A mystery is solved: Gravitational waves generate the constant hum of the earth

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25 years ago, weak permanent oscillations were discovered in the records of gravimeters, which are not excited by earthquakes and are also not natural resonances of the earth. Proposed causes like wind and ocean waves are ruled out because of their unreliability. The oscillation at 836.69 μ Hz exhibits most typical modulations expected for gravitational waves. Their frequency stability can also explain the surprising, previously unknown phase coherence discovered in the records of eight gravimeters. The gravitational waves emitted by the countless binary star systems in our galaxy are probably the cause of the strikingly strong background noise of gravimeters.

1 Introduction

25 years ago some very weak spectral lines (earth hum) were discovered in the longterm records of the very quietly located gravimeter Syowa (Antarctica), for which so far no convincing cause could be found [1]. The earth is an elastic body and can oscillate with different natural frequencies, which are excited again and again by earthquakes and decay with half-lives of hours or days. The enigmatic spectral lines, on the other hand, are constantly measurable and depend neither on seasons nor on earthquakes [2]. Modes with such high Q factors – that is, extremely long half-lives – are not found in the solid Earth, the oceans, or the atmosphere. They bear no relation to the known natural frequencies of the earth [6].

Our galaxy hosts double star systems with orbital periods of several minutes, which are extraordinarily constant [3]. All observations can be fully explained if one assumes that the emitted continuous gravitational waves (GW) excite the earth to forced oscillations [4]: The frequencies (in the mHz range) are very stable, have much smaller half-widths than the natural resonances of the Earth [5] (see also Figur 1) and their amplitude remains constant for years.

2 Instrumentation and noise

Because of the frequent disturbances caused by earthquakes, the Power Spectral Density (PSD) of superconducting gravimeters in the range around 1 mHz is about 10 $^{-18} \frac{m^2}{s^4 Hz}$ [13] [14]. To reduce noise, signals always go through filters during their processing. The measurable noise amplitude after the filter is calculated according to the equation

$$A_{noise} = \sqrt{PSD \cdot BW} \tag{1}$$

where BW is the bandwidth. One cannot narrow the bandwidth of the signal processing arbitrarily in order to eliminate the disturbing noise. Because that increases the necessary time span T_{min} that the filter needs to settle. The relationship (2) was first formulated by Küpfmüller [15] and is reminiscent of the Heisenberg uncertainty principle.

$$T_{min} \cdot BW \ge 0.5 \tag{2}$$

The frequency-time spectrogram of the original measurement [1] consists of 1015 spectra calculated from short data sets of only three days duration each. This increases the frequency resolution to about 2 μ Hz (equation (2)) and the noise level to 1.4×10^{-12} m/s² (equation (1)). In order to be able to analyze weak GW, the following investigation is based on data sets spanning ten years. This lowers the frequency resolution (= filter width) to 1.6 nHz and the noise level after filtering to 4×10^{-14} m/s².

3 Receiving and preparing the data

Gravity data have been recorded for decades and IGETS Potsdam [17] stores correspondingly long data series. Previously used for earthquake research, the gravimeters are mounted directly on the ground and therefore respond to distant, minute ground motions. Stronger earthquakes overload the sensors and cause data gaps of several minutes. Before release, the gaps are filled by artificial tidal signals, then the signals from the sensors pass through low-pass filters with a cutoff frequency of 8.3 mHz. When searching for GW in the frequency range around 1 mHz, the influence of the variable air mass above the gravimeter should be taken into account.

In [1] and [6] some frequencies are given in the vicinity of which conspicuous spectral lines of unclear origin were discovered. If these are generated by GW, it should be possible to detect characteristic modulations (for data sets spanning several years):

- The frequency of GW is well defined (narrow linewidth) and increases slowly. The natural frequencies of the earth are damped oscillations and differ from GW by considerably larger half-widths.
- Amplitude and direction of polarization of continuous GW are constant. Because the gravimeters are directionally sensitive antennas that are constantly moving, the received signal may be amplitude modulated.
- The frequency of GW is phase modulated (PM) with $f_{year} = 31.69$ nHz because the earth orbits the sun. This modulation creates additional spectral lines at a distance of $n \cdot f_{year}$. The modulation index of the PM can be calculated from the amplitudes of these sidebands.
- The reception frequency of GW is probably phase-modulated with $f_{day} = 11.6 \ \mu \text{Hz}$ because the earth rotates around its own axis in 23.93447192 hours. After earthquakes, one observes that the natural frequencies of the earth split into five singlets

 $f, f \pm f_{day}$ and $f \pm 2 \cdot f_{day}$ [7]. There are various reasons for this: The Coriolis force as a result of the Earth's rotation produces splitting because deformations of the Earth's crust spread faster in the direction of the rotation than in the opposite direction. Then there is the elliptical shape of the earth: the path across the poles is 67 km shorter than the equator. Whether this also applies to GW still needs to be investigated.

