# A Proposal for More Economic Fuel Use at Lagrange Points

**Filip Kozarski***

### Abstract

This paper demonstrates why – regarding fuel consumption – it is more senseful to perform stationkeeping at Lagrange point as often as possible, i.e. when thrust needed is greater than $12\ \mathrm{cm/s}$ for the James Webb Space Telescope. Fuel can be saved by striving to correct the orbit each time as early as manageable.

## Introduction

An interesting placement of telescopes in space are Lagrange points. One of them, namely Sun-Earth's L2 point has been chosen as most appropriate for the JWST, James Webb Space Telescope (NASA 2022). This stationary position is unstable (Goldstein 2002), therefore small perturbations increase with time. For this reason station keeping is required and fuel is needed. It is not yet realistic to expect refueling of JWST's fuel reservoir, so its lifetime is limited by fuel consumption. In case of JSWT the expected lifetime is around a decade due to Telescope's very successful deployment (NASA 2021).

Current station keeping plan is for the maneuver to happen every three weeks and skipped if telescope's orbit is still close enough to L2 (Donald J. Dichmann 2014). Such plan can be seen as rigid.

The goal of this paper is to propose a more economic way of fuel consumption. As an argument, the fuel needed per some time period is compared between two basic station keeping plans, namely an instant impulse by fuel kick at fixed distances from the stationary point.

## Evaluation

Let us consider the potential around the stationary point L2. The effective potential is best described in the rotating frame, taking gravitation and centrifugal force into account, without Coriolis force for simplicity (Bhatta 2020).

To estimate needed fuel, the test body of mass $m$ – i.e. the telescope – is first left for some time to diverge away from the starting position close to the stationary point L2, where it starts at rest. At a predetermined point some fuel is burnt to kick it back to the initial almost-stationary-position. To return there it needs velocity $v$, if $-v$ was the velocity away from L2 before the kick. The velocity $|v|$ is computed using $\Delta T + \Delta V = 0$, energy conservation in effective potential $V$,

$$\frac{mv^2}{2} = \Delta T = -\Delta V = -(V - V_0) = V_0 - V.$$

Next, the needed fuel impulse $F\delta t$ is computed through required momentum change $\Delta\Gamma$, taking $\Delta v = v - (-v) = 2v$,

$$F\delta t = \Delta\Gamma = m\ \Delta v = 2mv = \sqrt{8m(V_0 - V)}.$$

To compare efficiency of fuel powered impulses at various divergence thresholds, the impulse should be normalised with time in which the test object freely diverges from the initial almost stationary point. This time can be evaluated solving the equation of motion. It is linear in the vicinity of L2, since effective potential there can be approximated with a quadratic function,

$$V(x) = -\frac{1}{2}\alpha x^2,$$

where $x = 0$ is taken to be a stationary point and $\alpha$ is a constant, in principle determined from the second order derivative of the effective potential. Solving the equation of motion, i.e. Newton's law

$$\ddot{x} = -\frac{1}{m}\frac{\mathrm{d}V}{\mathrm{d}x} = \frac{\alpha}{m}x$$

for $x = x(t)$ gives exponential relation. The time at which the test object gets to some point $x$ can then be derived by inverting the solution,

$$t = \sqrt{\frac{m}{\alpha}}\ \log\frac{x}{x_\mathrm{m}},$$

with $x_\mathrm{m}$ a parameter chosen to be small enough for the quadratic approximation of the effective potential to still be appropriate in the considered region from $x_\mathrm{m}$ to the stationary point at 0. Furthermore at $t = 0$ the test object would get to $x_\mathrm{m}$. Time starts at some negative value, when $x$ is a small deviation from L2 which is at 0. As time $t$ increases, the test object moves toward a chosen threshold $x_i$. After the momentary fuel burn, the same amount of time is needed for

*mail name.surname at gmail. Template thanks to AAAI

the test object to return to the initial point, due to reversibility of the differential equation's solutions.

Fuel consumption can be compared between plans of different $x_i$ via time normalized impulses, which are computed as

$$\frac{F\delta t}{2\,\Delta t} = \frac{\sqrt{8m(V_0 - V)}}{2\sqrt{\frac{m}{\alpha}}\left(\log\frac{x_i}{x_\mathrm{m}} - \log\frac{x_0}{x_\mathrm{m}}\right)} = \frac{\alpha\ x_i}{\log\frac{x_i}{x_\mathrm{m}} - \log\frac{x_0}{x_\mathrm{m}}},$$

where $x_i$ are turning points and $x_0 \approx 0$ the starting point close to stationary, which is also why $V_0$ is taken as $0$. Let us set $x_0 = 10^{-3}x_\mathrm{m}$ and furthermore compare $x_1 = 10^{-1}x_\mathrm{m}$ and $x_2 = 2 \cdot x_\mathrm{m} = 2 \cdot 10^{-1}x_\mathrm{m}$.

The ratio of time-normalized-fuel-consumption between station-keeping plans of fuel-boost at $x_1$ and $x_2 = 2x_1$ is

$$\frac{x_1}{x_2}\frac{\log\frac{x_2}{x_\mathrm{m}} - \log\frac{x_0}{x_\mathrm{m}}}{\log\frac{x_1}{x_\mathrm{m}} - \log\frac{x_0}{x_\mathrm{m}}} = \frac{1}{2}\frac{\log 2\cdot 10^{-1} - \log 10^{-3}}{\log 10^{-1} - \log 10^{-3}} = \frac{1}{2}\frac{\log_{10} 2 - 1 - (-3)}{-1-(-3)} \approx 0.58$$

and is lower than 1. This means the fuel consumption is lower if the boost is made earlier, i.e. more often.

## Conclusion

It can be concluded that it is more economic to plan station-keeping as early as possible, considering of course that running the engine also has some initial cost to it. The threshold for using JWST's thruster is $\Delta v = 12\frac{\mathrm{cm}}{\mathrm{s}}$ (Donald J. Dichmann 2014). Fuel consumption increases if the free movement is undergone for a longer period. An improved plan *regarding fuel use* would be to use the thruster as soon as the boost needed to return it closer to a desired orbit is over the above $\Delta v$ threshold.

Waiting can be costly ...

Costs should be seen as investments.

Early bir gets the worm.

## References


Bhatta, S. 2020. Perturbation of Lagrange Points in Effective Potential Simulations.

Donald J. Dichmann, W. H. Y., Cassandra M. Alberding. 2014. STATIONKEEPING MONTE CARLO SIMULATION FOR THE JAMES WEBB SPACE TELESCOPE.

Goldstein, S., Poole, ed. 2002. *Classical Mechanics, 3rd*. Pearson.

NASA. 2021. JWST Deployment. https://blogs.nasa.gov/webb/2021/12/29/nasa-says-webbs-excess-fuel-likely-to-extend-its-lifetime-expectations/. Accessed: 2022-08-09.

NASA. 2022. JWST Orbit. https://jwst-docs.stsci.edu/jwst-observatory-characteristics/jwst-orbit. Accessed: 2022-08-09.