All possible roles for the TV host: the Monty Hall problem as a statistical experiment

Alessandro Renzi[1]

Abstract

I reflect on the Monty Hall problem (the probability puzzle, loosely based on the American television game show Let's Make a Deal), and some variant of it, believing that a specific point of view can present some useful connections with the fundamental problems about quantum mechanics from the point of view of the great questions posed by EPR paradox.

Keywords: Monty Hall; conditional probability; entanglement

1 Introduction

21 is a 2008 film based on the events of the "MIT Blackjack Team", a group of MIT students who between 1980 and 1990 broke through numerous casinos by resorting to card counting in blackjack.

In one scene from the film the professor (Kevin Spacey) challenges his students to answer the so-called Monty Hall problem correctly, and the brightest student and protagonist in the film (Jim Sturgess) answers correctly (using the concepts of conditional probability) by indicating the correct probability of winning a car rather than a… goat!

In this little discursive article, I share with those who will read me some of my reflections on the Monty Hall problem, reflecting on a variant of it, which I believe to be of some interest due to its possible connections with some fundamental problems about quantum mechanics from the point of view of the great questions posed by EPR paradox.

This paper is organized as follows:

- The classical Monty Hall problem
- The player's point of view
- A particular player arrives
- The TV host must come up with something!
- Trying to draw conclusions
- Then?

[1] Master's Degree in Physics at the University of Rome Tor Vergata, independent researcher and “trainee” citizen scientist, contacts are welcome at axrenzi71@gmail.com

2 The classical Monty Hall problem

The Monty Hall problem (or Monty Hall paradox) is a famous probability theory problem, related to the US prize game Let's Make a Deal. It takes its name from that of the TV show's host, Maurice Halprin, known under the pseudonym of Monty Hall. The problem is also known as the Monty Hall paradox, since the solution may appear counterintuitive, but it is not a true antinomy, as it does not generate logical contradictions.

In the game three closed doors are shown to the player; behind one is a car, while each of the other two hides a goat. The player can choose one of the three doors, winning the corresponding prize. After the player has selected a door, but hasn't opened it yet, the TV host - who knows what is behind each door - opens one of the other two, revealing one of the two goats, and offers the player the chance to change his initial choice, passing to the only remaining door; changing the door improves the player's chances of winning the car, taking them from 1/3 to 2/3. Although intuition may lead to considering the probability of victory linked to the initial choice of the player unchanged, this can be demonstrated in a rigorous way with a logical-mathematical analysis of the scenarios resulting from the initial choice of the player, and how the possible final events (car or goat) undergo a restriction according to the action of the TV host who opens one of the doors not chosen by the player.

In the Monty Hall problem, the probability of winning therefore rises from the initial 33.3% to 66.7% if the initial choice is changed. If the player keeps the choice, the probability of winning remains at 33.3%.

In summary, the initial choice is worth 33.3%, the conductor's action resets the probability of a gate by redistributing 100% on the remaining two, and this 100 is distributed unevenly between the gate initially chosen (which remains at 33.3 %) and the option available for the change (which is worth 66.7%).

In a variant of the problem, the TV host does not know what is behind the doors. After the player’s choice, the host opens one of the two remaining doors. Since he doesn't know what's behind it, with probability 1/3 he finds the car and the game is over. With probability 2/3 he finds the goat and can ask the player if he wants to make the change with the door closed. In this case, accepting the trade does not increase the player’s probability of winning which at this point is 1/2 regardless of his decision.

In the changed Monty Hall problem, the probability of victory therefore rises to 50%, but remains the same regardless of any change made by the player. In summary, the initial choice is 33.3%, the host’s action can reset the probability of a gate by redistributing 100% on the remaining two, and this 100 is distributed evenly between the gate initially chosen and the one that can be optioned with the possible change. We have a probability of victory - if the player can evaluate to make the change - given at 50% whether or not the change of the initial choice actually takes place.

3 The player's point of view

Let's consider the scenario in which the player does not know whether or not the TV host knows what is behind the three doors. Furthermore, we also place the host behind the three doors, so that the player knows nothing about the host. Clearly in the case of the "standard" problem we assume that the host can see what is behind the three doors and therefore can adjust accordingly when he opens one. In the "variant" to the problem, however, we assume that the host, despite being behind the doors with respect to the player - and therefore invisible to the latter - does not have the opportunity to know what is behind each door; and let's imagine that in order to choose which door to open, a roll of the dice is used: if an even number comes out, it will open the door on the left between the two not chosen by the player, if it comes out odd it will open the one on the right.

The player participates in the game without knowing anything about the host, knowing only that he will intervene at some point, but ignoring - the player - whether or not the host knows what is behind the doors.

