A NOTE ON ELECTROMAGNETIC ENERGY IN THE CONTEXT OF COSMOLOGY.

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Abstract. The density of electromagnetic energy and its flux are given by the well-known formulas which are widely used in classical electrodynamics. We rederive these formulas in the framework of special relativity and then extend them to the context of cosmology with the Big Bang.

1. Introduction.

In classical physics an electromagnetic field in vacuum is described by two vectors $\mathbf{E}$ and $\mathbf{H}$. They present the intensity of an electric field and the intensity of a magnetic field respectively. These vectors obey the Maxwell equations (see § 26 and § 30 in Chapter V of [1] or § 1 in Chapter II of [2]):

$$\operatorname{div}\mathbf{H}=0, \qquad \operatorname{rot}\mathbf{E}=-\frac{1}{c}\,\frac{\partial\mathbf{H}}{\partial t}, \tag{1.1}$$

$$\operatorname{div}\mathbf{E}=4\pi\rho, \qquad \operatorname{rot}\mathbf{H}=\frac{4\pi}{c}\,\mathbf{j}+\frac{1}{c}\,\frac{\partial\mathbf{E}}{\partial t}. \tag{1.2}$$

Here ρ is the charge density and $\mathbf{j}$ is the density of current. They obey the equation

$$\frac{\partial\rho}{\partial t}+\operatorname{div}\mathbf{j}=0, \tag{1.3}$$

which is called the charge conservation law (see § 29 in Chapter V of [1] or § 5 in Chapter I of [2]).

Being a form of materia, an electromagnetic field can store energy, can transport energy at a distance, and can transfer energy to other forms of materia. These features of an electromagnetic field are expressed by the following formulas[1]:

$$\varepsilon=\frac{|\mathbf{E}|^2+|\mathbf{H}|^2}{8\,\pi}, \qquad \mathbf{S}=\frac{c}{4\,\pi}\,[\mathbf{E},\mathbf{H}], \qquad w=(\mathbf{E},\mathbf{j}). \tag{1.4}$$

The quantity ε in (1.4) is the density of energy of an electromagnetic field (see § 2 in Chapter II of [2]). The vector quantity $\mathbf{S}$ in (1.4) is the flux density for the energy

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[1] Through square brackets with comma in (1.4) and throughout the paper we denote the cross product of three-dimensional vectors: $[\mathbf{E},\mathbf{H}]=\mathbf{E}\times\mathbf{H}$. Similarly, through round brackets with comma we denote the dot product of three-dimensional vectors $(\mathbf{E},\mathbf{j})=\mathbf{E}\cdot\mathbf{j}$.

flow of an electromagnetic field. It is also known as Umov-Poynting vector (see [3]). The scalar quantity w in (1.4) is called the specific rate of energy loss. If $w > 0$, it indicates the amount of electromagnetic energy transferred to charged particles in the unit volume per the unit of time. Otherwise, if $w < 0$, this quantity indicates the specific amount of energy pumped back into electromagnetic field by charged particles. The quantities ε, $\mathbf{S}$, and w from (1.4) obey the following equation (see § 2 in Chapter II of [2]) similar to (1.3):

$$\frac{\partial\varepsilon}{\partial t}+\operatorname{div}\mathbf{S}=-w. \tag{1.5}$$

Passing from classical physics to special relativity, we replace the three-dimensional space by the four-dimensional one, which is called the spacetime or the Minkowski space. The time variable t multiplied by the speed of light constant c becomes a coordinate in the Minkowski space

$$x^0=c\,t \tag{1.6}$$

along with three other coordinates x^1, x^2, x^3. In (1.6) and below throughout the paper we use upper and lower indices according to Einstein's tensorial notations (see [4] and § 20 in Chapter I of [5]).

In special and general relativity the quantities ρ and $\mathbf{j}$ from (1.3) are incorporated into a single four-dimensional vector:

$$\mathbf{j}=\begin{Vmatrix} j^{\,0}\\ j^1\\ j^2\\ j^3\end{Vmatrix},\quad\text{where}\quad j^{\,0}=c\,\rho. \tag{1.7}$$

Due to (1.6) and (1.7) the equation (1.3) is written as

$$\sum_{q=0}^{3}\nabla_{q}\,j^{\,q}=0. \tag{1.8}$$

In special relativity the covariant derivatives ∇_i coincide with the partial derivatives: $\nabla_i=\partial/\partial x^i$. In general relativity they are calculated in a more complicated way, see formula (5.12) in § 5 of Chapter III in [6].

