2;3;4;5;");SetAxis(Y_AXIS,0,0,100,"Brightness;;;;;;;");
DrawFrame(FRAME_SCALE,1,0xafffaf); Plot("OVALFILL,0,37,XY,5,5,",D);
SetPen(2,0x0000ff); Polyline(37,D); TextHang(0,-20,0,"cosmic microwave background");
}#v07=?>A
```

According to the blackbody theory, our universe has a mean temperature T with a standard blackbody spectrum in the cavity $r<D$, experimental observations confirmed the profile of cosmic microwave background radiation to be an exact blackbody radiation spectrum at the temperature T=2.725K, as shown in Fig.6.

Now, let us test the position equality space using the Hubble law. Consider an atom at far distance $x=r$, emitting an electromagnetic wave of wavelength λ. All stars are in the position equality space, so actually we live in the new coordinate system (x',y',z',d'), what we see is

$$\begin{aligned} x' &= \frac{x}{\sqrt{1-kr/D^2-r^2/D^2}} \qquad dx' \simeq \left(1+\frac{kr}{2D^2}\right)dx \\ y' &= y = 0 \\ z' &= z = 0 \\ d' &= \frac{iD}{\sqrt{1-kr/D^2-r^2/D^2}} \end{aligned} \tag{25}$$

Thus, we at the origin receive the wave length λ' of the electromagnetic wave as

$$\lambda' \simeq \left(1+\frac{kr}{2D^2}\right)\lambda \tag{26}$$

This is the Hubble law for far stars, the $2D^2/k$ equals to the Hubble constant, in fact the Hubble law happens in all directions in the sky like cosmic microwave background in all directions. The advantage of the position equality space is that it is not necessary for stars to recede as the prediction by the Doppler effect theory for frequency shift. For last many decades, our vision to the cosmology has been misguided by the abuse of the Doppler effect for electromagnetic wave over the Hubble law, the later leads to the expansion of the universe and the big bang hypothesis. Now, sleeping in the position equality space, it is time for the universe to become quiet [28].

5. How 2D matter wave to obtain spin

Dimension is defined as the number of independent parameters in a mathematical space. In the field of physics, dimension is defined as the number of independent space-time coordinates. 0D is an infinitesimal point with no length. 1D is an infinite line, only length. 2D is a plane, which is composed of length and width. 3D is 2D plus height component, has volume.

In this section we at first discuss how to measure dimension by wave. In Fig.7(a), one puts earphone into ear, one gets 1D wave in the ear tunnel.

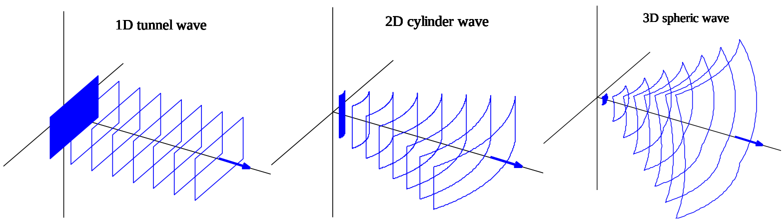
$$1D: \quad y = A\sin(kr - \omega t) = \frac{A}{r^0}\sin(kr - \omega t)\,. \tag{27}$$

where r is the distance between the wave emitter and the receiver. In Fig.7(b), one touches a guitar spring, one gets 2D cylinder wave.

$$2D: \quad y = \frac{A}{r^{1/2}}\sin(kr - \omega t)\,. \tag{28}$$

In Fig.7(c), one turns on a music speaker, one gets 3D spherical wave.

$$3D: \quad y = \frac{A}{r}\sin(kr - \omega t)\,. \tag{29}$$



(a) 1D tunnel wave (b) 2D cylinder wave (c) 3D spherical wave

Fig.7 The wave behavior in various dimensional spaces.

