Attempt to unify quantum mechanics and general relativity on the basis of causal fermion systems

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ABSTRACT

The unification of quantum mechanics and general relativity requires a rethinking of all physical considerations since Newton's theory of gravitation. The presented theory is based on the causal fermial system and the simplest possible law for the energy $E = 2^n i^t$. The natural numbers n and t cause cohesion and replace gravity. A distinction is made between this law of nature and, on the other hand, our world view with 3 isotropic dimensions x, y and z and rotations with pi. From the torque for an observer and two objects, a common constant of h, G, and c can be derived:

$$hGc^5\, s^8/m^{10}\, \sqrt{(pi^4 - pi^2 - 1/pi - 1/pi^3)} = 1{,}00000$$

Numerous calculations on planetary systems, the hydrogen atom and elementary particles are given, e.g. the exact calculation of the proton mass:

$$m_{proton} = (2pi)^4 + (2pi)^3 + (2pi)^2 - (2pi)^1 - 3 - 2/pi - 2/pi^6 + 4/pi^8 + 4/pi^{10} + 4/pi^{12} + 8/pi^{13} - 1/pi^{14} \;\; m_e \;=\; 1836.15267343\, m_e$$

In this way, the energies of objects are determined directly. The quantum field theory (QFT), on the other hand, uses the operating principle with the Euler-Lagrange formula with the consequence of virtual particles in the vacuum.

1. INTRODUCTION

Numerous experiments for the Bell nonlocality [1] have shown the violation of the inequality for entangled particle pairs and thus confirmed the predictions of quantum mechanics [2, 3, 4, 5]. Calculations with the QM, as well as the quantum field theory (QFT) [6, 7, 8], however, fall back on ideas from the GR or Newton's theory of gravitation, on the 4-dimensional space and forces. Since Newton every body has been assigned a mass in kg and a center of gravity. Almost all mathematical physics is based on this concept, with calculations based on gravity. The gravitational constant has the units $m^3/kg/s^2$. This alone shows that gravity only leads to finally observable measurements in m after several steps. The mass with the unit kg is not a directly observable quantity. Newton's law of gravitation, like the GR, does not give any indication of the diameters and orbits of celestial bodies.

The causal fermion systems introduced by Felix Finster [9,10,11,12,13] overcame the limitation of physical objects of space and time in favor of the underlying elementary particles. So it is a fundamental concept that all space-time structures (geometry, topology, fields, wave functions, etc.) are encoded in linear operators on a Hilbert space [14,15,16,17,18,19]. More precisely, each space time corresponds to an operator. The relationships between different space-time points are encoded in the corresponding operator products. The physical equations are formulated using a nonlinear variational principle, the causal action principle. The theory of causal fermion systems is a proposal for quantum geometry and an approximation to quantum gravity [20]. It offers a union of weak, strong and electromagnetic forces with gravity at the level of classical field theory.

Robert B. Laughlin, disputes the fundamental possibility that a theory could explain complex issues such as emergence and self-organization.[21,22] .

In this respect, many questions in physics are still unanswered. This also means putting all fundamental physical principles to the test, such as the theory of action at a distance, the inertial system, isotropic space, the vacuum, the importance of constants and dimensions in the universe, the difference between matter and antimatter. This article examines to what extent meaningful results on the basis of causal fermion systems are possible, even without the principle of action and Euler-Lagrange equations.

2. REPRESENTATION OF QUANTUM MECHANICS AS A ONE-DIMENSIONAL SYSTEM

Quantum numbers are the most compact information for a system. This leads to the assumption that nature is only made up of quantum numbers. For physical calculations, all the particles in a system must be assigned a single number. This is part of the universe. The natural numbers cause cohesion and thus replace gravity. The structure of the system is given by alternating states (e.g. a series of 1, 1, -1, 0, 0, 1, -1, -1). An object is a divider of this system. The simplest possible formula for calculations in physics based on natural units is therefore a single dimension, a single type of particle with universal speed and a single law of nature. This refers to the energy $\boldsymbol{E = 2^r i^t} \;\; \boldsymbol{r} \in \mathbb{R}, \boldsymbol{t} \in \mathbb{N}$ (1) as the most compact information.

In contrast, our conception of the world is one with 3 isotropic dimensions x, y and z. A comparison of x, y and z is physically very problematic. Each ruler is rotated for comparison and subjected to the Coriolis force. Therefore, only polar coordinates are used in the following. If one calls **r the large radius, xy the small radius and z the deviation**, there are only ratios like wie

$r/z = n/m \quad xy/z = l/m \quad n > l >| m |> 0 \quad n, l, m \in N$.r, xy and z cannot be the same. As in a Turing machine, each variable has a defined storage space. n, l and m refer to the ratios of the orbital times. **2pi** is just the appropriate conversion factor from radius r to circumference and orbital period OP according to our view of the world. This is a consequence of evolution. The polynomial $E_{(Object,Surface)} = (r(t) + (2pi)xy(t) + (2pi)^2 z(t))$ is the combination of the 3 dimensions of an object into **one Dimension**. Starting from the middle, there is a clear order according to the sizes r > xy > z, after all there are no 100% circular orbits (n < l + spin) in QM either. This makes Heisenberg's inequality obsolete. Polynomials can be treated mathematically as vectors. Schrödinger's wave theory is based on $\Psi = Ae^{-i/\hbar(Et+mrdr/dt)} = Ae^{-i(wt+r/\lambda)} = Ae^{-ik} \; mit\, k \in \mathbb{Z}$ (3). **Nature knows no transcendent numbers!** The more particles N there are in a system, the finer is the resolution (w and λ) with the spatial coordinate r and the time t.

$$r_{Orbit} = A(2pi)^{-iwt}(2pi)^{r/\lambda} = \pm \mathrm{R}_{Center}(w)\,(2pi)^{r(0)+n/l\lambda} \quad (4)$$

This also corresponds to quantum mechanics. But the most important point is:

1. A complete, computable system always requires 2 objects O_1 (N_1) and O_2 (N_2) **and** an observer O_B (N_B)..
2. Ignoring O_B (N_B) the idea of the universe is a continuum. This also means that 3 further parameters c , h , G are required for 3 dimensions and leads to discrepancies between the micro- and macrocosm, to vacuum, Euler-Lagrange, QFT and so on.