Despite the striking similarity of the equations (1.3) and (1.5), in special and general relativity the quantities (1.4) are not considered at all. The main goal of this paper is to discuss a way for introducing the analogs of the quantities (1.4) in the framework of special and general relativity and then to construct their extension to cosmology with the Big Bang.

2. The case of special relativity.

In special relativity the spacetime is a flat manifold topologically equivalent to $\mathbb{R}^4$ and equipped with the structure of an affine space and with a pseudo-Euclidean metric $\mathbf{g}$ of the signature $(+\,-\,-\,-)$. This metric is known as the Minkowski metric. Due to the structure of an affine space the spacetime of special relativity admits Cartesian coordinates. Those of them which are orthonormal with respect to the Minkowski metric $\mathbf{g}$ are physically interpreted as inertial coordinate systems. Coordinates of any two inertial coordinate systems are related to each other by Lorentz transformations. In any inertial coordinate system the metric tensor and the inverse metric tensor of the metric $\mathbf{g}$ are given by the following matrix:

$$g_{ij} = g^{ij} = \begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{Vmatrix}. \tag{2.1}$$

An electromagnetic field in special relativity is presented by a skew-symmetric tensor $\mathbf{F}$ whose components are given by the following matrices:

$$F^{pq} = \begin{Vmatrix} 0 & -E^1 & -E^2 & -E^3 \\ E^1 & 0 & -H^3 & H^2 \\ E^2 & H^3 & 0 & -H^1 \\ E^3 & -H^2 & H^1 & 0 \end{Vmatrix}, \qquad F_{pq} = \begin{Vmatrix} 0 & E^1 & E^2 & E^3 \\ -E^1 & 0 & -H^3 & H^2 \\ -E^2 & H^3 & 0 & -H^1 \\ -E^3 & -H^2 & H^1 & 0 \end{Vmatrix}. \tag{2.2}$$

In terms of the matrices (2.2) the Maxwell equations (1.2) are written as

$$\sum_{q=0}^{3} \frac{\partial F^{pq}}{\partial r^q} = -\frac{4\pi}{c}\, j^p. \tag{2.3}$$

The other two Maxwell equations (1.1) are written similarly

$$\sum_{q=0}^{3}\sum_{k=0}^{3}\sum_{s=0}^{3} \varepsilon^{pqks}\,\frac{\partial F_{ks}}{\partial r^q} = 0. \tag{2.4}$$

Here ε^{pqks} is the four-dimensional analog of the Levi-Civita symbol:

$$\varepsilon_{pqks} = \varepsilon^{pqks} = \begin{cases} 0, & \text{if among } p,\ q,\ k,\ s \text{ there are at} \\ & \text{least two equal numbers;} \\ 1, & \text{if } (p\,q\,k\,s) \text{ is an even permutation} \\ & \text{of the numbers } (0\,1\,2\,3); \\ -1, & \text{if } (p\,q\,k\,s) \text{ is an odd permutation} \\ & \text{of the numbers } (0\,1\,2\,3). \end{cases} \tag{2.5}$$

The above formulas (2.2), (2.3), (2.4), and (2.5), can be found in § 8 and § 9 of Chapter III in the book [2].

The energy and momentum stored in any form of materia both are described by its energy-momentum tensor $\mathbf{T}$ (see [7]). In the case of an electromagnetic field the components of its energy-momentum tensor is given by the formula

$$T^{qj} = -\frac{1}{4\pi}\sum_{p=0}^{3}\sum_{i=0}^{3}\left(F^{pq}\,g_{pi}\,F^{ij} - \frac{1}{4}F_{pi}\,F^{pi}\,g^{qj}\right), \tag{2.6}$$

see formula (94.8) in § 94 of Chapter XI in [1] or formula (4.5) in § 4 of Chapter V in [2]. The tensor (2.6) is symmetric and traceless. It obeys the following equality

$$\sum_{s=0}^{3}\nabla_s T^{ps} = -\frac{1}{c}\sum_{s=0}^{3} F^{ps}\,j_s, \tag{2.7}$$

see formula (4.6) in § 4 of Chapter V in [2]. Remember that in special relativity the covariant derivatives ∇_s in (2.7) coincide with the partial derivatives: $\nabla_s = \partial/\partial x^s$. The equality (2.7) encloses the equality (1.5) in the case $p = 0$, but it does not reduce to (1.5) as a whole.

