Debugging Relativity: Analyzing Special Relativity Theory's Zero Day Defect

Steven B. Bryant, College of Computing, Georgia Institute of Technology, Atlanta, GA
steven.bryant@gatech.edu

Abstract

Computing researchers perform detailed reviews and conduct independent analyses to identify Zero Day Defects in computing solutions and software products, defects that may exist despite the observation that the system is functioning properly and is otherwise producing correct results. This paper demonstrates that the same review and analytical skills used to identify Zero Day Defects in software reveal a Zero Day Defect in Einstein's Special Relativity Theory. Specifically, it places Einstein's expressions and equations into a visual mathematical context that illuminates their relationships and facilitates the unambiguous identification of the defect and its root cause. Like Zero Day software defects, the Special Relativity Zero Day defect only manifests in certain conditions, but otherwise the theory's equations produce good results. This analysis demonstrates the strength and importance of Zero Day Defect analysis to identify previously undetected problems, while simultaneously extending this important computational analysis technique to other scientific disciplines.

Introduction

Accepted as a scientific field in the 1950s, Computer Science is a relatively new discipline when compared to other disciplines like physics, biology, and chemistry. [Denning 2005, Dodig-Crnkovic 2002] It is built upon a foundation of "[mathematics,] computational science, systems, engineering, and design." [Denning 2005] Computer Science is unique in that the equations and theories of those other disciplines are often modeled or otherwise embedded into computing systems and software packages. [Denning 2005]

As software is developed, it undergoes some amount of testing (i.e., validation and verification) to ensure its reliability and fitness. [Adrion et al. 1982] Despite the degree to which a solution is validated and verified, computing researchers generally recognized the possibility of yet–to–be–discovered defects, which are commonly called bugs. [Bennett and Wennberg 2005, Bilge and Dumitraş 2012] While this belief is analogous to the idea that all theories of fallible – a belief held by scientists in other disciplines – computing takes this further by defining tools, techniques, and processes solely designed to identify defects before, during *and* *after* a solution is deployed. Some computing professionals are rewarded (e.g., "bragging rights", bounties) for identifying previously unknown software defects, while others may attempt to exploit such defects for nefarious purposes. [Krishnamurthy and Tripathi 2006, Sprague and Wagner 2018]

Computing researchers leverage detailed reviews and analytical tools, techniques, and processes to identify Zero Day Defects – bugs that have not yet been exploited and have not yet caused a visible failure in the solution. Zero Day Defects are difficult to detect because they are bugs in a system that has been tested and where everything appears to work as intended. Yet the defect is present and, in the right situation, will result in a failure or nefarious exploit.

This paper demonstrates the power and strength of Zero Day Defect Analysis by using its tools and techniques to reveal a Zero Day Defect in Einstein's Special Relativity Theory (SRT). Specifically, it presents an analytical tool to identify the problem, analyze where and why the defect exists, and explain why the defect has not resulted in a noticeable failure of the theory. Similar to how Zero Day software defect analysis does not require an intimate understanding of the software being evaluated, the Zero Day Defect analysis presented herein does not require an intimate understanding of Einstein's Special Relativity Theory.

Discussion

Introduced in 1905, Einstein's Special Theory of Relativity is one of the most well–recognized scientific theories ever created. [Einstein 1905] It has been widely reviewed and enjoys wide–ranging experimental support, and its equations have been derived in multiple ways, leading to a belief that the theory is mathematically sound and free of significant defect. Researchers have repeatedly demonstrated the theory's mathematical predictive usefulness. Additionally, some proponents believe that it (or one of its related derivatives) is the only theory capable of explaining certain experiments and observations, such as those related to Einstein's energy equation: $E = mc^2$. [Einstein 1905, Pakman 2012] With more than a century of support and no recognized experimental failure, some proponents no longer believe that the theory is fallible. However, as is the case with Zero Day software defects, prior validation and verification does not mean the theory is defect free.

Zero Day Defect Detection

Detection of software defects does not require the researcher to understand each line of written code. Similarly, the examination of Einstein's theory for a Zero Day Defect does not require the mastery of, or a thorough understanding of Special Relativity Theory. What is required is a disciplined approach that uses recognized, accepted tools and techniques. The primary tool that will be used in this analysis is the *arithmetic mean*, more generally referred to as a mathematical average.