Correct and ultimately easier is to consider the number of particles $N_{1,2,B}$ with O_B in a system. The ratio N_1/N_{total} to N_2/N_{total} is relevant for the radii R_{Orbit}, r_1 and r_2. In this way, the apoapsis and periapsis of celestial bodies can also be calculated. Example (see below) $R\_moon/(R\_earth + R\_moon) = 2\text{^}3/2pi = 4/pi$. Relative Error = 1.00011. This is the smallest possible quantum number since the moon has a locked rotation!

Pi is simply a tool to distinguish between inside and outside. It's just a question of the number system. Nature works dual. For our understanding of space, the base is 2pi, one complete revolution. The barrier between an object and another object is a circle. Either the object is inside or outside, matter or antimatter, in time before or in time after. Exactly on the circle the energy is zero. Regardless of how epicycles are built from circles, the barrier remains. It doesn't matter whether the physics consists of 3 or 11 spatial dimensions; the length of the polynomials is man-made, from our idea of a 3-dimensional space.

Photons consist of an electron and an anti-electron. In nature, these are two immediately adjacent particles. They cannot be separated and observed except by emission or absorption or by pairing with a 3rd object. The pair formation shows the consequence of the decay and results in an electron moving towards the center and an anti-electron moving in the opposite direction.

A photon has exactly the properties of an electron paired with an antielectron.

$spin1 = spin½ + spin½ \quad E_{ges} = E_{Electron+}E_{Antielectron}$

$N_{Electron} = -N_{Antielectron} = 1 \;\; E_{Electron} > 0 \;\; E_{Antielectron} < 0$. Bosons consist of an even number of particles and fermions of an odd number. The speed $pi\ c$ (see below) allows interaction between 2 entangled photons solely via the angular momentum. This applies to all entangled objects.

For calculations in physics, an observer and two objects 1, 2 are essential, with the respective numbers of particles N_B, N_1 and N_2. Basically, physical laws result from the respective conditions, the torques and a corresponding formula for the time or frequencies.

$N_B/r_B = N_1/r_1 = N_2/r_2$ (5) $\qquad N_B/w_B = N_1/w_1 = N_2/w_2$ (6)

These simple relationships correspond to Kepler's laws. $(T_1/T_2)^2 = (a_1/a_2)^3$ results from our idea of the 3 dimensions. (5) and (6) and apply to celestial bodies as to an atom. Elementary particles also consist of one or more particles.

There is a centroid for 2 objects and an observer. The lever law of classical physics applies as follows:

$M_{12B} = N_1(r_1 + 2pixy_1 + 4pi^2 z_1) + N_2(r_2 + 2pixy_2 + 4pi^2 z_2) + N_2(r_B + 2pixy_B + 4pi^2 z_B) = 0$ (7)

$L_{1,2,B} = \mathrm{N}_1\, w_1 + N_2\, w_2 + \mathrm{N}_B\, w_B = 0$ (8)

Torque M and angular momentum L are appropriate for these formulas with N particles. According to Gauss' integral theorem, what is inside an object does not matter, whether it is a solid body or a complex system of a center and satellites. According to classical mechanics, the center of gravity corresponds to M = 0 and L = 0. **According to quantum mechanics, the energy can only be calculated when the 3 objects interact with the same smallest center of gravity = Q. Q stands for a single quantum n = 1.**

$N_1/\,\mathrm{N}_B(r_1/r_B + xy_1/xy_B + z_1/z_B) + N_2/N_B\,(r_2/r_B + xy_2/xy_B + z_2/z_B) = -1 \pm pi \pm pi^2 \pm pi^3$

(9) with (1)

$$(r_1^2/r_B^2 + xy_1^2/xy_B^2 + z_1^2/z_B^2) + (r_2^2/r_B^2 + xy_2^2/xy_B^2 + z_2^2/z_B^2) = -1 \pm pi \pm pi^2 \pm pi^3$$

$E_{1,2} = (\mathrm{r}_1\, v_{1,r} + xy_1\, v_{1,xy} + z_1 v_{1,z})c + (\mathrm{r}_2\, v_{2,r} + xy_2\, v_{1,xy} + z_2 v_{1,z})c = \sqrt{(-1 \pm pi \pm pi^2 \pm pi^3)}c^2$

(10)

$Q^2 = -1 \pm pi \pm pi^2 \pm pi^3$ (11)

$$E_{(1,2)} = E_1 + E_2 + E_W = Qc^2$$

The pi^a summands in Q are the connections to other members of a larger system. This means that 2 chains can be combined into a larger system using the 2 end links.
Usually the energy is calculated with the mass: $E^2 = x^2\, \mathrm{p}_x^2\, c^2 + y^2\, \mathrm{p}_y^2\, c^2 + z^2\, \mathrm{p}_z^2\, c^2 - m^2\, c^4$
However, it is also necessary to include the observer's measurement. The measurement takes over the recoil. The mass m naturally has no unit, it is simply a ratio. The masses result from the interaction or the torque of the three bodies. Simply by assuming a particle number > 2, the mass is superfluous. With $\mathrm{N}_1\, w_1 + N_2\, w_2 + \mathrm{N}_B\, w_B = 0$ applying to each system:
$\mathrm{N}_1\, w_1/(N_B w_B) + N_2\, w_2/(N_B w_B) = -1$
$N_1^2 - N_2^2 = N_B^2 \quad w_1^2/w_B^2 + w_2^2/w_B^2 = -1 \quad w_1^2 + w_2^2 = -w_B^2$ (12)