```
<Clet2020 Script>// Clet is a C compiler [26]
int i,j,k,type,nP[10]; double D[20],S[1000];
int main(){SetViewAngle("temp0,theta60,phi-30");SetAxis(X_AXIS,0,0,200,"X;0;200;");
DrawFrame(FRAME_LINE,1,0xafffaf); type=2;SetPen(1,0x00ff);
for(i=10;i<160;i+=20){D[0]=i;D[1]=0;D[2]=0;D[3]=i+5;D[4]=0;D[5]=0;D[6]=i;D[7]=10;D[8]=0;
if(type==0) {D[9]=4;D[10]=40;D[11]=20;D[12]=i;TextHang(50,0,100,"1D tunnel wave");k=CARD;}
else if(type==1) {D[9]=200;D[10]=i/2;D[11]=20;D[12]=i;TextHang(50,0,100,"2D cylinder wave");k=50;}
else {D[9]=200;D[10]=i/2;D[11]=i/2;D[12]=i;TextHang(50,0,100,"3D spheric wave");k=40;}
Lattice(k,D,S);nP[0]=POLYGON;nP[1]=0;nP[2]=200;nP[3]=XYZ;
if(i==10) nP[1]=3; if(type==0) nP[2]=4; Plot(nP,S[9]);}
j=30;D[3]=D[0]+j*S[0];D[4]=D[1]+j*S[1];D[5]=D[2]+j*S[2];
SetPen(3,0x00ff);Draw("ARROW,0,2,XYZ,10",D);}
#v07=?>A
```

In general, we can write a wave in the form

$$y = \frac{A}{r^w}\sin(kr - \omega t)\,. \tag{30}$$

It is easy to get the dimension of the space in where the wave lives, the dimension is D=2w+1. Nevertheless, wave can be used to measure the dimension of space, just by determining the parameter w.

Waves all contain a core oscillation (vibration invariance)

$$\frac{d^2y}{dr^2}+k^2y=0\text{,} \tag{31}$$

Substituting y into the core oscillation, we obtain the radial wave equation

$$\frac{d^2y}{dr^2}+\frac{2w}{r}\frac{dy}{dr}+(k^2+\frac{w(w-1)}{r^2})y=0\cdot \tag{32}$$

This equation expresses the wave behavior modulated by the spatial dimension parameter w. For 1D wave w=0, it is trivial, but for 2D wave w=1/2, it reduces to the Bessel equation in a cylinder coordinate system (r,φ)

$$\frac{d^2y}{dr^2}+\frac{1}{r}\frac{dy}{dr}+(k^2-\frac{1}{4r^2})y=0 \quad \text{(2D wave)}$$

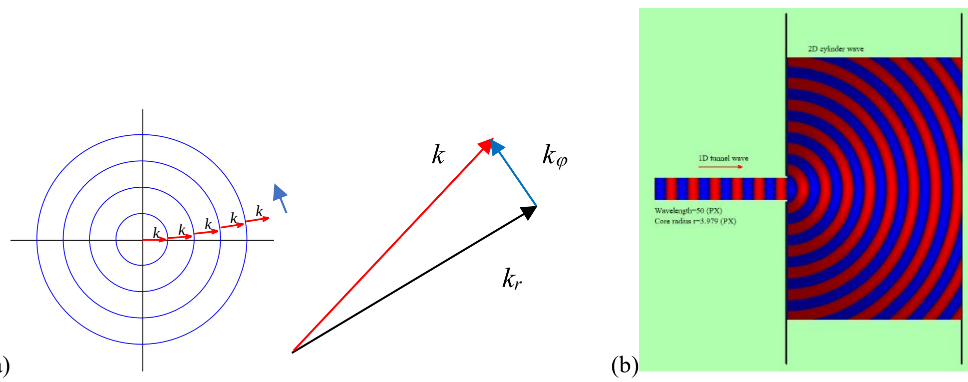
comparing to the Schrodinger's equation: . (33)

$$\frac{d^2R(r)}{dr^2}+\frac{2}{r}\frac{dR(r)}{dr}+[k^2-\frac{l(l+1)}{r^2}]R(r)=0$$

In quantum mechanics, y is an electronic wave function, comparing to the Schrodinger radial wave equation in textbooks, we find that the $-1/4r^2$ term represents the electronic spin effect. However, here according to the above radial Bessel equation we can simply conclude that sound wave, electromagnetic wave, or any wave can have spin effect in 2D space! Let us use ***k*** denote the wave-vector, then the above 2D wave equation tells us

$$k_r^{\ 2}=k^2-k_\varphi^{\ 2};\quad k=\frac{2\pi}{\lambda};\quad k_\varphi=\pm\frac{1}{2r}\cdot \tag{34}$$

The k_φ causes the 2D wave-vector ***k*** to spin little by little as illustrated Fig.8. The positive and negative k_φ corresponds to spin up and spin down respectively; as r goes to the infinity, the spin effect vanishes off.