Our goal in this section is to separate the equation (1.5) from other equations enclosed in (2.7). Assuming that some inertial coordinate system is chosen and fixed, we take the unit vector along its time axis:

$$\mathbf{n} = \mathbf{e}_0 = \begin{Vmatrix} n^0 \\ n^1 \\ n^2 \\ n^3 \end{Vmatrix} = \begin{Vmatrix} 1 \\ 0 \\ 0 \\ 0 \end{Vmatrix}. \tag{2.8}$$

Then from (2.8) we derive

$$\nabla_s n^p = 0, \qquad\qquad \nabla_s n_p = 0, \tag{2.9}$$

since in special relativity covariant derivatives coincide with partial derivatives. Applying (2.9), from (2.8) we derive

$$\sum_{p=0}^{3}\sum_{s=0}^{3}\nabla_s(c\,T^{ps}\,n_p) = c\sum_{p=0}^{3}\sum_{s=0}^{3}\nabla_s(T^{ps})\,n_p = -\sum_{p=0}^{3}\sum_{s=0}^{3}F^{ps}\,j_s\,n_p. \tag{2.10}$$

Due to (1.7), (2.1), (2.2), and (2.8) the right hand side of (2.10) is transformed as

$$-\sum_{p=0}^{3}\sum_{s=0}^{3}F^{ps}\,j_s\,n_p = -\sum_{s=1}^{3}E^s\,j^s = -(\mathbf{E},\mathbf{j}) = -w. \tag{2.11}$$

Looking at the left hand side of the equality (2.10), we define the four-dimensional vector $\mathbf{J}$ with the following components:

$$J^s = \sum_{p=0}^{3} c\,T^{ps}\,n_p. \tag{2.12}$$

Using (2.1), (2.2), (2.6) and (2.8), we can calculate the components of the vector (2.12) explicitly. It turns out that

$$\mathbf{J} = \begin{Vmatrix} J^0 \\ J^1 \\ J^2 \\ J^3 \end{Vmatrix} = \begin{Vmatrix} c\,\varepsilon \\ S^1 \\ S^2 \\ S^3 \end{Vmatrix}, \tag{2.13}$$

where ε is the density of electromagnetic energy from (1.4) and S^1, S^2, S^3 are the components of the Umov-Poynting vector from (1.4). Due to (2.11) and (2.12) the equality (2.10) is written as the following equation:

$$\sum_{s=0}^{3} \nabla_s J^s = -w. \tag{2.14}$$

It is easy to see that the equation (2.14) is an analog of the equation (1.8). Due to (1.6) and (2.13) the equation (2.14) is a four-dimensional version of the three-dimensional equation (1.5).

Thus, we have found a four-dimensional presentation for the equation (1.5) which is analogous to the four-dimensional presentation (1.8) of the equation (1.3). However this presentation has no proper interpretation in special relativity. The matter is that the choice of the vector $\mathbf{n}$ in (2.8) is bound to the choice of an inertial coordinate system. There are infinitely many inertial coordinate systems in special relativity, but none of them is preferable as compared to any other.

3. The case of general relativity.

The spacetime of general relativity is an arbitrary manifold equipped with a pseudo-Euclidean metric $\mathbf{g}$ of the signature $(+\,-\,-\,-)$. Typically this metric is not flat. Therefore typically in general relativity we have no Cartesian coordinate systems and no inertial coordinate systems either. Moreover, covariant derivatives in general relativity do not coincide with partial derivatives. Nevertheless we could repeat the above arguments from section 2 provided we could find a vector field $\mathbf{n}$ whose components satisfy the equations (2.9). Such a vector field is called covariantly constant. The matter is that typically in general relativity there are no covariantly constant vector fields. Therefore in general relativity, as well as in special relativity, we cannot solve our problem of finding proper four-dimensional presentation of the equation (1.5) similar to the four-dimensional presentation (1.8) of the equation (1.3).