Mathematically, the arithmetic mean, τ, of two operands, t and s, is $\tau = \frac{1}{2}(t+s)$. Since $\frac{1}{2}t = t - \frac{1}{2}t$, this arithmetic mean can be equivalently written as: $\tau = t - \frac{1}{2}(t-s)$. Both equations are equivalent, even if the second equation is less familiar and not immediately recognized as an arithmetic mean. The former τ equation is called the *addition mean equation* because it uses the addition operator. The latter τ equation, which Sutton and Barto [2018] refer to as the incremental mean with $n = 2$, is herein called the *subtraction mean equation* because it uses the subtraction operator. [Bryant 2016]

Once the tool is identified, it must be calibrated and/or validated. Since the arithmetic mean equations are well–recognized and accepted, and this analysis does not evaluate a dataset, calibration is not required. Validation is performed by demonstrating that both arithmetic mean equations will properly solve a recognizable and easily verified problem.

Tool Validation

The following scenario is adapted from Bryant [2022]. Imagine a street; placed on the street is a bus of length x'; also placed on the street next to the vehicle's rear bumper is a jogger. When the bus is stationary and the jogger moves at a constant velocity c, the time for the jogger to travel from the bus's rear bumper to its front bumper is $\frac{x\prime}{c}$. This is the Jogger's *baseline transit time*.

When the bus moves forward at a constant velocity v and the jogger moves at constant velocity c, the time for the jogger to travel from the bus's rear bumper to its front bumper is greater than the baseline transit time. Mathematically, this time is found using the expression $x'/(c-v)$.

Similarly, when the vehicle is moving forward at velocity v, the time for the jogger to travel from the vehicle's front bumper to its rear bumper is less than the baseline transit time. Mathematically, this time is found using the expression $x'/(c+v)$.

Notice that above c is simply the variable that represents the jogger's velocity and does not suggest that the jogger is moving at the speed of light. Additionally, we consider the case where $v < c$ alone such that the range of v is $[0, c)$. Validation is performed by confirming that both equations will correctly produce the arithmetic mean given the expressions above.

Question 1: What is the average transit time?

We must now validate the use of the tool by confirming its ability to properly answer questions about the expressions just presented. Independent of any meaning associated with the expressions, when $t \leftarrow x'/(c-v)$ and $s \leftarrow x'/(c+v)$ are substituted into the addition mean equation presented earlier, the equation is simplified as: $\tau = cx'/(c^2 - v^2)$. When x', v, and c, are known, this equation is concretely solved. While we will arrive at the same equation using the subtraction mean equation, we will intentionally solve it in two explicit steps. First, we solve the expression $\frac{1}{2}(t-s)$, which is: $vx'/(c^2 - v^2)$ [**Eq. 1**], followed by rewriting the partially solved subtraction mean equation as: $\tau = t - vx'/(c^2 - v^2)$ [**Eq. 2**]. Second, we replace t (in the partially solved equation) with its corresponding expression $x'/(c-v)$ [**Eq. 3**]. As expected, when simplified, the subtraction mean equation also yields $\tau = cx'/(c^2 - v^2)$.

Question 2: What is the average transit length?

This average transit length question is solved by multiplying the mean time τ by the jogger's velocity c. Since $\xi = c\tau$ [**Eq. 4**], this average transit length ξ is readily found as: $\xi = c^2x'/(c^2 - v^2)$ [**Eq. 5**].

Notice that this tool will properly answer questions if additional complexity is introduced. For example, if we are not explicitly given the length of the bus as x' but are instead given that 1) at time 0 the rear of the bus was a position 0, and 2) at time t the position of the front of the bus is at x. Thus, the length of the bus is found as $x' = x - vt$. While t is an overloaded variable because it was used here and in both mean equations discussed above, we can safely use it here since we have already found the arithmetic mean and the variables t and s are no longer in use. Despite this final complexity, the equality of the statement $\xi = c\tau$ is always maintained, as required by mathematical rules, regardless of the values of x, c, v, and t. It is important to notice that the creation and validation of this assessment tool required no knowledge, understanding, or application of Special Relativity Theory.