(a) (b)

Fig.8 (a) 2D wave-vector k spins little by little in the cylinder coordinates (r,φ). (b) from 1D to 2D, the spin works to split the electron beam due to double-value k_φ.

```
<Clet2020 Script>// Clet is a C compiler [26]
int i,j,k;double r,x,y,a,D[100];
int main(){DrawFrame(FRAME_LINE,1,0xafffaf); SetPen(1,0x0000ff);
for(i=0;i<90;i+=20) {D[0]=-i;D[1]=-i;D[2]=i;D[3]=i; Draw("ELLIPSE,0,2,XY,0",D);}
for(i=0;i<90;i+=20) {a=0.2*(i-i*i/200)*PI/180;r=i;D[0]=r*cos(a);D[1]=r*sin(a);
r+=18;D[2]=r*cos(a);D[3]=r*sin(a); SetPen(2,0xff0000);
Draw("ARROW,0,2,XY,8",D);TextHang(D[2]-10,D[3]+5,0,"#ifk");}
}#v07=?>A
```

If the 2D wave is the de Broglie matter wave for a particle beam, in a cylinder coordinate (r,φ), then the matter wave has a spin angular momentum given by

$$k_r = \frac{p_r}{\hbar};\quad k_\varphi = \frac{p_\varphi}{\hbar} = \frac{J_\varphi}{r\hbar};\quad J_\varphi = \pm\frac{1}{2}\hbar \cdot \tag{35}$$

According to the angular momentum formula in general physics, it is recognized that the particle total momentum p is a constant given by

$$\begin{aligned} &\left(\frac{p}{\hbar}\right)^2 = \left(\frac{p_r}{\hbar}\right)^2 + \left(\frac{p_\varphi}{\hbar}\right)^2 . \\ &k^2 = k_r{}^2 + k_\varphi{}^2 = const. \end{aligned} \tag{36}$$

Since the particle total wave vector k is a constant, the wave-vector k_r must vary as r changes. The wave-vector in the radial direction would change as the wave attenuates.

6. How matter wave to obtain atomic spin

As shown in Fig.9(a), if the coherent length of the electron in a hydrogen atom is long enough, it will wind around the time axis, the matter wave in the circle will overlap, and interfere with itself at every location. The overlapping number N depends on the coherent length L; if $N=\infty$, the matter wave has the interference like the Fabry-Perot interference in optics.

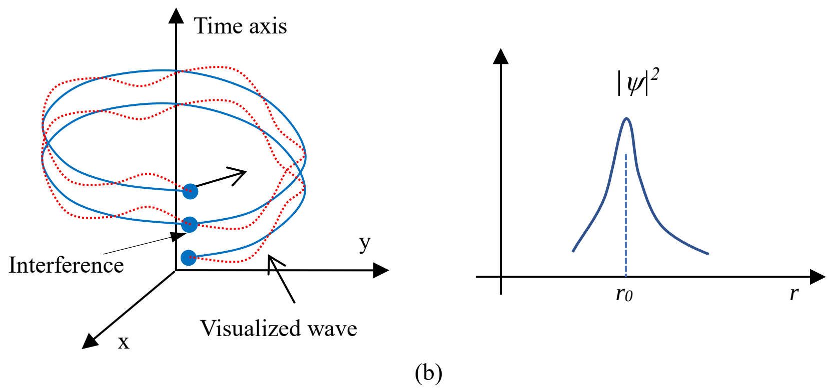


Fig.9 (a)The matter wave winds around the time axis, overlap and interfere with itself. (b)The distribution of overlapped 2D matter wave about its Bohr' radius.