What is the importance of the frequency of an object in quantum mechanics and TOE?
Three polar coordinates are summarized in the TOE: $r_{Object} = (r(t) + (2pi)xy(t) + (2pi)^2 z(t))$ xy corresponds to the transverse plane of rotation and applies to all objects in a system. The longitudinal direction of propagation is given by the ratio r/z. The properties of a photon can only be determined in relation to a third body. w is not the frequency f that is usually assigned to an elementary particle. f is the frequency of recoil after emission or absorption and depends on the detector, observer and the mass of the earth. The interaction $E_W = pi^2 c = hf$ can be included in the square root $Q/c^2 = \sqrt{(-1 \pm pi \pm pi^2 \pm pi^3)}$ in the pi^2 term. Only when 2 objects no longer emit energy, regardless of if they are particles, electromagnetic waves or gravitational waves, is a basic state reached in the entire system:

$$Q/c^2 = \sqrt{(\pm 1 \pm pi \pm pi^3)} \qquad (13)$$

2.1. Gravitational constant

With the product G h, the mass is eliminated and can only be calculated as a single unit. In 3 dimensions, the volume is limited to a particle $V_e = pi^2 c^3$. N particles have a volume of $V_r = Nr^3$. No single particle will occupy the same position after a complete revolution of the whole system $\sqrt{(1 \pm pi \pm pi^3)}$, and thus the relation $V_N = Npi^2 c^3$ (14) holds and leads with (13) to $Ghc^3 pi^2 Quantum = Ghc^5 pi^2 \sqrt{(\pm 1 \pm pi \pm pi^3)} \approx \pm 1$. All quanta have a charge as an electron or antielectron. Gravity is the difference from the smallest possible distance between two quanta. Two quanta result in a graviton. Cohesion corresponds to the interaction of a photon. The ratio of the 2 quanta results from the direct sequence in the series $1/pi^3 + 1/pi + 0 * 1 + 0 * pi + pi^2 + pi^4$. The polynomial $0 * 1 + 0 * pi$ is again part of the interaction between the 2 quanta. $Graviton = \sqrt{(1 + pi(pi + pi^3) - (1 + 1/pi + 1/pi^3))} = \sqrt{(pi^4 + pi^2 + 1/pi + 1/pi^3)}$ This results in

$$hGc^5s^8/m^{10}\sqrt{(pi^4-pi^2-1/pi–1/pi^3)} = 0.999991 \qquad (15)$$

h, G and c form a unit and are defined by this formula. The units meters and seconds **must** appear in this formula. Three objects can be used as standard units of measure if at least two measures are specified: orbital period, diameter, and/or particle count. The value of G is only known up to the fifth digit. In this respect, the result can be assumed to be 1. h and c are already exactly defined. The only parameter left to be determined by measurement is G. The only force holding the world together are the natural numbers, and they appear as centrifugal and centripetal forces. The only parameter that is not directly observable is the mass with kg! We measure everything that has to do with kg as a digital display, i.e. in m. Mass is not an observable. Two objects with 3 dimensions need 3^2 parameters plus the total number of particles and 10 equations. The ART with 16 equations, only 10 of which are independent, is redundant. This also means that the GR should be fully formulatable without the gravitational constant, but with the particle numbers N.

2.2. H0 and the gravitational constant

The equation for the graviton $hGc^5s^8/m^{10}\sqrt{(pi^4-pi^2-1/pi–1/pi^3)} = 0.999991$ can also be differently formulated by dividing the volume by the number of particles $V_N = Npi^2c^3$:

$$G_{Universe}/V_N = hGc^5s^8/m^{10}\sqrt{(pi^4-pi^2-1/pi–1/pi^3)}/pi^2/c^3 = hGc^2s^5/m^7\sqrt{(1-1/pi^2....)}$$

If you multiply $G_{Universe}/V_N$ by 2 c, then you obtain the orthogonal component of speed of light c, which is the expansion of the universe H0.

$hGc^32\sqrt{(1-2/pi^2)}s^5/m^8 = 2.13 \cdot 10^{-18}/s$ (17) Measurements: $H0 = 2.1910^{-18}/s$ **All interactions could be attributed solely to the expansion of the universe.**

2.4. Calculations for the sun - earth - moon

For the 3 spatial dimensions, $2^3 = 8$ is the basis for the rotation or frequency ratios. This is also reflected in the periodicity of 8 in the periodic table. The largest possible stable ratio of radii of celestial bodies is that of the earth and the moon. This results in the following ratios of the diameters of the earth/(earth + moon):

$$R_{moon}/(R_{earth}+R_{moon}) = 2^3/(2pi) = 4/pi.$$

Calculated: $R_{moon} = 6356.75\ km(4/pi-1) = 1736.9\ km$ related to the pole diameters. Relative error = 1.00011.