Analysis & Vulnerability Detection

Like the creation of the assessment tool, the application of the assessment tool does not require an understanding of Special Relativity Theory. Instead, it requires observation of mathematical steps and an understanding of the rules of mathematics embedded in the creation of the assessment tool above. Discussed previously, this analytical approach mirrors techniques that computational researchers use to detect Zero Day software defects. As shown in Fig. 1, each of the five equations and expressions identified above while validating the assessment tool also explicitly appear in Einstein's derivation. Thus, prior to his implicit final adjustment where Einstein multiplies each equation by $\sqrt{1 - \frac{v^2}{c^2}}$, the variable ξ (Fig. 1, Circle 5) is simply the arithmetic mean of two lengths, sc and tc, found using the subtraction mean equation, as used above to validate the assessment tool. [Bryant 2022, Einstein 1905]

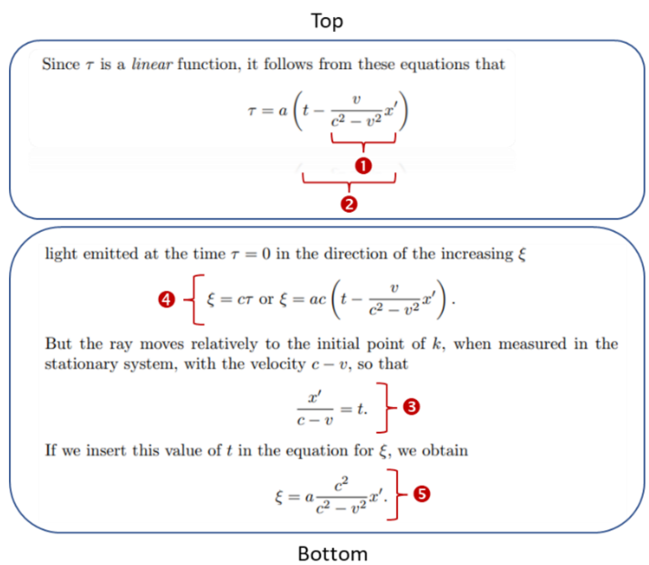

Figure 1. The five expressions and equations used to find ξ using the subtraction mean equation as discussed in the text are explicitly found in Einstein's Special Relativity Theory derivation, revealing that ξ is the average of sc and tc. Source: On The Electrodynamics of Moving Systems.[Einstein 1905] Translation from https://www.physics.umd.edu/courses/Phys606/spring_2011/einstein_electrodynamics_of_moving_bodies.pdf

Discussed during tool assessment validation, ξ can be found directly using the addition mean equation as, $\xi = \frac{1}{2}(tc + sc)$. As shown in Fig. 1, we have demonstrated that the critical elements of Einstein's relativity derivation can be solved using the arithmetic mean equations alone, which should raise an important question: *If the assessment tool yields the* *same equation as found in Einstein's derivation, doesn't that serve as validation of the theory?* To answer this question, consider Eq. 4, (see Fig. 1, Circle 4) which is a statement of "creation". Specifically, it says that ξ is *created* as a representation of the product $c\tau$. An important implication is that, mathematically, ξ must *always* equal $c\tau$. Adherence to this mandatory relationship requirement was demonstrated when the assessment tool was validated, above.

In contrast, when Einstein's final equations, shown in Fig. 2, are evaluated with $x = 1, v = 0$, and $t = 0$, they produce $\xi = 1$ and $\tau = 0$. [Bryant 2022] Critically important is that the mathematical relationship, whereby ξ must *always* equal $c\tau$, is not maintained. In this specific case, the equation $\xi = c\tau$ evaluates to $1 = 0$, which is a mathematical contradiction that represents the Special Relativity Theory Zero Day Defect. [Bryant 2022]

It follows from this relation and the one previously found that $\phi(v) = 1$, so that the transformation equations which have been found become

$$\begin{aligned} \tau &= \beta(t - vx/c^2), \\ \xi &= \beta(x - vt), \\ \eta &= y, \\ \zeta &= z, \end{aligned}$$

where

$$\beta = 1/\sqrt{1 - v^2/c^2}.$$

Figure 2. The final Special Relativity Theory equations. [Einstein 1905] When $x = 1, v = 0$, and $t = 0$, the system of equations produce $\xi = 1$ and $\tau = 0$, resulting in the $1 = 0$ contradiction. Translation from https://www.physics.umd.edu/courses/Phys606/spring_2011/einstein_electrodynamics_of_moving_bodies.pdf

An implication of this finding is that when $\xi = c\tau$ is maintained, the one–to–one correspondence between (x, y, z, t) and (ξ, η, ζ, τ) is not satisfied, invalidating the validation and proof found in section 3 of the SRT derivation. [Einstein 1905, Feynman *et al.* 2011] However, the existence of this defect and corresponding validation failure does not result in a degradation of the theory's predictive power but instead highlights the need to understand the root cause of the defect and identify an experimental edge case under which the defect can be observed.