The overlapped matter wave is given by

$$\begin{aligned} &\psi = w + we^{i\delta} + we^{i2\delta} + ...we^{i(N-1)\delta} = w\frac{1-\exp(iN\delta)}{1-\exp(i\delta)} . \\ &\delta = \frac{1}{\hbar}\oint_L pdl \end{aligned} \tag{37}$$

where, δ is the phase shift after one circle retardation for the matter wave. Obviously, the Bohr orbits can survive only if the denominator of the above equation is satisfied by

$$\delta = \frac{1}{\hbar}\oint_L pdl = 2\pi n;\quad n = 1, 2, 3, ... \cdot \tag{38}$$

For a 2D Bohr orbit, in the φ direction and in the r direction, the 2D wave function expresses as $\psi=\Phi(\varphi)R(r)$. The self-interference of the winding matter wave would enhance its attenuation about the Bohr radius r_0 in the radial direction, shown in Fig.9(b). For the slope-up side and the slope-down side about the r_0, as shown in Fig.10, it easily estimates the wave function in a composite state as follows

$$\begin{aligned} r \le r_0: \quad & R(r,t) \approx (\frac{r_0}{r})^{-j_1} (\frac{r_u}{r})^{1/2} \sin(kr - \omega t) \\ r > r_0: \quad & R(r,t) \approx (\frac{r_0}{r})^{j_2} (\frac{r_u}{r})^{1/2} \sin(kr - \omega t) \end{aligned} \quad . \tag{39}$$

where the parameter j determines the slope of the attenuation in the r direction, the half number 1/2 is prepared for the spin concept.

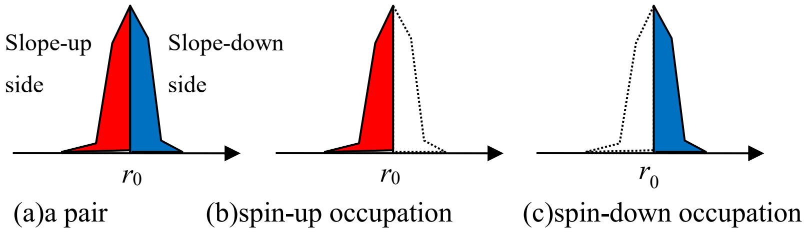


Fig.10 The orbit accommodates one spin-up and another spin-down electrons.

To note that the vibration-function is invariant core oscillation

$$(\frac{d^2}{dr^2} + k^2)R = 0 \quad . \tag{40}$$

which holds vibration invariance by keeping k=constant. Substituting R into it, these wave functions actually have to satisfy the Bessel-like equations:

$$\begin{aligned} & r \le r_0 : w = -j_1 + \frac{1}{2}, \\ & \frac{d^2R}{dr^2} + \frac{2(-j_1 + \frac{1}{2})}{r}\frac{dR}{dr} + [k^2 + \frac{(j_1 + \frac{1}{2})(j_1 - \frac{1}{2})}{r^2}]R = 0 \\ & r > r_0 : w = j_2 + \frac{1}{2}, \\ & \frac{d^2R}{dr^2} + \frac{2(j_2 + \frac{1}{2})}{r}\frac{dR}{dr} + [k^2 + \frac{(j_2 + \frac{1}{2})(j_2 - \frac{1}{2})}{r^2}]R = 0 \end{aligned} \quad . \tag{41}$$

Because the total wave vector k^2 is a constant, these equations become governing equations that dominate the wave behavior. Their solutions are approximately given by

$r \le r_0$:

$$R(r,t) = (\frac{r_0}{r})^{-j_1} (\frac{r_u}{r})^{-1/2} \sin(r\sqrt{k_r^{\ 2} - \frac{(j_1+\frac{1}{2})(j_1-\frac{1}{2})}{r^2}} - \omega t + \phi) + O(\frac{1}{r^{3+j}})$$

$r > r_0$:

$$R(r,t) = (\frac{r_0}{r})^{j_2} (\frac{r_u}{r})^{1/2} \sin(r\sqrt{k_r^{\ 2} - \frac{(j_2+\frac{1}{2})(j_2-\frac{1}{2})}{r^2}} - \omega t) + O(\frac{1}{r^{3+j}})$$

(42)

The slope-up range accommodates the spin-up electron; the slope-down range accommodates the spin-down electron; thus, one orbit accommodates a pair of electrons.

Comparing to the Schrödinger wave equation, it is easy to find that j takes place at the site "orbital angular momentum quantum number". In a spherical 3D coordinate (r,θ,φ), we find that j was called as the quantum number in the θ direction. Indeed, here the attenuating parameter j would automatically be quantized in this quantum system as it plays the role of angular momentum in the above equations.