This unique relationship between the sun, earth and the first moon in the planetary system explains why the moon fits pretty much exactly into the sun during a solar eclipse. The distances between all bodies can also be the result of the expansion of the entire universe $H0 = 2.1910^{-18}/s$. $\frac{d}{dt} distance(Moon) = 38.2\ mm/384400\ km/1\ year = 3.1510^{-18}/s$. $(1 - 1/pi)3.1510^{-18}/s \approx H0$

2.4. Calculation of the speed of light c from the radius of the earth and 1 day

The sun, the earth and the moon together form a system with special relationships of the orbital periods. This also means that the speed of light c should be in the greatest possible ratio. pi^2 in $Q = \sqrt{(\pm 1 \pm pi \pm pi^2 \pm pi^3)}c^2$ corresponds to the rotation in the transverse plane. For a photon this is $E_W = pi^2 c^2 = hf$. The factors pi and pi^3 relate to the longitudinal direction of propagation and the spin. c' = pi c is the speed at which an electron rotates around its geodesic line. For the relative velocity of electrons in a photon compared to electrons on the surface of a macroscopic object, the orbital period is: $OP = r/\omega = 2/(pi\ c\ m) r^2$. This simply yields the conservation of angular momentum. $L_{e-} + L_{e+}$ in the photon = L_{-e} the e_- on the surface at a distance r from the center of the earth. If you insert the radius of the earth's surface of 6378.626 km and the orbital period of one day into this formula, the result is the speed of light c.

$$r = \sqrt{(pi/2\ c\ m\ day)} = 6378626m \quad r^2/day/m\ 2/pi = c \qquad (18)$$

The radius at the equator is 6378137 m (GSM 80), with a difference of 489 m. This consideration relates in particular to the system sun, earth, moon and its bound rotation. It is a ground state after a long evolution throughout the planetary system.

2.5. Transfer of the equations to elementary particles

The masses of elementary particles are energies expressed by polynomials. Each summand represents one of the 3 dimensions. Composite particles are sums of two polynomials. Every polynomial of an elementary particle starts with the 3 coefficients for r, xy and z:

$$E_{Tripel} = a_r(2pi)^d + a_{xy}\ (2pi)^{d-1} + a_z(2pi)^{d-2} \qquad a_{r,xy,z},\ d \in \mathbb{Z} \qquad (19) \qquad E = E_{kin} + E_{Tripel}$$

For stable particles, the coefficient a_r is 1 for matter or -1 for antimatter. a_{xy} describes the angular momentum with quantum number L. a_z describes m + spin. The energies of all elementary particles are related to that of the electron.

$$E_{e-} = 1 \pm (pi)^{-1} \qquad E_{e+} = -1 \pm (pi)^{-1} \qquad (20)$$

For the polynomials, operators apply as in QM. In our imagination, 2 full revolutions and a radius r > 0 are required to capture all the information of an object. I.e. **after each triple there is a parity change P-**. The ladder operator J_{+-} produces a basis of $(2pi)^{d+-1}$ from a basis $(2pi)^d$

2.5.1. Calculation of the mass of a proton

The calculation of the proton mass starts with 2 polynomials (19):

$E_{p,1} = (2pi)^4 + (2pi)^3 + (2pi)^2 - E_W$ and the antiparticle $E_{p,2} = -((2pi)^1 + (2pi)^0) - E_W$.

E_W corresponds to the interaction or binding energy with a first estimate

$E_p = (2pi)^4 + (2pi)^3 + (2pi)^2 - (2pi)^1 - (2pi)^0 - 2E_W = 1838.79090228 - 2E_W$

Or with the parity operator P

$$E_p = (2pi)^4 + (2pi)^3 + (2pi)^2 + P_-((2pi)^1 + (2pi)^0 + 2E_W) = 1838.79090228 - 2E_W$$

2.5.2 Calculation of the interactions in the proton

As described under 2., the descendant places of E_W should also depend on an observer, i.e. the environment of the proton. This would be compatible with inversion of the polynomials $\ldots + (2pi)^d + \cdots$ with reflection on the unit circle and thus to bases of $(1/pi)^d$. It is relatively easy to find the right factors, especially since a pattern emerges. It must be remembered that high-energy experiments cannot directly reveal the inner workings of a proton. Each factor must define at least one quantum property.

The number 12 of safe descendant digits $(1/pi)^d$ should depend on the combinations of dimensions 3-d and 4-d.

It should also be considered that in the micro-world the dimensions are reduced and predominantly occur in steps of $1/pi^{2d}$. For now, the factors are speculative.

Mass of proton $m_P = (2pi)^4 + (2pi)^3 + (2pi)^2 - (2pi)^1 - 1 - 2 - 2/pi - 2/pi^6(1 - 2/pi^2 - 2/pi^4 - 2/pi^6(1 + 1/pi^2(2pi - 1/4))) =$

= $(2pi)^4 + (2pi)^3 + (2pi)^2 + P_-((2pi)^1 + 3 + 2/pi + 2/pi^6) + 4/pi^8 + 4/pi^{10} + 4/pi^{12} + 8/pi^{13} - 1/pi^{14}$

Theory: 1836.15267343 m_e Measured: 1836.15267343(11) m_e (21)

The last factor $1/pi^2(2pi - 1/4)$ deviates from the rule. It describes the particle that is closest to the center of gravity of the atom. $1/4 = (1/2\ Spin\ of\ e)^2$. This is at least a reasonable assumption, although this is already within error.

The gap from $2/pi$ to $2/pi^6$ is a placeholder for electrons in atoms.

$(2pi)^4 + (2pi)^3 + (2pi)^2 - (2pi)^1$ is reflected in $-2/pi^6 + 4/pi^8 + 4/pi^{10} + 4/pi^{12}$.