Thus, the criteria to be tested are fixed: frequency spacing of the spectral lines, amplitude and phase ratios, frequency drift. The database consists of the records of eight gravimeters (PE, OS, BF1+2, MB, ST, CO and CB), which recorded particularly low-noise data between the years 2009 and 2018. To ensure high frequency resolution, 120 consecutive records from each gravimeter (one month each) are concatenated into a single long data chain and all tide signals below 100 μ Hz are removed by a high-pass filter. In a last step the center frequency of the search range is reduced from 840 μ Hz to 150 μ Hz (Superhet method).

4 The search for GW

The goal of this investigation is: Do we find spectral lines in the vicinity of 840 μ Hz that fulfill the above list of requirements and are therefore probably produced by a GW?

No single spectrum of a gravimeter shows anomalies with sufficient S/N to indicate GW. Therefore, the available long-term records are phase-coherently added to improve the S/N. The procedure is based on the following consideration: Seven gravimeters are located – closely adjacent – in Central Europe and one on the opposite side of the earth in Canberra/Australia. The shortest distance L is about 12×10^6 m long and passes almost through the center of the earth. When a GW passes the Earth, L oscillates in the same rhythm and sinusoidally with the maximum amplitude ΔL . Since L is much smaller than the assumed wavelength of the GW ($\lambda \approx 3.6 \times 10^{11}$ m), the strain h is calculated with the approach

$$h = \Delta L/L = h_0 \cdot \sin(\omega t) \tag{3}$$

Previous estimates give values $h_0 \approx 10^{-19}$ for binary systems in our Galaxy. Thus the change of the local gravity \ddot{L} near the Earth surface is

$$\ddot{L} = L \cdot \omega^2 \cdot h_0 = 12 \times 10^6 \ m \cdot (2\pi \cdot 837 \times 10^{-6} \ \text{Hz})^2 \cdot 10^{-19} \approx 3.3 \times 10^{-17} \ \frac{m}{s^2}$$
(4)

This value undercuts the sensitivity of superconducting gravimeters by a factor of 1000 (see section 2). In fact, no spectral lines with S/N > 2 are found in the vicinity of 840 μ Hz.

In the investigation [8] it is shown that the S/N improves considerably if one removes all phase modulations [9]. Therefore, one looks for spectral lines in the vicinity of 840 μ Hz whose amplitude increases when one compensates the PM with f_{year} . Any GW from outside our solar system exhibits this PM. A single solution was found at $f_{GW} = 836.69 \ \mu \text{Hz}$, to which all further investigations refer.

5 Influence of the PM at 836.69 μ Hz

On earth one can receive only modulated GW, no constant frequencies. The probability that we have found a GW increases with the number of signatures (see Section 3). Any kind of modulation generates sidebands (spectral lines next to f_{GW}) which affect the reception of the GW:

- In PM, the sidebands take their energy from the central frequency. If one compensates the PM, the amplitude of the central frequency increases.
- The bandwidth of the signal processing must be at least wide enough to contain all essential sidebands of the PM. This increases the noise level (see equation (1)). If all modulations are compensated, bandwidth and noise can be reduced drastically.