We argue that there is not motion in the θ direction in our 2D planet model in the cylinder coordinates $(r,\ \varphi)$; the j is just an attenuating parameter of the composite matter wave about its Bohr radius r_0, depending on the overlapping number N. So that the attenuating parameter j is a disguised freedom, because of this disguised freedom, the electron motion in atoms was mistakenly explained as 3D motion for a hundred years.

Experimental evidence shows: magnetic needles would flip in a magnetic field; the magnetic moment corresponding to the motion in the θ direction in the 3D mode cannot been flipped in external magnetic field, this experimental observation indicates that the atomic angular momentum in the θ direction in 3D model does not exist; only the attenuating parameter j enable to allow the magnetic moment no-flipping in external field, because the j represents a disguised freedom.

Electron in atoms is 2D motion like in planetary orbits, but for a hundred years the electron motion in atoms was mistakenly explained as 3D motion in modern physics, actually this big mistake had made serious harm to the modern physics.

How to calculate atomic magnetic moment? If the orbit accommodates a pair of electrons, the two electrons must chase each other in the φ direction, because heading to each other would collide with each other frequently and exhaustedly. The total angular momentum should take both contributions into account as follows

$$\begin{aligned} J_\varphi &= \sqrt{(j_1+\frac{1}{2})(j_1-\frac{1}{2})}\hbar \pm \sqrt{(j_2+\frac{1}{2})(j_2-\frac{1}{2})}\hbar \\ &= m\hbar; \quad m = 0, \pm 1, \ldots \end{aligned} \tag{43}$$

Experiments tell us there is not magnetic moment for the pair of electrons, that is

$$J_\varphi = 0; \quad j_1 = j_2; \quad m = 0; \tag{44}$$

Although the total angular momentum and total magnetic moment are zero, the electrons both still have their chasing motion in the φ direction (if parameter $j_2+1/2$ is defined as the positive direction, then parameter $-j_1+1/2$ is defined as the negative direction), i.e. their combined spin angular momentum is invisible for external magnetic field.

If the orbit accommodates a single spin-down electron, its angular momentum is

$$J_{\varphi}=\sqrt{(j_2+\frac{1}{2})(j_2-\frac{1}{2})}\hbar=m\hbar\ . \tag{45}$$

where, the electron must subject to the orbital quantization conditions

$$\begin{aligned}&\frac{2}{\hbar}\int_{r_{\min}}^{r_{\max}}p_r dx=2\pi n_r,\quad n_r=0,1,2,...\\&\frac{1}{\hbar}\int_0^{2\pi}p_{\varphi}rd\varphi=2\pi m,\quad m=\pm1,\pm2,...\end{aligned}\ . \tag{46}$$

The single electron spin is towed up by the dimension parameter j_2. The atomic magnetic moment is given by

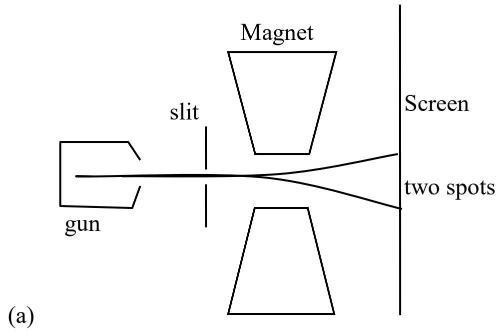
$$\mu=\frac{e}{2m_e}J_{total}=\frac{e}{2m_e}J_{\varphi}=\frac{e}{2m_e}m\hbar\ . \tag{47}$$

$$J_{total}=J_{\varphi}=m\hbar=[(m-\frac{1}{2})\hbar+\frac{1}{2}\hbar]=[(m-\frac{1}{2})\hbar+|J_{spin}|]\ . \tag{48}$$

The spin angular momentum is enveloped by the total angular momentum.

7. Stern-Gerlach experiment and Lander factor

In general, it is hard in macroscopic scale to separate the left-turn electrons and right-turn electrons in a 2D matter wave, except under certain conditions in some deliberately designed magnetic apparatus. In Stern-Gerlach experiments, as shown in Fig.11(a), silver (Ag) atoms are heated in an oven, the oven has a small hole through which some silver atoms escape. The silver vapor busts out the oven and go through a slit as the collimator and is then subjected to an inhomogeneous magnetic field. In this experiment, the single valent election of silver atom moves in its Bohr orbit, as shown in Fig.11(b), we adopt a cylinder coordinates (r,φ) with the origin at the sliver center.



