Extension of an imaginary triangle through Complex Variables

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Abstract: Complex Variables has a link to general geometry in placing the geometry of squares. Given is a problem in geometry where a short-cut is taken to solve what the angle is in the given situation.

Question: If the angle opposite 45° and the angle opposite \( x \) when added is equal to \( x^4 + y^4 + z^4 = (a + b) \) given that \( x^2 + y^2 + z^2 = \sqrt{\pi} \) and \( x + y + z = 0 \), find \( x \) and organize the triangles to form a proof of the Pythagorean Theorem.

Answer: Given a sphere \( x^2 + y^2 + z^2 = r^2 \), where \( r = \pi^{1/4} \) we have \( x^2 + y^2 + z^2 = \sqrt{\pi} \).

If \( x + y + z = 0 \), what is \( (a + b) \) from \( x^4 + y^4 + z^4 = (a + b) \), if we use understanding from complex variables we know that \( x + y + z = 0 \), \( (a + b) \) from \( x^4 + y^4 + z^4 = (a + b) \).

\( \pi/2 = x, -\pi/2 = y, 0 = z \), so \( x + y + z = 0 \), then \( x^2 + y^2 + z^2 = \frac{\sqrt{\pi}}{2} + \frac{\sqrt{\pi}}{2} + 0 = \sqrt{\pi} \), this satisfies the condition so \( x^4 + y^4 + z^4 = \frac{\pi}{4} + \frac{\pi}{4} + 0 = \pi/2 = a + b \). Complex Variables is used since \( u \) and \( v \) cancel, the \( i \) part implies negation.

Triangle A: \( 45^\circ + 3x + a = 180^\circ \), Triangle B: \( 135^\circ + x + b = 180^\circ \), \( a + b = \pi/2 = 90^\circ \)

Triangle B+Triangle A= \( 270^\circ + 4x = 360^\circ \), so \( x = \frac{\pi}{4} \) or \( \frac{45}{2} \), then \( x = 22.5^\circ \)

Final Part 1 Answer: Angle \( a + b = \frac{\pi}{2} \), so \( x = 22.5^\circ \) or \( \frac{\pi}{8} \)

How would we check this?

Check: \( \text{Triangle A} = 180^\circ + 45^\circ + 3 \cdot 22.5^\circ + a, a = 67.5^\circ, \text{Triangle B} = 135^\circ + 22.5^\circ + b = 180^\circ \)

Part 2 Proof: Rearranging the structure of triangles (A and B), we create this diagram. An imaginary triangle (C) is inverted.

Using the law of similar trapezoid areas

\( \frac{1}{2}(a + b)(a + b) = ab + \frac{1}{2}c^2 \)

\( \frac{1}{2}(a^2 + 2ab + b^2) = ab + \frac{1}{2}c^2 \)

\( (a^2 + 2ab + b^2) = 2ab + c^2 \)

\( a^2 + b^2 = c^2 \)

Thus the Pythagorean Theorem is proved by law of similar Trapezoids