If the player is well versed in probability, or perhaps if he has seen film ***21***, he will know that he has a different probability of victory in the face of the same starting situation and in the face of the same sequence of events (his initial choice, action of the host) up to the final decisive phase, and that this different probability depends on the level of knowledge or ignorance that the host has with respect to the given starting situation. But the player does not know this and therefore, although it is in any case convenient to change the choice to win the car (the change would be irrelevant if it goes wrong), he will not be able to know if the host knew or did not know what was behind the doors.

4 A particular player arrives

It happens that the player of the episode of the day, arriving at the TV studio, quickly loses interest in the stakes (the machine) and begins to reflect on the question of whether he is able to find out if the host knows or does not know what is behind the three doors.

He then challenges the host: I can understand whether or not you know what's behind the doors, but you have to accept to do what I propose; clearly I will not investigate you to find out if you can see what is behind the doors nor will I investigate how you choose to open one door or the other; I will simply play your program but I will place one and only one condition that it respects 100% the rules of your program and that you must accept in advance. The host, sure of his facts (and trusting in a surge in the television audience of his program) unwisely accepts! At which the player expresses his optional condition: to repeat the bet n times, where n is a statistically significant number.

Having accepted, the host cannot hold back… he is forced to play.

Consequently, the player, having played the program n times (always in the same way, i.e., either changing or not changing his initial choice but using the exact same strategy in each of the n times), is finally able to experimentally calculate the statistical frequency of his wins (33.3%, 50%, 66.7% the possible results) and can say with certainty whether or not the host knew what was behind the three doors.

5 The TV host must come up with something!

The host is in crisis, together with his program, given the setback suffered ... he must invent something to recover from the TV audience ... idea? The host decides to change the rules of the game, that is, to go to a point in the studio where he cannot see what the player chooses.

In the "standard" version of the new game the host still knows what's behind the doors, and when he opens his door (one of those that doesn't hide the car) two things can happen:

> 1) opens the door that was selected by the player, then the game ends; the probability of this happening is calculable; the competitor loses because the door certainly hides a goat,
>
> 2) opens a door other than the one that was selected by the competitor, then the game continues; also, the probability of this happening is calculable; the competitor can decide whether to change his initial choice and then open the door and see if it hides a goat or the car.

In the "variant" version - also of the new game – the host doesn't even know what's behind the doors, and when he opens his door two things can happen:

1) opens the door that was selected by the player ... car or goat the game ends; the probability of this happening is calculable,
2) opens a door other than the one that was selected by the player ... also the probability of this happening is calculable; the game continues unless the door opened by the host does not hide the car (defeat for the player).

Let's think about the cases where the game can continue after the action of the TV host:

• in the "standard" version of the new game we arrived at the situation in which the host opened a door that hides a goat and by chance wanted it not to be the one chosen by the competitor; we arrived at a situation similar to the standard version let's say "in schedule" until the day before the arrival of the particular competitor ... but we know that it was not the only possible situation, since the game could end early - with the defeat of the competitor - if the handler had accidentally opened the door selected by the competitor.

• In the "variant" version of the new game we have reached the situation in which the host has opened a door that could hide the car; if this were the case, the game could end prematurely with the player's victory.

6 Trying to draw conclusions

We have four different situations, with different and distinguishable statistical results in case of repetition of the episode N times:

The host knows the contents of the gates and sees the player’s choice	The host does not know the contents of the gates and sees the player's choice	The host knows the contents of the gates and does not see the player's choice	The host does not know the contents of the gates and does not see the player's choice
The game cannot end before the player's second choice; probability of victory prevailing in the event of a "change"	The game cannot end before the player's second choice; probability of victory unchanged in case of "change"	The game can end before the player's second choice; in this case the player would surely lose	The game can end before the player's second choice; in this case the player would surely win
deterministic scenario with bond (determinism + entanglement)	*probabilistic scenario with bond (probability + entanglement)*	*deterministic scenario without bond (determinism + no entanglement)*	*probabilistic scenario with bond (probability + no entanglement)*

In essence, therefore, we can deduct from a statistical measure whether the system is governed by a deterministic rule (the same type of process, repeated on the same system with identical starting conditions, produces identical results) or not (the same type of process, repeated on the same system with identical starting conditions, can lead to different results). And whether the system has entanglement between the two actors within the system itself.

7 Then?

The question at this point is the following: is it conceivable a physical system in which to experimentally represent the above situation to be able to claim to be able to perform a test on the role in the system by an "actor" with respect to the observer, an "actor" who may act in a hidden way on the system being observed by the observer? In other words, to test whether the system - impenetrable to us - is governed by deterministic or inherently probabilistic mechanisms? And with the verification of a possible bond (entanglement) between actor and observer?