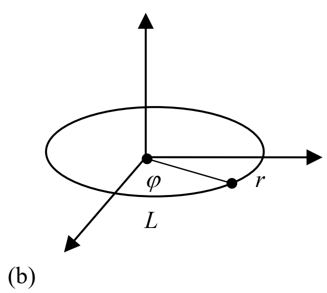


Fig.11 The Stern-Gerlach experiment apparatus.

As shown in Fig.12(a), if the coherent length of the electron in a hydrogen atom is long enough, it will wind around the time axis, the matter wave in the circle will overlap, and interfere with itself at every location. The overlapping number N depends on the coherent length L; if $N=\infty$, the matter wave has the interference like the Fabry-Perot interference in optics.

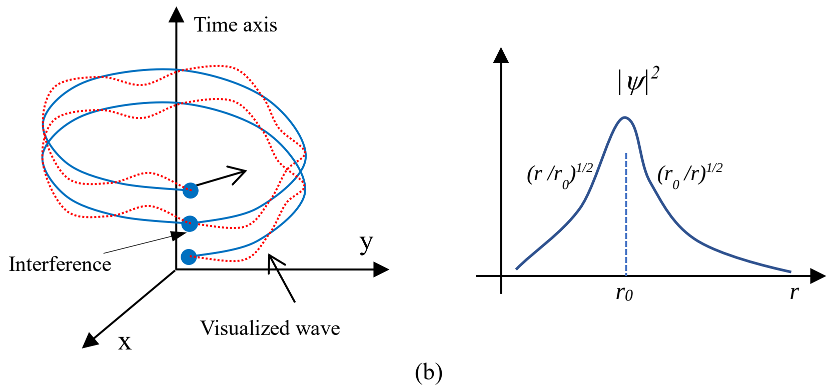


Fig.12 (a)The matter wave winds around the time axis, overlap and interfere with itself. (b)The distribution of overlapped 2D matter wave about its Bohr' radius.

The overlapped matter wave is given by

$$\psi = w + we^{i\delta} + we^{i2\delta} + ...we^{i(N-1)\delta} = w\frac{1-\exp(iN\delta)}{1-\exp(i\delta)} . \quad (49)$$
$$\delta = \frac{1}{\hbar}\oint_L pdl$$

where, δ is the phase shift after one circle retardation for the matter wave. Obviously, the Bohr orbits can survive only if the denominator of the above equation is satisfied by

$$\delta = 2\pi n; \quad n = 1, 2, 3, ... \quad (50)$$

For a 2D Bohr orbit, in the φ direction and in the r direction, the 2D wave function is written as $\psi=\Phi(\varphi)R(r)$, consider a simplest case as shown in Fig.12(b), the overlapped matter wave of a silver atom with a finite N is approximately given by

$$\Phi(\varphi) = \exp(\frac{ip_\varphi}{\hbar}r\varphi); \quad R(r) = \begin{cases} \sqrt{\frac{r_0}{r}}\exp(\frac{ip_r}{\hbar}r) = \sqrt{\frac{r_0}{r}} & p_r \simeq 0 \quad r > r_0 \\ \sqrt{\frac{r}{r_0}}\exp(\frac{ip_r}{\hbar}r) = \sqrt{\frac{r}{r_0}} & p_r \simeq 0 \quad r < r_0 \end{cases} . \quad (51)$$

Although $p_r=0$, the fact was omitted by almost all of us, the wave function in a 2D cylinder wave ($r>r_0$) satisfies

$$\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr} + [k^2 + \frac{\frac{1}{2}(\frac{1}{2}-1)}{r^2}]R = 0 . \quad (52)$$

Because the single valent election of silver atom lives in a 2D orbital space, acquiring the extra spin angular momentum, the Ag atom has a magnetic moment

$$J_{spin} = \pm\frac{1}{2}\hbar \quad \Rightarrow \quad \mu = g\frac{e}{2m}J_{spin} . \quad (53)$$

where g is the Lander factor, g=2. Because the interaction energy of the magnetic moment with the magnetic field is just -μB, the z-component of the force experienced by the silver atom is given by

$$f_z = \frac{\partial}{\partial z}(\mu B) = \mu \frac{\partial B}{\partial z}. \tag{54}$$

where we have ignored the components of B in direction other than the z- direction. In the z direction, the silver atom beams, 50% atoms experience an upward force, and other 50% atoms experience a downward force, thus on the screen we get the view there are two spots, in agreement with the theoretical prediction.