Theoretically, a PM signal occupies infinite bandwidth. In practice, one chooses only the strongest sideband frequencies in the vicinity of f_{GW} . This range is called the Carson bandwidth, which is calculated using the modulation index η . If the source of GW is close to the ecliptic, the maximum frequency shift due to the Doppler effect reaches the value

$$\Delta f = f_{GW} \cdot \left(\sqrt{\frac{c + v_{Earth}}{c - v_{Earth}}} - 1 \right) \approx 84 \ n\text{Hz}.$$
 (5)

Then the modulation index is

$$\eta = \frac{\Delta f}{f_{modulation}} = \frac{\Delta f}{f_{year}} = 2.64 \tag{6}$$

and the signal must be processed with bandwidth

$$BW_{Carson} = 2 \cdot f_{year}(\eta + 1) = 231 \mu \text{Hz}.$$
(7)

A technical remark: The Bessel function $J_0(2.4)$ has the value zero. This means: For $\eta = 2.4$ the central frequency f_{GW} is not detectable, because all the energy is in the sidebands. The result of equation (6) is so close to the critical value that the amplitude of f_{GW} almost vanishes. The multiple zeros of $J_0(\eta)$ may also be the cause that some unidentified lines in figure 2 of [1] look strikingly broad. For certain values of η , the sidebands have higher amplitudes than f_{GW} .

To determine the central frequency f_{GW} , one must eliminate the PM. For $\eta = 2.64$ it follows from formulas (12) and (13) in [9]:

- If one compensates the PM, the associated sidebands disappear and the central spectral line at f_{GW} doubles its amplitude.
- Therefore, one may reduce the bandwidth of the signal processing from 231 μ Hz (equation (7)) to 2 μ Hz (equation (2)), which decreases the amplitude of the noise by a factor of 10.7.

Both effects multiply and improve the S/N by a factor of 460 and thus the probability of detecting GW.

6 A peculiar phase relation

According to theory, GW change the length of scales. Therefore in section 4 it was assumed that each GW changes the diameter of the earth periodically. Since gravimeters measure only the (locally) vertical component, the instruments in Europe and Australia should measure *equal* acceleration. To test this conjecture, we compare records from two geographically distant regions:

- Seven gravimeters PE, OS, BF1, BF2, MB, ST, and CO measure in Europe. Because they are closely spaced and measure only the (locally) vertical component, we may assume that they measure in-phase accelerations. Their measured values are added and given the name EU.
- The only instrument in Australia is located in Canberra. To adjust the amplitude to the European sum value, we multiply the measured values of CB by a factor of seven.

Combining the data of these two groups in different ways and then comparing the amplitudes of adjacent spectral lines, we obtain a most surprising result. One spectral line is the Earth's natural resonance 0S0, an isotropic mode of vibration excited by earthquakes and weakly damped ($Q \approx 5500$ [12]). Therefore, a sharp spectral line can be measured everywhere on the Earth's surface at $f_{0S0} = 814.39 \ \mu$ Hz. The other spectral line is at $f_{GW} = 836.68949(4) \ \mu$ Hz, has an even smaller half-width and is probably generated by a GW (The description of the measurement follows in section 7). From the table 1 we take the following statements:

- As expected, the eigenresonance 0S0 can be detected with the same amplitude in Europe and Australia. (lines 1+2)
- The acceleration caused by the eigenresonance 0S0 in Europe and Australia is *in phase*. This is shown by adding the measured values from Europe and Australia (line 3). Cause: all gravimeters decrease or increase synchronously their distance from the center of the earth. If one subtracts these measured values, the amplitudes of the eigenresonance 0S0 compensate almost completely (line 4). This confirms the good calibration of all gravimeters.

Gravimeter	Amp(0S0)	S/N	Amp(GW)	S/N	Remarks
	$f = 814 \ \mu \text{Hz}$		$f_{GW} = 836 \ \mu \text{Hz}$		
EU	3290	552	302	4	7 gravimeters
CB	3323	163	434	4	one gravimeter
EU + CB	6580	481	525	1.6	very noisy
EU – CB	587	4	611	9.4	prominent line

Table 1): Comparison of amplitudes of adjacent spectral lines depending on the combination of data sources from Europe and Australia. The amplitudes in the 4th column are determined in section 7.

- For the suspected GW at 836.69 μ Hz the opposite is true: The S/N worsens when adding the readings from Europe and Australia. Then the spectral line is hardly detectable. Obviously, the exciting GW *does not* cause the diameter of the earth to change periodically (line 3).
- If one subtracts the records from Europe and Australia, the amplitude of the spectral line of the suspected GW at 836.69 μ Hz and the S/N increase (line 4). The simplest explanation is: the GW does not (or very little) change the diameter of the Earth, but it pushes the Earth back and forth. Just as a surface wave does not deform swimming ducks but makes them dance up and down.