Each term $\boldsymbol{n\ pi^r}$ $\boldsymbol{n, r} \in \mathbb{Z}$ of the polynomial is **unique** since pi is a transcendental number. No term can be replaced by another basis. The arrangement of the factors alone has a system. For each ground state, the absolute values are $|a_{d+1}(2pi)^{d+1}| > |a_d(2pi)^d| > |a_{d-1}(2pi)^{d-1}|$ The sequence is monotomous and converges**. Every calculation of the proton mass with the QFT must ultimately, the bottom line, lead to exactly this result!**

The smallest fraction of energy with the smallest orbit should be an electron neutrino.

$$E_{Neutrino} = \mathrm{E}_{Electron}\, 2/pi^6\, 2pi^6 2/pi^2 1/4 = (2pi-1)pi^{14}/2 = 1.1510^{-6}eV \quad (22)$$

2.5.3. Neutron

The gap in $-2/pi - 2/pi^6 + \cdots$ is a placeholder for further interactions between further protons or neutrons to build up the periodic table.

$m_{neutron} = (2pi)^4 + (2pi)^3 + (2pi)^2 - E_W$

$m_{neutron} = E_W + (2pi)^1 + (2pi)^0 \qquad 1/pi^2 : 1/pi^4 \quad E_W = 1/2/pi^2(1 + 1/pi^2)$

$m_{neutron} \approx (2pi)^4 + (2pi)^3 + (2pi)^2 - (2pi)^1 - (2pi)^0 - 1/pi^2 - 1/pi^4 = 1838.68$

Speculation for now:

$m_{neutron}$=$(2pi)^4 + (2pi)^3 + (2pi)^2 - (2pi)^1 - 1 - 1/pi^2 - 1/pi^4 + 2/pi^6(2 + 1/pi^2 - 1/pi^4 - 1/pi^6(1 + 1/pi^2(2pi – 1/4)))$

$$m_{neutron} = (2pi)^4 + (2pi)^3 + (2pi)^2 - (2pi)^1 - 1 - pi^{-2} - pi^{-4} + 4pi^{-6} + 2pi^{-8} - 2pi^{-10} - 2pi^{-12} - 4pi^{-13} + pi^{-14}$$

Theory: 1838.6836617 m_e Measured: 1838.68366173(89) m_e (23)

In contrast to the proton, $4pi^{-6}$ is unstable. The further away this potential breaking point is from the beginning of the polynomial, the lower the decay probability. This makes the neutron unstable. The ratio $(2pi^{-6})/1$ should be proportional to the decay rate of a neutron into a proton, electron and electron antineutrino. The 3 interactions, i.e. the electromagnetic force, weak force and strong force, result from the 3 dimensions (r, xy, z).

The energy difference between a proton and a neutron essentially corresponds to the energy of two electrons: $E_n – E_p = (-1 - pi^{-2} - pi^{-4}) - (-1 - 2 - 2pi^{-1}) = (2 + 2pi^{-1}) - pi^{-2} - pi^{-4} = 2(1 + pi^{-1}) + E_W$ (24)

2.5.4. Muon

The calculation is analogous to that for a proton.

$$m_{muon} = (2pi)^3 + E_W \qquad m_{muon} = -E_W + (2pi)^2 \; E_W \approx 1 – 1/pi$$

$$m_{muon} = (2pi)^3 - (2pi)^2 – 2E_W = (2pi)^3 - (2pi)^2 – 2 - 2/pi = 205.93m_e$$

The muon is an unstable particle. The comparison with the calculation of the proton mass is only an estimate. Due to the instability, $E_W \approx 1 - 1/pi^2$ is more likely.

$$m_{muon} = (2pi)^3 - (2pi)^2 – 2E_W^2 = (2pi)^3 - (2pi)^2 – 2 - 2/pi^2 = 206.77m_e \qquad (25)$$

Theory: 206.77 m_e Measured: 206.7682830(46) m_e

2.5.5. Tauon

A tauon consists of many particles, as can be seen from the numerous decay channels. The first particle with factor $(2pi)^4$ is a proton. So the tauon should have the factor $2(2pi)^4$.

First estimate for the mass of a tauon:

$$m_{Tauon} = 2(2pi)^4 = 3117.0m_e$$

Without the factors $(2pi)^3$ and $(2pi)^2$, the tauon, like the proton, cannot exist.

$$m_{Tauon} = 2(2pi)^4 + (2pi)^3 + (2pi)^2 = 3404.61m_e$$

Spekulation: $2(2pi)^4 + (2pi)^3 + 3(2pi)^2 - (2pi)^1 = 3477.29$ mit 2 x 3 Teilchen

$$m_{Tauon} = 2(2pi)^4 + (2pi)^3 + 3(2pi)^2 + E_W$$

$m_{Tauon} = -E_W + 2pi$ Interaction: $E_W = 1/pi^3(1 – 1/pi^2)$

$$m_{Tauon} = 2(2pi)^4 + (2pi)^3 + 3(2pi)^2 - 2pi – 1/pi^3(2 – 2/pi^2) = 3477.235m_e \qquad (26)$$

Theory: 3477.23 m_e Measured: 3477.23 m_e

L. Hydrogen atom

The energies in the TOE depend on the center, i.e., on E_{proton}. All energies have the unit c^2. $c^2$is a ratio between two objects from $E_1 = (2pi)^n i^{t/n}$ and $E_2 = (2pi)^l i^{t/l}$. That is, the parameters e,e_0, m, h, and c are already present in the energies as polynomials. This seems unusual for the traditional calculations with the Bohr atomic model. Only the E_{proton} polynomial is required to calculate the energy levels of the hydrogen atom.

$$E_{proton} = (2pi)^4 + (2pi)^3 + (2pi)^2 - (2pi)^1 - 1 - 2 - 2/pi - 2/pi^6 + 4/pi^8 + 4/pi^{10} + 4/pi^{12} + 8/pi^{13} - 1/pi^{14}$$

$E_e = E_{kin} + 1 \pm 1/(2pi)chR_\infty = 0.00002662568m_e$ is the ionization energy ER.