Why did physics introduce the Lander factor? In fact, there are two kinds of angular momentum we should consider in the φ direction

$$\begin{aligned} J_{total} &= J_{\varphi} = \pm\hbar \\ J_{spin} &= \pm\frac{1}{2}\hbar \end{aligned}. \tag{55}$$

The spin needs to occupy the half of the total angular momentum in this case, in other words the total angular momentum contains the spin angular momentum, that is

$$J_{total} = J_{\varphi} = \pm(\frac{1}{2}\hbar + \frac{1}{2}\hbar) = \pm(\frac{1}{2}\hbar + |J_{spin}|). \tag{56}$$

In other words, the spin angular momentum is enveloped by the total angular momentum. Thus, we obtain the normal magnetic moment which should be

$$\begin{aligned} &\mu = \frac{e}{2m} J_{total} = \frac{e}{2m} J_{\varphi} = \frac{e}{2m} 2J_{spin} = \frac{e}{2m} gJ_{spin}. \\ &\therefore \quad g = 2 \end{aligned} \tag{57}$$

The external magnetic field can only probe the total magnetic moment, but fails to directly detect the enveloped spin angular momentum, we need the Lander factor to grip on the spin concept.

8. Conclusions

This year is 99th anniversary of the initiative of de Broglie's matter wave, it is a good time for rediscovering the matter wave. In analogy with the ultimate speed c, there is an ultimate acceleration β, nobody's acceleration can exceed this limit β, for electrons and quarks, β=2.327421e+29(m/s2). Because this ultimate acceleration is a large number, any effect connecting to β will become easy to test. In this paper, an approach is put forward to connect the ultimate acceleration with quantum theory, consequently, the matter wave can bear the spin concept. This paper also carefully explains how the matter wave to display its spin effect in Stern-Gerlach experiments. It is completely a new aspect to quantum mechanics for the relativistic matter wave to contain spin.

References

[1]C. Marletto, and V. Vedral, Gravitationally Induced Entanglement between Two Massive Particles is Sufficient Evidence of Quantum Effects in Gravity, Phys. Rev. Lett., 119, 240402 (2017)

[2]T. Guerreiro, Quantum effects in gravity waves, Classical and Quantum Gravity, 37 (2020) 155001 (13pp).

[3]S. Carlip, D. Chiou, W. Ni, R. Woodard, Quantum Gravity: A Brief History of Ideas and Some Prospects, International Journal of Modern Physics D, V,24,14,2015,1530028. DOI:10.1142/S0218271815300281.

[4]de Broglie, L., CRAS,175(1922):811-813, translated in 2012 by H. C. Shen in Selected works of de Broglie.

[5]de Broglie, Waves and quanta, Nature, 112, 2815(1923): 540.

[6]de Broglie, Recherches sur la théorie des Quanta, translated in 2004 by A. F. Kracklauer as De Broglie, Louis, On the Theory of Quanta. 1925.

[7]NASA, https://solarscience.msfc.nasa.gov/interior.shtml.

[8]NASA, https://nssdc.gsfc.nasa.gov/planetary/factsheet/marsfact.html.

[9]B. Ryden Introduction to Cosmology, Cambridge University Press, 2019, 2nd edition.

[10]D. Valencia, D. D. Sasselov,R. J. O'Connell, Radius and structure models of the first super-earth planet, The Astrophysical Journal, 656:545-551, 2007, February 10.
[11]D. Valencia, D. D. Sasselov,R. J. O'Connell, Detailed models of super-earths: how well can we infer bulk properties? The Astrophysical Journal, 665:1413–1420, 2007 August 20.
