Proof of Hawking radiation thanks to the Hubble sphere.

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Abstract/Introduction

We establish a simple relationship between the Hawking temperature and the mass of the Hubble sphere via the Hubble time which could be a proof of the validity of Stephen Hawking’s theory of black hole evaporation.

The Hawking temperature $T_{Haw}$ of a black hole of mass $M_{Haw}$ has been defined by Stephen Hawking as follows:

$$T_{Haw} = \frac{1}{8\pi M_{Haw}} \frac{\hbar c^3}{Gk_B} \quad (1)$$

On the other hand, a Schwarzschild black hole of mass $M_S$ has an associated Schwarzschild radius $R_S$ such that:

$$R_S = \frac{2GM_S}{c^2} \quad (2)$$

Now the Hubble sphere of mass $M_H$ and radius $R_H = c/H$, where $H$ is the Hubble constant, can be assimilated to a Schwarzschild black hole because $R_H = R_S$. With the understanding that the Hubble mass is defined as the mass of dark energy plus the mass of dark and baryonic matter, although it is more usual to speak in terms of energy rather than mass in this context.

For $M_{Haw} = M_S = M_H$ ans with $R_H = R_S$, we find the associated Hubble time $t_H = \frac{1}{H}$ thus:

$$t_H = \frac{2GM_{Haw}}{c^3} \quad (3)$$
It becomes simple to show with the definitions of Planck time, \( t_P = \sqrt{\frac{\hbar G}{c^3}} \) and Planck temperature, \( T_P = \sqrt{\frac{\hbar c^5}{G k_B^2}} \) that:

\[
\frac{T_P}{T_{\text{Haw}}} = \frac{4\pi t_H}{t_P}
\]  

(Eq. 4)

This writing is not new: it simply reflects (Eq.1) expressed in Planck units the well-known expression,

\[
\frac{T_{\text{Haw}}}{T_P} = \frac{M_P}{8\pi M_H}
\]  

(Eq. 5)

where is the Planck mass, \( M_P \), replacing the notion of mass of (Eq.5) by the notion of time (Eq.4).

The only novelty of this article is to consider the mass of the universe at the Hubble radius for the Hawking temperature and to be able to verify it numerically with the equations (Eq.4) and (Eq.5) In other words, to consider the universe at the Hubble radius as a relevant laboratory to validate the theory of Stephen Hawking.