$$E_H = E_p + 1 + E_W = E_p + E_e + chE_H$$
$$= (2pi)^4 + (2pi)^3 + (2pi)^2 - (2pi)^1 - 2 - 2pi^{-1} - 2pi^{-6} + 4pi^{-8} + 4pi^{-10} + 4pi^{-12} + 8pi^{-13} - 1/pi^{-14} + E_W$$

Calculating H in this way is initially limited by the inaccuracy of the proton mass:

$$m_H = (1836{,}15267343(11) + 1 + 0.00002662568)m_e = 1837.15270006(11)m_e$$

Unlike the neutron calculation, the interaction of the electron is in the final part after the $-2pi^{-6}$

gap. If even exponents occur in the form $-2pi^{-6}$, a polynomial result:

$$0.00002662568 = 2pi^{-10} + 4pi^{-12} + 7pi^{-14} + \ldots (residual\ value\ 1.7 * 10^{7})$$

$$E_H = (2pi)^4 + (2pi)^3 + (2pi)^2 - (2pi)^1 - 2 - 2pi^{-1} - 2pi^{-6} + 4pi^{-8} + 6pi^{-10} + 8pi^{-12} + 8pi^{-13} + 6pi^{-14} + \ldots$$

$E_H = 1837.15269989$ is therefore in the range of possible errors.

In the equation for the graviton $hGc^5s^8/m^{10}\sqrt{\left(pi^4 - pi^2 - \frac{1}{pi} - \frac{1}{pi^3}\right)} = 1.00000$, there are groups of even or odd exponents. The interaction $E_W = pi^2c = hf$ in the root $Q/c^2 = \sqrt{(-1 \pm pi \pm pi^2 \pm pi^3)}$ means that the respective mismatched terms become 0 by radiating energy. This can again be found in the formula of E_H with the polynomial

$$delta(E_H - E_p) + E_{observer} = 1 + 0 * (2 - 2pi^{-1} - 2pi^{-6} + 4pi^{-8}) + 2pi^{-10} + 4pi^{-12} + 7pi^{-14} + \ldots + E_{observer} = 0 \qquad (27)$$

Emission or absorption involves a system of 3 objects with energies $E_e, E_p,$ and $E_{observer}$. Between $-2pi^{-6}$ and $+4pi^{-8}$, there is thus the possibility of absorption of a photon = electron + antielectron. (27) can be calculated using classical physics. The impulse is divided into 3 objects. The energy depends on the mass of the detector and ultimately on the mass of the earth. The detector dominates the recoil and thus the energy $E_{observer} = hf$. In the entire system of the electron, proton and observer, revolutions are exchanged while conserving energy and angular momentum.

Thus far, the ground state at time t = 0 has been considered. Each factor $1/pi^n$ also has a time component $1/pi^n(1 + i^{t/n})$. According to the ratios $N_e/w_e = N_p/w_p = N_{obs.}/w_{obs.}$, the total angular momentum is constant. The energy differences result in the energy of the photon with principal quantum number n. $N_e^2 - N_p^2 = N_{observer}^2 \quad w_1^2/w_B^2 + w_2^2/w_B^2 = -1 \; w_e^2 + w_p^2 = -w_{observer}^2$ (see 12 above)

$$pi^{-6}(1 + i^{(t/-6)/n2} - i^{(t/-6)/n1}) = hf/m_e \quad (1 + 1/n_2^2 - 1/n_1^2)E_H = E_{Erde} + hf$$

$$(1/n_2^2 - 1/n_1^2)R_\infty = hf \qquad (28)$$

The term $i^{(t/-6)(1/n2-1/n1)} \propto f$ is a beat and has to be converted by a trigonomic formula into a real part in the r direction and an orthogonal part in the time direction. The frequency f, which is normally assigned to a photon or electron, is only the frequency between an excited atom and a receiver, as every transmitter needs a ground. The gap between $-2pi^{-6}$ and $+4pi^{-8}$ dominates the kinetic energy with n_1/n_2pi^{-7}. **The quantum property only arises when 3 objects have a common center of gravity.**

For the time being, this example is only intended to show how polynomials enable a second way of calculating the energies in an overall system, atom or molecule. Ultimately, grouping molecules into a single and unique polynomial should be possible.

2.7. Calculations of the orbits in the solar system

The sun with radius r_{sun} is orbited by smaller objects with radius r_{orbit}. In this respect, the particle number N in $\mathrm{E} = (2\mathrm{pi})^{(\mathrm{N/d})}\mathrm{i}^{(\mathrm{t/N})}$ can be replaced by the dividers n, l and m and is related to the center $\mathrm{r_{center}}$. E can again be represented as a polynomial with at least 6 terms: $E_{(n,l,m,s)} = r_{center}^2(2pi)^{nlms}i^{t/(nlms)}$. n, l, m and s are only placeholders for the time being and must be determined more precisely.

Resonances in the solar system should result from rotation and revolution time ratios (period $2^3 = 8$). In the inertial system, from the center of gravity, the orbital times are divided between the rotation around the center and the orbital period of the orbit, giving the factor 1/2.