[12]T. Guillot, A. P. Showman, Evolution of "51Pegasusb-like" planets, Astronomy & Astrophysics,2002, 385,156-165, DOI: 10.1051/0004-6361:20011624
[13]T. Guillot, A. P. Showman, Atmospheric circulation and tides of "51Pegasusb-like" planets, Astronomy & Astrophysics,2002, 385,166-180, DOI: 10.1051/0004-6361:20020101
[14]L.N. Fletcher,Y.Kaspi,T. Guillot, A.P. Showman, How Well Do We Understand the Belt/Zone Circulation of Giant Planet Atmospheres? Space Sci Rev, 2020,216:30. https://doi.org/10.1007/s11214-019-0631-9
[15]Y. Kaspi, E. Galanti, A.P. Showman, D. J. Stevenson, T. Guillot, L. Iess, S.J. Bolton, Comparison of the Deep Atmospheric Dynamics of Jupiter and Saturn in Light of the Juno and Cassini Gravity Measurements,Space Sci Rev, 2020, 216:84. https://doi.org/10.1007/s11214-020-00705-7
[16]Orbital Debris Program Office, HISTORY OF ON-ORBIT SATELLITE FRAGMENTATIONS, National Aeronautics and Space Administration, 2018, 15 th Edition.
[17]M. Mulrooney, The NASA Liquid Mirror Telescope, Orbital Debris Quarterly News, 2007, April,v11i2,
[18]Orbital Debris Program Office, Chinese Anti-satellite Test Creates Most Severe Orbital Debris Cloud in History, Orbital Debris Quarterly News, 2007, April,v11i2,
[19]A. MANIS, M. MATNEY, A.VAVRIN, D. GATES, J. SEAGO, P. ANZ-MEADOR, Comparison of the NASA ORDEM 3.1 and ESA MASTER-8 Models, Orbital Debris Quarterly News, 2021, Sept,v25i3.
[20]D. Wright, Space debris, Physics today,2007,10,35-40.
[21]TANG Zhi-mei, DING Zong-hua, DAI Lian-dong, WU Jian, XU Zheng-wen, "The Statistics Analysis of Space Debris in Beam Parking Model in 78° North Latitude Regions," Space Debris Research, 2017, 17,3, 1-7.
[22]TANG Zhimei DlNG, Zonghua, YANG Song, DAI Liandong, XU Zhengwen, WU Jian The statistics analysis 0f space debris in beam parking model based On the Arctic 500 MHz incoherent scattering radar, CHINESE JOURNAL 0F RADIO SCIENCE, 2018, 25,5, 537-542
[23]TANG Zhimei，DING, Zonghua，DAI Liandong，WU Jian，XU Zhengwen, Comparative analysis of space debris gaze detection based on the two incoherent scattering radars located at 69N and 78N, Chin．J．Space Sci, 2018 38,1, 73-78. DOI:10.11728/cjss2018.01.073
[24]DING Zong-hua, YANG Song, JIANG hai, DAI Lian-dong, TANG Zhi-mei, XU Zheng-wen, WU Jian, The Data Analysis of the Space Debris Observation by the Qujing Incoherent Scatter Radar, Space Debris Research, 2018, 18,1, 12-19.
[25]YANG Song, DING Zonghua, Xu Zhengwe, WU Jian, Statistical analysis on the space posture, distribution, and scattering characteristic of debris by incoherent scattering radar in Qujing, Chinese Journal of Radio science, 2018 33,6 648-654, DOI:10.13443/j.cjors.2017112301
[26]Clet Lab, Clet: a C compiler, download at https://drive.google.com/file/d/1OjKqANcgZ-9V56rgcoMtOu9w4rP49sgN/view?usp=sharing
[27]Huaiyang Cui, Relativistic Matter Wave and Its Explanation to Superconductivity: Based on the Equality Principle, Modern Physics, 10,3(2020)35-52. https://doi.org/10.12677/MP.2020.103005
[28]Huaiyang Cui, Relativistic Matter Wave and Quantum Computer, Amazon Kindle eBook, 2021.
[29]Huaiyang Cui, Evidence of Planck-Constant-Like Constant in Five Planetary Systems and Its Significances, viXra:2204.0133, 2022.
[30]Huaiyang Cui, Approach to enhance quantum gravity effects by ultimate acceleration, viXra:2205.0053, 2022.
[31]N.Cox, Allen's Astrophysical Quantities, Springer-Verlag, 2001, 4th ed..
[32] S. E. Schneider, T. T. Arny, Pathways to Astronomy, McGraw-Hill Education, 2018, 5th ed.
[33] Huaiyang Cui, Ultimate Acceleration in Quantum Mechanics to Obtain Spin, viXra:2207.0149, 2022.
[34]Y. A. Cengel, J. M. Cimbala, Fluid Mechanics: Fundamentals and Applications, McGraw-Hill, 2014, 3th ed.