As shown in Fig. 3, the root cause of this defect is that Einstein properly replaces t with $x'/(c - v)$ in the subtraction mean equation when completing and simplifying ξ, but fails to perform this replacement when completing and simplifying his stand–alone τ function. [Bryant 2016, Bryant 2022] This leads to Einstein conflating the overloaded variable t when simplifying the partially solved subtraction mean equation τ (where it represents one of the operands in the equation for the arithmetic mean) with the x' equation (where it is an independent variable). Overloaded variables are an important concept in the Computer Science Body of Knowledge because the scope rules of specific programming languages combined with the use of overloaded variables are often the source of difficult to locate software defects.

Since $\xi = c\tau$ must be true regardless of whether τ is found as a stand–alone equation or as the stand–alone ξ equation divided by c, this analytical technique confirms the Zero Day Defect in other SRT derivations, including: Einstein [1961] where the equation is presented as $x' - ct' = 0$, Steinmetz [1923] where it is presented as $x' = ct'$, and Einstein [2003] where it is presented as $\sqrt{x'^2 + y'^2 + z'^2} = ct'$. While the defect is present in other derivations such as Feynman, Leighton and Sands [2011] and Serway and Jewett [2004], its identification cannot be explicitly highlighted because they do not show all of the derivation's steps and the necessary mathematical statement is implied rather than explicitly stated. Other derivations, such as found in Mermin [2005], are sufficiently different from Einstein's original derivation that a different analytical tool than presented herein is required to identify the Zero Day Defect.