For the time being, the following are speculative:

Orbital period for the lunar orbit: $1/2(8^2 - 8^1 - 1) = 27.5\ d$ *Measured: 27.322 d* (30)

Orbital time for the Venus orbit: $1/2(8^3 - 8^2 + 1) = 224.5$ *224.70 d* (31)

Orbital time for the earth orbit: $1/2(8^3 + 3(8^2 + 8 + 1)) = 365.5$ *365.25 d* (32)

Orbital time for the Mercury orbit relative to the sun's rotation *of 25.38 d*

$1/2(8 - 1 - 1/21/8) = 88.03$ 87.969 d (33)

If the times are set relative to the sun's rotation $xy_{sun}i^{4t}$, then there is a complete revolution for every whole number t.

$$E = r_{center}^2\left(\mathrm{r}_{sat}\, i^{t/n} + xy_{sat}\, i^{t/n-1} + \underset{sat}{\mathrm{z}}\, i^{t/n-2} + r_{sun}\, i^{4t} + \underset{sun}{\mathrm{x}}\, i^{4t-1} + z_{sun}\, i^{4t-2}\right)$$

For Mercury with n = 1, this results in the following equation:

$$E_{(n,l,m,s)} = \mathrm{r}_{center}^2\left(32pi^5i^t + 16pi^4i^{t-1} + 8pi^3i^{t-2} + 4pi^2i^{4t} + 2pii^{(4t-1)} + i^{(4t-2)}\right) \quad (34)$$

The radial component r_{sat} $32pi^5i^t$ mainly corresponds to the potential energy. xy_{sat} $16pi^4i^{t-l}$ is orthogonal to r_{sat} and mostly corresponds to the kinetic energy. $\overrightarrow{ryx} \propto t$ directly yields Kepler's 2nd law. $8pi^3i^t$ corresponds to the ecliptic and is therefore orthogonal to the distance from the sun and can be set to $z_{sat} = 8pi^3$ in the energy formula. However, it requires other relations between the real part and other i^t terms.

$$E_{(n,l,m,s)} = \mathrm{r}_{sun}^2(32pi^5 1/4(3 + i^t) - 16pi^4 1/2(1 + i^{t-1}) + 8pi^3 + 4pi^2 1/4(3 + i^{4t}) - 2pi1/2(1 + i^{(4t-1)}) + 1) \quad (35)$$

From this, we obtain the following for **Mercury:**

Periapsis:

$$r_{Orbit} = 696342\ km\sqrt{(1 + 0 * pi + 4pi^2 + 8pi^3 - 0 * 16pi^4 + 32pi^5)} = 69916199\ km \quad (36)$$

Measured: $AU = 149.610^6 km$ $0.4667\ AU = 69.8110^6 km$ Relative error = 1.0015

Apoapsis:

$$r_{Orbit} = 696342\ km\sqrt{(1 - 2/2pi + 2pi^2 + 8pi^3 - 16/2pi^4 + 32/2pi^5)} = 46114001\ km \quad (37)$$

Measured: $0.3075\ AU = 46.00210^6 km$ Relative error = 1.0024

N. Orbits of all planets in the solar system

The energies or radii of the orbits are approximately calculated for the entire solar system.

$E_{total} = \mathrm{R}_{sun}^2\, pi^3/2$ (Planet + Apo/Periapsis moon + sun)

$$E_{(n,l,m,s)} = \mathrm{R}_{Sun}^2\, pi^3/2\, ((4pi^2 3^n 2^l) + (4pi^2 3^m 2^{s/2}) + (1 + 2pi + 4pi^2)) \quad (38)$$

E_{total} is a multiple of $pi\char`^3\ /2$ (cf. 11) and is divided into 3 objects. All energies are multiples of $4pi\char`^2$. Beginning at the surface of the sun, the quantum properties of the solar system come into play. The definition of the surface results from the coincidence of the body when it rotates. Thus, there is no exact limit for the surface. The energies $E_{(n,l,m,s)}$ can be inserted in a single line of a program. Everything else is only necessary for our contemplation of the world. There are 4 loops for the 4 parameters n, l, m and s. n, l and m depend on the parameters r, xy, and z. s describes the large moons. The following table therefore also contains values of ½ or ¼; n, l and m are not directly comparable with the quantum numbers in QM. The inner planets predominantly depend on $4pi^2 3^n 2^l$, and the outer planets predominantly depend on $4pi^2 3^m 2^{s/2}$. Each run requires a unit of time. Apoapsis and periapsis result. These are the limit values of two different quantum combinations (n,l,m,s). Kepler's laws are used for graphics, with 2 orthogonal circles for apoapsis and periapsis, i.e., an ellipse. Another circle gives the deviation. The advantage of the solar system over atoms or elementary particles is that the orbits can be directly observed.

All calculations of the radii in the solar system cannot be exact. The only exact laws are those Galileo, without pi. The orbits are derived from rational numbers during the formation of the solar system. The fractal nature of the solar system also means coincidence. Pi is the geometric mean in chaos.

$$r_{orbit} = 696342\ km\sqrt{(pi^3/2((4pi^2 3^n 2^l) + (4pi^2 3^m 2^{s/2}) + (1 + 2pi + 4pi^2)))} \quad (39)$$

Example

Mercury:

n = 1: l = 0: m = 1: s = 0

$$Apoapsis = 696342\sqrt{(pi^3/2(((2pi)^2 3^1 2^0) + ((2pi)^2 3^1 2^{(0/4)}) + (1 + 2pi + (2pi)^2)))}$$

Apoapsis = 46175339

n = 1: l = 2: m = 2: s = 0

$$Periapsis = 696342\sqrt{(pi^3/2(((2pi)^2 3^1 2^2) + ((2pi)^2 3^1 2^{(0/4)}) + (1 + 2pi + (2pi)^2)))}$$

Periapsis = 69304544

The results in Table I show possible orbits.