That would explain, why so far – despite intensive search – nobody could prove continuous GW with LIGO: The distances between the mirrors of the interferometers do not change, they move as a rigid unit. Maybe LIGO should not measure the distance of the mirrors but the acceleration? If this explanation is correct, the previous assumption would have to be corrected that GW only change the lengths of scales (strain $h = \Delta L/L$) and not their position.

7 The compensation of the phase modulation

In the first step one combines the records of eight low-noise gravimeters as described in section 6: The data combination PE+OS+BF1+BF2+MB+ST+CO - 7*CB passes through an IQ bandpass filter which reduces the center frequency 836 μ Hz to 150 μ Hz. The bandwidth 260 μ Hz is sufficient to allow all sideband frequencies of the suspected PM to pass. The subsequent decimation of the file length by a factor of 24 accelerates the data processing.

Then a time-consuming iteration eliminates all the characteristics of the presumed PM (see section 3). The MSH procedure used in this process [8] improves the S/N of the GW so much that the remaining spectral line at f_{GW} becomes unmistakable. The iteration ends when the PM of the auxiliary oscillator (in the MSH process) matches that of the original GW signal.

The direction of incidence of a GW is presumably unrelated to the orbit and axis of rotation of the Earth. The EU-Australia link can be considered as a linear receiving antenna moving in the wave field of the GW and constantly changing its angle with respect to the direction of the GW. These effects modulate the received signal and the PM is modeled using the two approaches:

$$\Phi_y = a_y \cdot \sin(p_y + t \cdot \omega_y) + a_{2y} \cdot \sin(p_{2y} + t \cdot 2\omega_y) + a_{3y} \cdot \sin(p_{3y} + t \cdot 3\omega_y) \tag{8}$$

Thereby applies $\omega_y = 2\pi \cdot f_{year}$;

 a_{2y} and p_{2y} are amplitude and phase of the double frequency,

 a_{3y} and p_{3y} are amplitude and phase of the triple frequency.

The evaluation shows that the PM at the rate of several hours should be modeled by four summands:

$$\Phi_d = \sum_{n=1}^4 a_{nd} \cdot \sin(p_{nd} + t \cdot n \cdot \omega_d) \tag{9}$$

Thereby applies $\omega_d = 2\pi \cdot f_{day}$;

 a_{nd} and p_{nd} are amplitude and phase of n times the frequency.

In addition, f_{GW} and the frequency drift are determined. A total of 16 parameters are optimized with the goal of maximizing the amplitude of the spectral line f_{GW} . Figure 1 shows the result.



Figure 1): Spectrum of the environment of fGW after compensation of all phase modulations and the drift. Before compensation of PM, the total energy of GW is distributed on many spectral lines and therefore disappears in the noise. After compensation, the energy is concentrated on a single line at $f_{GW} = 836.69 \ \mu Hz$. 0S0 is always a sharp single line because it is weakly attenuated ($Q \approx 5500$). The group 0S5 is a bundle of eleven closely spaced lines.

Because the earth rotates, the eigenresonance 0S5 splits into eleven closely spaced lines and occupies the frequency range between 835.4 μ Hz and 843 μ Hz [10]. Q of this eigenresonance is 355 [11] and causes line widths of about 2400 nHz. The sharp spectral line GW in figure 1 is different: Its half-width 2 μ Hz is extraordinarily small. The corresponding Q factor is $\approx 4 \times 10^5$ and far above the range measured for natural resonances of the Earth. The frequency drift of f_{GW} is 1.69 nHz per year. This striking line cannot be explained by resonance effects.

8 PM in annual rhythm

With the approach (8) one determines the slow PM of the period duration 365 days. From the measured parameters a_{ny} and p_{ny} , the Bessel coefficients of the PM ($J_0 = 0.6377$, $J_1 = 0.5144$, $J_2 = 0.175$ and $J_3 = 0.038$) and the modulation index $\eta = 1.266$ are calculated. From this follows the maximum frequency deviation $\Delta f_{year} = \eta \cdot f_{year} =$ 40.11 nHz, which the Doppler effect produces due to the orbital speed of the earth. Since Δf_{year} is much smaller than the maximum value 83.67 nHz (equation (5)), the ecliptic latitude of the source of GW is $\pm 28.65^{\circ}$.