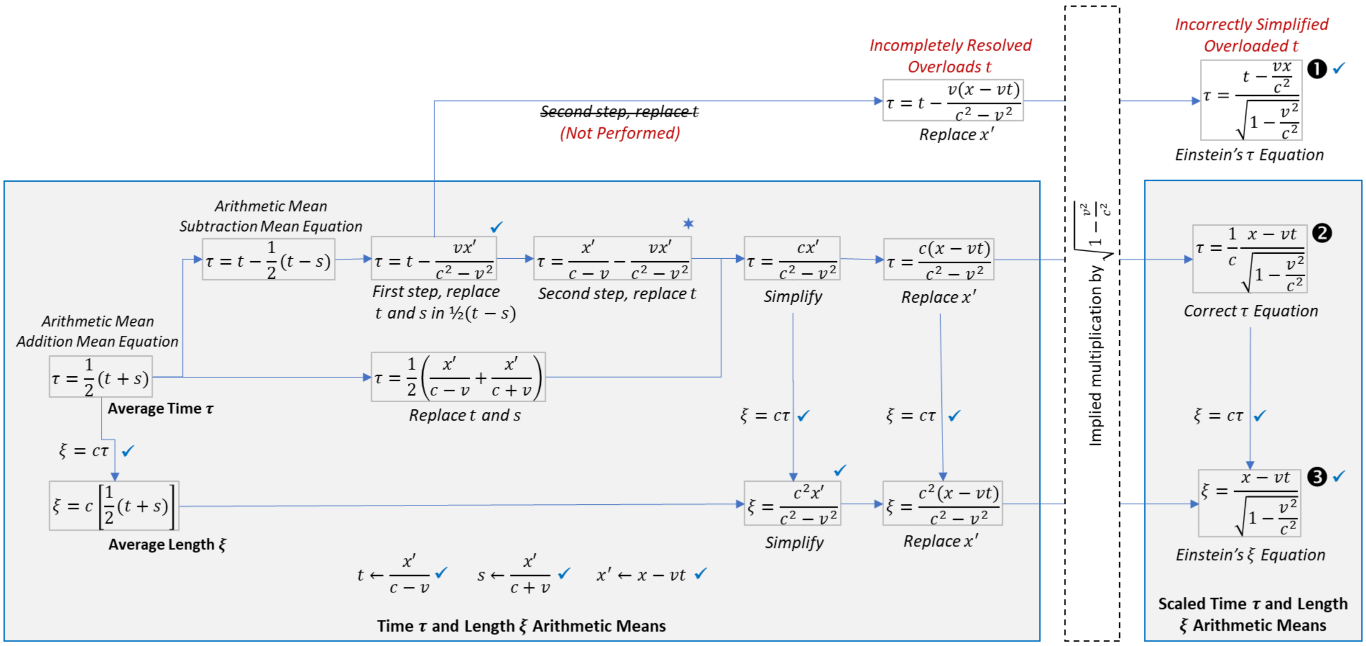

Figure 3. Within the same system of equations, Einstein's stand–alone scaled τ equation (❶) is mathematically inconsistent with the scaled ξ stand–alone equation (❸) created as $\xi = c\tau$. Specifically, when evaluated with $x = 1$, $v = 0$, and $t = 0$, the Zero Day Defect manifests in Einstein's scaled τ equation resulting in 0, while the correct scaled τ equation (❷) produces $\frac{1}{c}$ which mathematically aligns with the scaled ξ equation result of 1. Highlighted with the star (✶), the source of the Zero Day Defect is Einstein's "[insertion of] the value t" in the τ equation when creating the ξ equation but failure to make the same insertion when solving the stand–alone τ equation. [Einstein 1905] Equations and expressions with checkmarks explicitly appear in Einstein's SRT derivation. Einstein performs the implied multiplication without explanation.

It can be argued that Einstein's τ equation (Fig. 3, Circle 1) can be transformed into the scaled τ equation (Fig. 3, Circle 2) when the equation $x = ct$ is introduced, allowing t in Einstein's τ equation to be replaced by $\frac{x}{c}$ and x to be replaced by ct. Notice, however, that such substitutions are not generalizable. Specifically, the equivalence of Einstein's τ equation and the corrected scaled τ equation is only satisfied in the specific case when $x = ct$. Additionally, if these corrective substitutions are made in Einstein's final τ equation and the flow of the arrows in Fig. 3 are reversed, it is visually shown that these substitutions are equivalent to performing the omitted second step in the subtraction mean equation.

Eluding Detection

Although this defect has been present since Special Relativity Theory's inception, its existence, like many Zero Day Defects, is difficult to detect experimentally. Adding to the difficulty is the widely held belief that the theory and its accompanying system of equations is the only one capable of mathematically explaining certain experiments and observations. As shown in Fig. 4, the author has shown how a classical mechanics–based theory makes equal predictions for all experiments associated with validating Einstein's energy equation, $E = mc^2$. [Bryant 2016, Bryant 2022]

Summarizing the derivation steps as performed by Einstein [1905] and shown in Fig. 4, notice that x' is replaced with L, and ξ is multiplied by $\sqrt{1 - \frac{v^2}{c^2}}$. [Bryant 2016, Einstein 1905] In contrast, in the Classical Mechanics–based alternative, x' is replaced with $\frac{L}{2}$ and the implied multiplication is not performed. [Bryant 2016, Bryant 2022] While not explicitly stated in his derivation, Einstein implicitly sets Δ to the kinetic energy expression $\frac{1}{2}mv^2$ and simplifies the resulting equation as $L = mc^2$. Finally, L is replaced by E in the final step by modern convention. [Bryant 2016, Bryant

2022, Einstein 1905] Both theories begin as similar but non–equivalent equations that result in $E = mc^2$ due to the truncation of each series, demonstrating how a solution that contains a Zero Day Defect can elude detection. This finding of multiple, non–equivalent starting equations resulting in the same final equation suggests that the energy equation is properly written as $E \approx mc^2$. [Bryant 2016, Bryant 2022] It also suggests that, without the accompanying increased experimental precision that results from the use of the untruncated equations, it is unlikely that experimentation alone will reveal this defect.