TABELLE I. Orbits vom Sonnensystem aus der Formel:

$$r_{orbit} = 696342\ km\sqrt{(pi^3/2((4pi^2 3^n 2^l) + (4pi^2 3^m 2^{s/2}) + (1 + 2pi + 4pi^2)))} \qquad (39)$$

			Quantum numbers	n	l	m	s
Sun	R = 696342.0						
Orbital P = 25.38		Measured: 25.38	RE: 0.000				
Rotational P = 25.4		Measured: 25.38	RE: 0.000				
Mercury	R = 2448.57	Measured: 2439.7	RE: 0.004				
Orbital P = 88.7		Measured: 87.969	RE: 0.008				
Rotation P = 59.0		Measured: 58.65	RE: 0.007				
Apoapsis = 46.2		Measured: 46.0	RE: 0.00	1	0	1	0
Periapsis = 69.3		Measured: 69.8	RE: -0.01	1	2	1	0
Inclination 7.14 °		Eccentricity 0.2003					
Venus	R = 6123.80	Measured: 6051.8	RE: 0.012				
Orbital P = 226		Measured: 224.701	RE: 0.007				
Rotation P = 245		Measured: 243.6	RE: 0.008				
Apoapsis = 106.5		Measured: 107.4	RE: - 0.01	2	2	0	0
Periapsis = 110.9		Measured: 108.9	RE: 0.02	2	2	1	1
Earth	R = 6954	Measured: 6378	RE: 0.090				

Orbital P = 368		Measured: 365.25	RE: 0.010				
Rotation P = 1.0		Measured: 1	RE: 0.000				
Apoapsis = 148.4		Measured: 147.1	RE: 0.01	2	3	0	0
Periapsis = 151.6		Measured: 152.1	RE: 0.00	2	3	1	1
Moon	R = 1900	Measured: 1737.4	RE: 0.094				
Orbital P = 27.38		Measured: 27.322	RE: 0.002				
Rotation P = 27.38		Measured: 27.322	RE: 0.002				
Apoapsis	0.3697	Measured: 0.363	RE: 0.02	2	3	0	0
Periapsis	0.416	Measured: 0.406	RE: 0.03	2	3	1	1
Mars	R = 2356	Measured: 3396.2	RE: -0.306				
Orbital P = 712.9		Measured: 686.98	RE: 0.038				
Apoapsis = 208.3		Measured: 206.6	RE: 0.01	2	4	0	0
Periapsis = 243.1		Measured: 249.2	RE: -0.02	2	4	3	2
Asteroids							
Apoapsis = 293.5		Measured: 299.2	RE: - 0.02	2	5	0	0
Periapsis = 510.4		Measured: 508.6	RE: 0.00	3	5	2	1
Jupiter	R = 71617	Measured: 71492	RE: 0.002				
Orbital P = 4510		Measured: 4332.75	RE: 0.041				
Apoapsis = 739.7		Measured: 740.5	RE: 0.00	3	6	4	1
Periapsis = 810.8		Measured: 816.7	RE: -0.01	3	6	5	2
Satellite Jo							
Apoapsis	0.37512	Measured: 0.42160	RE: -0.11	3	6	4	1/2
Satellite Europa							
Apoapsis	0.6192	Measured: 0.67090	RE: -0.08	3	6	4	2/4
Satellite Ganymede							
Apoapsis	0.97469	Measured: 1.07000	RE: -0.09	3	6	4	3/4
Satellite Callisto							
Apoapsis	1.68404	Measured: 1.88300	RE: -0.11	3	6	4	4/4

Saturn	R = 59505	Measured: 60268	RE: -0.013				
Orbital P = 11659		Measured: 10759.1	RE: 0.084				
Apoapsis = 1394.2		Measured: 1352.5	RE: 0.03	3	7	7	1
Periapsis = 1524.5		Measured: 1514.6	RE: 0.01	3	7	7	2
Uranus	R = 25187	Measured: 25559	RE: -0.015				
Orbital P = 31668		Measured: 30685	RE: 0.032				
Apoapsis = 2659.3		Measured: 2741.3	RE: - 0.03	4	8	7	1
Periapsis = 2984.6		Measured: 3003.7	RE: -0.01	4	8	8	1
Neptune	R = 22354	Measured: 24341	RE: -0.082				
Orbital P = 60927		Measured: 60189	RE: 0.012				
Apoapsis = 4402.3		Measured: 4444.5	RE: - 0.01	5	8	7	1
Periapsis = 4517.6		Measured: 4545.6	RE: -0.01	5	8	8	0
Pluto	R = 3054	Measured: 1188	RE: 1.571				
Orbital P = 110383		Measured: 90559.7	RE: 0.219				
Apoapsis = 4402.3		Measured: 4436.8	RE: - 0.01	5	8	7	1
Periapsis = 7485.6		Measured: 7375.9	RE: 0.01	6	8	7	0

The given planetary radii are not corrected by moons. The orbital periods are calculated from the radii and therefore do not have to correspond exactly to Newton's laws.

III. SUMMARY

Physics can be explained mathematically. The basis of the theory are the natural numbers. The simplest assumptions result in one-dimensional energies $\boldsymbol{E} = \boldsymbol{2}^{r}\boldsymbol{i}^{t} \quad \boldsymbol{r} \in \mathbb{R},\ \boldsymbol{t} \in \mathbb{N}$ for a system consisting of at least 2 objects and one observer. These are the raw data that nature gives us. Our idea of the world is that of a 4-dimensional space-time. It is characterized by rotations and generates polynomials with the base 2pi from rational numbers. This results in relationships between the units h, c, G and meters and seconds:

$$hGc^5\ s^8/m^{10}\sqrt{(pi^4 - pi^2 - 1/pi - 1/pi^3)} = 1{,}00000 \text{ und } r = \sqrt{(\tfrac{pi}{2} c\ Day\ m)} = 6378626\ m.$$

Important results can be expected for elementary particle physics, atomic theory and celestial mechanics. The polynomials based on the transcendent number pi should also allow for complex situations such as emergence and self-organization. Some is still speculative. However, the previous considerations should be meaningful enough to further pursue the connection of causal fermion system, QFT of GR and to further expand the theory.

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