4. The case of cosmology.

Cosmology differs from general relativity in that it studies not an abstract spacetime, but spacetimes associated with our real universe. Homogeneous and isotropic cosmological models are based on the Friedmann-Robertson-Walker metric:

$$ds^2 = (dx^0)^2 - R(x^0)^2 \biggl(\frac{dr^2}{1 - K\,r^2} + r^2\,d\theta^2 + r^2\,\sin^2\theta\,d\phi^2\biggr). \tag{4.1}$$

The metric (4.1) incorporates three models with different values of the constant K:

$$K = \begin{cases} +1 & \text{for the closed model,} \\ \ \ 0 & \text{for the flat model,} \\ -1 & \text{for the hyperbolic model,} \end{cases} \tag{4.2}$$

see Section 1.1.3 in [8]. The metric (4.1) means that the metric tensor $\mathbf{g}$ is presented by a diagonal matrix with the following diagonal entries:

$$\begin{aligned} &g_{00} = 1, && g_{11} = -\frac{R^2}{1 - K\,r^2}, \\ &g_{22} = -R^2\,r^2, && g_{33} = -R^2\,r^2\,\sin^2\theta. \end{aligned} \tag{4.3}$$

The inverse metric tensor is also given by a diagonal matrix. Its diagonal components are inverse to the components (4.3):

$$\begin{aligned} &g^{00}=1, && g^{11}=-\frac{1-K\,r^2}{R^2},\\ &g^{22}=-\frac{1}{R^2\,r^2}, && g^{33}=-\frac{1}{R^2\,r^2\,\sin^2\theta}. \end{aligned}\tag{4.4}$$

The constant K in (4.3) and (4.4) is given by the formula (4.2), while $R=R(x^0)$ is some function of the variable x^0. Let's denote

$$u^0=x^0,\qquad u^1=r,\qquad u^2=\theta,\qquad u^3=\phi.\tag{4.5}$$

In terms of the coordinates (4.5) the components of the metric connection for the metric (4.1) with the components (4.3) and (4.4) are given by the following well-known formula (see § 7 in Chapter III of [6]):

$$\Gamma^k_{ij}=\frac{1}{2}\sum_{s=0}^{3}g^{ks}\left(\frac{\partial g_{sj}}{\partial u^i}+\frac{\partial g_{is}}{\partial u^j}-\frac{\partial g_{ij}}{\partial u^s}\right).\tag{4.6}$$

Here is the list of nonzero components of the metric connection (4.6):

$$\begin{aligned} &\Gamma^1_{01}=\Gamma^1_{10}=\frac{R'}{R}, && \Gamma^2_{02}=\Gamma^2_{02}=\frac{R'}{R},\\ &\Gamma^3_{03}=\Gamma^3_{30}=\frac{R'}{R}, && \Gamma^2_{12}=\Gamma^2_{21}=\frac{1}{r},\\ &\Gamma^3_{13}=\Gamma^3_{31}=\frac{1}{r}, && \Gamma^0_{11}=\frac{R\,R'}{1-K\,r^2},\\ &\Gamma^1_{11}=\frac{K\,r}{1-K\,r^2}, && \Gamma^0_{22}=R\,R',r^2,\\ &\Gamma^1_{22}=-(1-K\,r^2)\,r, && \Gamma^0_{33}=R\,R'\,r^2\,\sin^2\theta,\\ &\Gamma^1_{33}=-(1-K\,r^2)\,r\,\sin^2\theta, && \text{where}\ \ R'=\frac{dR}{dx^0}. \end{aligned}\tag{4.7}$$

Now, using the coordinates (4.5), we define the following parametric lines:

$$\begin{aligned} &u^0(\tau)=\tau, && u^1(\tau)=r=\text{const},\\ &u^2(\tau)=\theta=\text{const}, && u^3(\tau)=\phi=\text{const}\,. \end{aligned}\tag{4.8}$$

Using (4.7), one can easily show that the lines (4.8) are geodesic lines for the metric (4.1). Indeed, they satisfy the equations of geodesic lines

$$\frac{d^2u^k}{d\tau^2}+\sum_{i=0}^{3}\sum_{j=0}^{3}\Gamma^k_{ij}\,\frac{du^i}{d\tau}\,\frac{du^j}{d\tau}=0,$$

see § 8 in Chapter III of [6]. We denote trough **n** the tangent vectors to the geodesic lines (4.8). Then in the coordinates (4.5) we have

$$\mathbf{n}=\begin{Vmatrix} n^0\\ n^1\\ n^2\\ n^3\end{Vmatrix}=\begin{Vmatrix} 1\\ 0\\ 0\\ 0\end{Vmatrix},\quad\text{where}\quad n^k=\frac{du^k}{d\tau}\quad\text{for}\quad k=0,\,1,\,2,\,3. \tag{4.9}$$

The formula (4.9) is similar to (2.8). However, unlike (2.8) the vector field **n** in (4.9) is canonically defined. It is canonically associated with the Friedmann-Robertson-Walker metric.