The frequency f_{GW} oscillates sinusoidally around the mean value in the course of a year, if one ignores the PM in the daily rhythm (see section 9). It reaches its minimum on the 90th day of the year and its maximum value on the 273rd day. From this follows the ecliptic longitude of the source of GW.

9 PM in daily rhythm

With the approach (9) one checks whether the GW is phase modulated with the period 24 hours (or shorter). From the measured parameters a_{nd} follow first the Bessel coefficients $(J_0 = -0.3342, J_1 = 0.2387, J_2 = 0.4808 \text{ and } J_3 = 0.352 \text{ and } J_4 = 0.1678)$ and the modulation index $\eta = 3.256$. The calculated maximum frequency deviation $\Delta f_{day} = \eta \cdot f_{day} = 37.79 \ \mu\text{Hz}$ is remarkable in several respects:

- The low peripheral speed of the earth at the equator of 460 m/s can produce a maximum frequency deviation of ± 1.3 nHz in the 24-hour rhythm. The measured frequency deviation Δf_{day} is 29000 times larger and changes in a 12-hour rhythm. This very strong PM is not generated by the Doppler effect.
- The Bessel coefficient J_2 outperforms its neighbors J_1 and J_3 . Therefore, the main rhythm of the fast PM is about 12 hours. The precise value is determined in the section 11.

10 Checking the amplitude constancy of the GW

A spectrum contains no information whether and how the amplitude of a signal changes over time. A phase-sensitive integrator acts like an extremely narrow-band filter and can answer this question. One possible method is based on the addition theorem of trigonometric functions and can measure only a single frequency f_{check} . The SG data are digitized and consist of discrete values. Between successive readings z_n and z_{n+1} of the signal, a certain time interval passes (the sampling time T_s). With each step, the phase angle increases by the value $\alpha = 2\pi T_s \cdot f_{check}$. If one wants to know whether the received noise z_n, z_{n+1}, z_{n+2} ... contains a signal of frequency f_{check} , one sums the amplitudes by alternately calculating the two formulas:

$$x_{n+1} = z_n + \cos(\alpha)x_n + \sin(\alpha)y_n \tag{10}$$

$$y_{n+1} = \cos(\alpha)y_n - \sin(\alpha)x_n \tag{11}$$

The sequence of values x_n and y_n depends on the choice of parameters:

- Without an injected signal and with the initial values $x_1 = 1$, $y_1 = 0$, the formulas calculate a table of values for $x = sin(2\pi tf)$ and $y = cos(2\pi tf)$ with constant amplitude.
- Setting $x_1 = y_1 = 0$ and injecting a monochromatic signal z_n whose frequency matches f_{check} , the formulas calculate an oscillation whose amplitude increases in proportion to time.
- If the injected frequency differs from f_{check} or if the injected signal z_n varies in phase or amplitude, the output signal of the integrator is small and not linear.
- If noise is injected, the formulas calculate a low-bandwidth frequency mixture in the vicinity of f_{check} , whose amplitude fluctuates irregularly.

Choosing $f_{check} = 836.69 \ \mu$ Hz and injecting the signal mixture recorded by the SG, we obtain the result shown in Figure 2. Since the integrated amplitude increases in proportion to time, the injected signal mixture contains a signal whose frequency and amplitude are constant throughout the ten-year period – as expected from a continuous GW.

The small deviations from the linear increase of the amplitude (see Figure 2) show the almost perfect constancy of amplitude and phase of the injected signal z_n over the entire period of ten years – a clear indication of a continuous signal.



Figure 2): After the signal of the GW is freed from all PM and frequency drift, the frequency should no longer change. The slightest phase fluctuations would produce noticeable deviations from linearity. Apparently, earthquakes cannot affect the signal even though the gravimeters are mounted directly on the ground.

11 Amplitude modulation (AM) in diurnal rhythm

The MSH procedure determines and compensates the characteristic values of the PM. Subsequently, there are no more sidebands that can be assigned to a PM. This does not exclude that further modulations are present. If the GW is additionally amplitude modulated, sidebands with corresponding frequencies still exist. If the signal is modulated with a *single* frequency f_{AM} , the spectrum consists of three spectral lines in the distance f_{AM} . This constellation can be tested with a matched filter that allows only these three frequencies to pass. Each passband has a bandwidth of 32 nHz in order to visualize a possible fine structure of the amplitude response (see figure 4).

