The Helmholtzian factorization shows that the heavy leptons and quarks are separated as:

$$(m, m, m, m) = (m, m, m, 0) + (0, 0, 0, m) = m + m.$$  

such that:

- $$(m + m)^2$$ satisfies the Dirac factoring, and:
- $$(m + m)^2$$ satisfies the Helmholtzian factorization, as a pythagorean quadruple [5][6], yielding the pythagorean quadruple.

As noted in [1], the light leptons - neutrinos are not so easily separated, meaning that:

- $$m, m, m, m$$ have characteristics beyond simply adding the heavy lepton plus quark characteristics.

a small table of pythagorean quadruples [7]:

<table>
<thead>
<tr>
<th>(a, b, c, d)</th>
<th>(a, b, c, d)</th>
<th>(a, b, c, d)</th>
<th>(a, b, c, d)</th>
<th>(a, b, c, d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1, 1, 1, 2)</td>
<td>(1, 1, 3, 5)</td>
<td>(1, 1, 7, 10)</td>
<td>(1, 2, 2, 5)</td>
<td>(1, 2, 4, 10, 11)</td>
</tr>
<tr>
<td>(1, 2, 2, 4, 5)</td>
<td>(1, 2, 4, 10, 11)</td>
<td>(1, 2, 8, 10, 13)</td>
<td>(1, 2, 8, 10, 13)</td>
<td></td>
</tr>
<tr>
<td>(2, 2, 2, 5)</td>
<td>(2, 2, 3, 9)</td>
<td>(2, 2, 3, 8, 9)</td>
<td>(2, 2, 4, 5, 7)</td>
<td>(2, 2, 7, 8, 11)</td>
</tr>
<tr>
<td>(2, 2, 7, 8, 11)</td>
<td>(2, 4, 5, 6, 9)</td>
<td>(2, 4, 7, 10, 13)</td>
<td>(4, 4, 4, 11, 13)</td>
<td>(4, 4, 5, 8, 11)</td>
</tr>
<tr>
<td>(4, 4, 5, 8, 11)</td>
<td>(4, 4, 8, 13)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As noted before [5]:

- in general, from any pythagorean quadruple $$(a, b, c, d)$$ holds $$(a + a + a + a)^2 = \left(\theta^2 \right)^2 = \left(\theta \right)^2$$:

$$m = (a, b, c, d) \times \frac{\theta^2}{a^2}$$.

Similarly:

- from any pythagorean quintuple $$(a, b, c, d, e)$$ holds $$(a + a + a + a + a)^2 = \left(\theta^2 \right)^2 = \left(\theta \right)^2$$:

$$m = (a, b, c, d, e) \times \frac{\theta^2}{a^2}$$.

For pythagorean triples $$(a, b, c)$$:

- the parametrization: $$(a, b, c) = \left(2, 2, 2\right)$$.

Now consider the fermions:

- $$m_e = m(3, 1)$$
- $$m_u = 5m_e$$
- $$m_e = 1 \cdot \left(\frac{2}{1150} \right)^2 m_e$$
- $$m_d = 10m_e$$

and likewise for neutrinos.
and quarks:

\[
\begin{align*}
    m_u &= (4, 10, 28) \frac{2}{3} m_c, &
    m_c &= (4, 10, 28) (\frac{23}{40}, \frac{7}{5}) \cdot m_u, &
    m_s &= (4, 10, 28) \sqrt[3]{\frac{5}{2}} \cdot m_u,
\end{align*}
\]

\[
\begin{align*}
    m_d &= (4, 10, 28) \frac{1}{3} m_u, &
    m_s &= (4, 10, 28) (\frac{23}{40}, \frac{7}{5}) \cdot m_d, &
    m_s &= (4, 10, 28) \sqrt[3]{\frac{5}{2}} \cdot m_d,
\end{align*}
\]

\[
\left(\frac{2}{3} \cdot \frac{5}{3} \cdot \frac{14}{3} \cdot \frac{1}{3} \right) = (4, 10, 28) \frac{1}{3} \Rightarrow 30 \cdot \frac{1}{3} = 5 = \frac{15}{3}
\]

\[
(4, 10, 28) \cdot 60k = 30 \cdot 2k = \frac{5}{3} \cdot 6 \cdot 2k = \frac{15}{3} \cdot 12k
\]

\[
\begin{align*}
    m_u &= (\frac{2}{3}, \frac{5}{3}, \frac{14}{3}, \frac{1}{3}) m_u, &
    m_c &= (\frac{2}{3}, \frac{5}{3}, \frac{14}{3}, \frac{1}{3}) 12k m_c, &
    m_s &= (\frac{2}{3}, \frac{5}{3}, \frac{14}{3}, \frac{1}{3}) \sqrt[3]{\frac{5}{2}} \cdot k m_u,
\end{align*}
\]

\[
\begin{align*}
    m_d &= (\frac{2}{3}, \frac{5}{3}, \frac{14}{3}, \frac{1}{3}) 4m_u, &
    m_s &= (\frac{2}{3}, \frac{5}{3}, \frac{14}{3}, \frac{1}{3}) 4k m_c, &
    m_s &= (\frac{2}{3}, \frac{5}{3}, \frac{14}{3}, \frac{1}{3}) \sqrt[3]{\frac{5}{2}} \cdot k m_u,
\end{align*}
\]

Note:

\[
\begin{align*}
    u^* &= (\frac{2}{3}, \frac{5}{3}, \frac{14}{3}, \frac{1}{3}) + \left(\frac{2}{3}, \frac{5}{3}, \frac{14}{3}, \frac{1}{3}\right) = (0, 0, 0) = (\frac{1}{3}, \frac{5}{3}, 0)
\end{align*}
\]

Note:

although the mass constituents may be in any order the addition is equivalent to aligning/rearranging the constituents from least to largest and adding and adding the resultant tuple, constituent-wise

so:

the primitive triples of the quark mass/color constituent pythagorean quadruples add up to 0 (mod 3), as color, and

the first/least of the primitive triples of the quark mass/color constituent pythagorean quadruples add up to 0 (mod 2) as charge, and

the total sum of the squares of the primitive triples is the square of the quark mass magnitude (the last/largest constituent pythagorean quadruple/quintuple)

Thus, the quark mass/color/charge constituent pythagorean quadruple/quintuple is fully determinant of

of the quark mass, color, and charge.

However, because each quark has a different multiple, the fermion decay/interaction combination of mass magnitudes is more complex.

These multiple corroborations attest to the quark primitive pythagorean triples/quadruples/quintuples

\[
\left(\frac{2}{3}, \frac{5}{3}, 0, \frac{5}{3}\right) \& \left(\frac{2}{3}, \frac{5}{3}, \frac{14}{3}, 0, 5\right)
\]

and to the Helmhotzian factorization yielding the fermion and quark architectures.
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