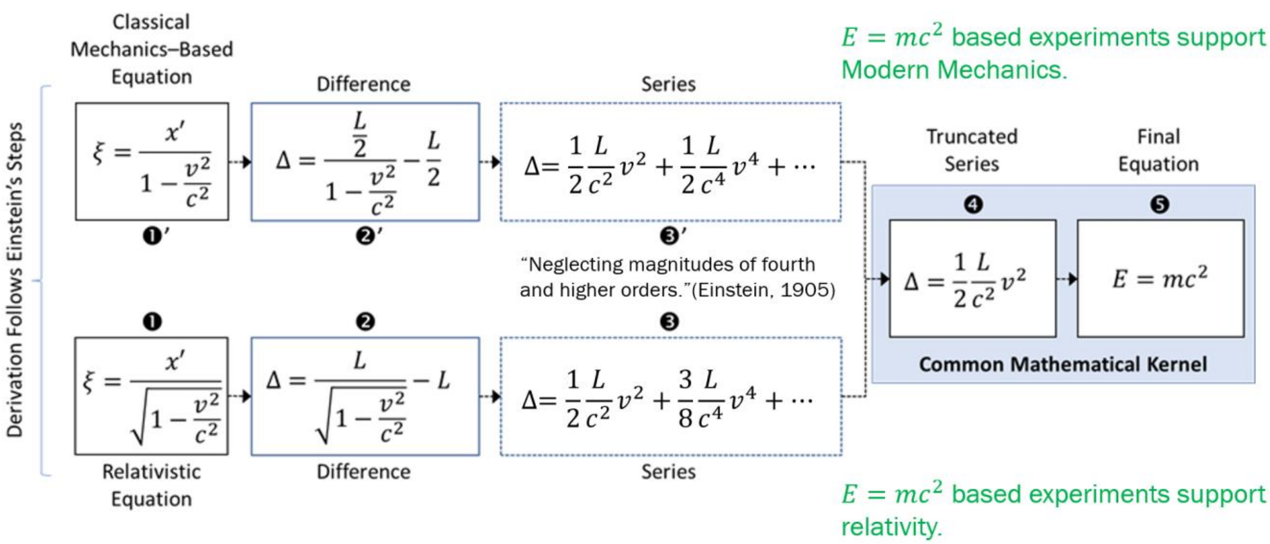

Figure 4. The Classical Mechanics based approach begins with the unscaled arithmetic mean length ξ and substitutes x' with $\frac{L}{2}$ while the relativistic equation begins with the scaled arithmetic mean length ξ and substitutes x' with L. Both equations then follow the steps in Einstein [1905] to arrive at the same final equation: $E = mc^2$. Source: Bryant [2022] Without the use of untruncated equations (prior to ❹), their predictions cannot be differentiated to determine if one of the theories can be experimentally excluded, further illustrating the difficulty of Zero Day Defect detection through experimentation alone.

Further notice that if an experiment uses $\frac{\xi}{c}$ or a variant of τ that does not explicitly include the exact numerator as found in Einstein's scaled τ equation, the defect can also elude detection. This ability for a solution – whether software or a scientific theory – to provide useful information and useful answers, despite containing a hidden defect, is the hallmark of a Zero Day Defect. However, as demonstrated above, the existence of a Zero Day Defect may not lead to an easily detectable or immediate failure or exploit without identification of the defect's root cause and creation of a specific edge case test or experiment.

Summary

This paper has demonstrated the importance of detailed reviews and independent analysis used in Computer Science by identifying a Zero Day Defect in Special Relativity Theory. It also illustrates a strength in the evaluation technique which emphasizes a reliance on understanding the assessment tool and analytical approach over an in–depth expert–level understanding of the material being examined. Specifically, this mathematical analysis did not require thorough expertise in the meaning of Special Relativity Theory. It instead required an understanding of the assessment tool, the ability to recognize mathematical equations as part of a detailed review, and the ability to adhere to recognized and accepted mathematical rules. In doing so, this paper found that, prior to scaling, Einstein's ξ equation is the arithmetic mean of two operands (specifically, two classical mechanics length equations) and that the numerator in the τ equation is incorrectly simplified due to an overloaded t variable.

Like many software Zero Day Defects, the Special Relativity Theory Zero Day Defect is hard to detect through experimental means alone unless a very exact and specific edge case results in a recognized and accepted failure. This finding also illustrates an important difference between the scientific method and Zero Day Defect detection. In the scientific method, expert review and experimental validation imply a defect free solution or theory, until demonstrated otherwise. In contrast, Zero Day Defect analysis presumes a defect exists in a validated, functioning solution or theory. This review and analysis technique specifically designed to uncover Zero Day Defects in functioning solutions and theories should be exported from Computer Science and incorporated into other scientific disciplines.

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