Testing whether the GW of frequency $f_{GW} = 836.69 \ \mu$ Hz is permanently amplitude modulated with one of the oscillation durations 6, 12, and 24 hours yields a result with good S/N only for $T_{mod} \approx 12$ h (figure 3). This modulation is not excited by earthquakes because there are no abrupt amplitude changes with exponential decay during the whole period. The mean amplitude is not constant, but shows peculiar fluctuations at four-year intervals. These are possibly caused by the GW source itself.





If one enlarges the first days of the amplitude modulation shown in figure 3, one recognizes fast amplitude changes (figure 4). After every 11.9678 ± 0.0002 hours – which is almost exactly half the sidereal day length – the amplitude maxima repeat. This rhythm remains constant throughout the ten-year investigation period. The striking deviation from synodic daylength suggests a GW source outside the solar system.

A possible explanation: If one interprets the straight line between Europe and Canberra/Australia as a linear GW antenna, it rotates by 180° per 11.9678 hours. As with all linear antennas, the reception amplitude depends on the angle between the orientation of this antenna and the propagation direction of the GW. Without knowledge of the directional pattern of this antenna, the direction to the source of the GW cannot be determined.



Figure 4): The stretched initial range of the envelope of Fig 3. The amplitude of GW changes at the rhythm of half the sidereal daylength. The modulation degree A_{max}/A_{min} is remarkably large. Time counting starts at 0 o'clock on 2009-01-01.

12 Summary and Discussion

In all records of the eight superconducting gravimeters (at different locations), one finds a spectral line at $f_{GW} = 836.68949(4) \ \mu \text{Hz}$ whose frequency and mean amplitude are strikingly constant throughout the ten-year measurement period. All modulations of this signal prove an excitation by a GW:

- The signal is phase modulated with f_{year} and the modulation index η is below the limit of the Doppler shift calculated from the motion of the Earth in orbit.
- The signal is also phase modulated at a much higher frequency, which matches very well with the half sidereal day duration. Therefore, it is very unlikely that the signal is generated in the Sun or in near-Earth space.
- The frequency deviation of this PM in a 12-hour rhythm far exceeds the limit that can be expected based on the low circumferential velocity at the equator. In addition, this PM would have to have a 24-hour rhythm.
- The signal is also amplitude modulated at the rhythm of half the sidereal day duration. Rotating dipole antennas provide such signals.
- The comparatively very small half-width and large amplitude of the spectral line of f_{GW} exclude that this line is an intrinsic resonance of the Earth, although it is in the frequency range of the group 0S5.
- The peculiar, experimentally secured and verifiable phase relations of the records from Europe and Australia (section 6) cannot be explained by the assumption that GW influence the length of scales. The assumption that GWs shift the scale would solve the puzzle why no continuous GWs have been detected with LIGO so far and despite the highest sensitivity. Installing some gravimeters in Australia or New Zealand could help solve this puzzle.
- Integration of the signal with a phase-sensitive method confirms an extremely small variation of the phase during the ten-year measurement period. This rules out excitation by non-synchronized earthquakes.

With the help of the MSH method [8], many GW could be detected in the records of gravimeters in the frequency range between 400 μ Hz and 60 Hz [19]. Probably there are so many binary star systems and pulsars in our galaxy that the sum of all GW causes the enormous background noise measured by superconducting gravimeters (compare Figure 1). Conjectures that the instruments themselves or even unknown processes in the Earth's interior produce this noise will be disproved at the latest when it is possible to apply the Hogbohm method of radio astronomy to the records of gravimeters.

At no point in this investigation is *stacking* of spectra performed, because in doing so the important phase relations are destroyed by magnitude formation of the FFT result. The experimental findings in section 6 cannot be explained without knowledge of the phase relations. Therefore, in this study, raw data are always linked *without* calculating magnitudes.

13 Data availability

The recordings of all gravimeters can be downloaded free of charge from GFZ Potsdam [17]. A detailed description of the IGETS data base, the IGETS products and the registration procedure can be found in [18]. The raw data are formatted as ASCII files and cover one month each. They may be requested as MATLAB files from the author.

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