Fig. 4.1 illustrates the universe in the case of the closed model (see (4.2)). The geodesic lines (4.8) are shown in yellow. They are called the *evolution lines* or

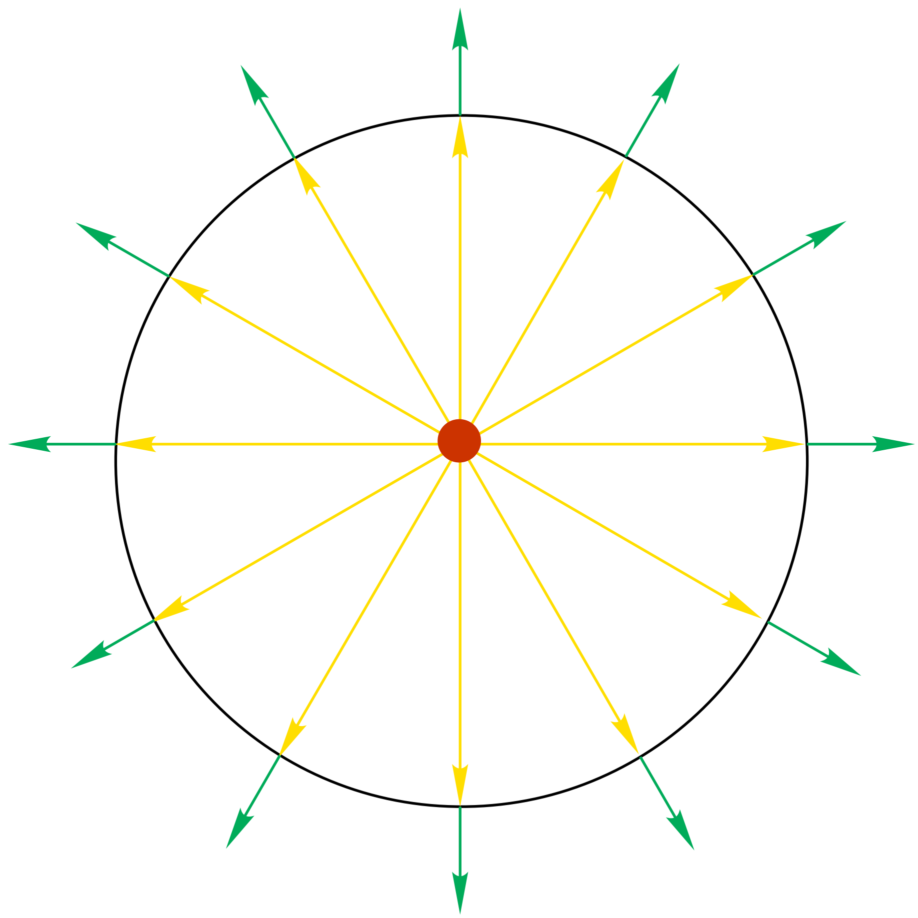

Fig. 4.1

evolution trajectories. The red point in the center represents the Big Bang. The black circle in Fig. 4.1 represents the current state of evolution. It is called the *evolution front*. The length l_0 of the evolution lines (4.5) measures the distance from the evolution front to the Big Bang. This distance is in temporal direction. Therefore the quantity

$$t_0 = \frac{l_0}{c} \tag{4.10}$$

is known as the *current age of the universe*. It is approximately 13.8 billion years according to our present knowledge (see [9]).

The vector field $\mathbf{n}$ from (4.9) is presented by green vectors in Fig. 4.1. They indicate the evolution direction. We choose the vector field (4.9) in order to use it in the formula (2.12). Unlike the vector field (2.8) in special relativity, the vector field (4.9) is not covariantly constant, i. e. it does not satisfy the equalities (2.9). But we can calculate the covariant derivatives in (2.9) explicitly:

$$\nabla_{\!s} n^p = D^p_s, \qquad\qquad \nabla_{\!s} n_p = D_{sp}. \tag{4.11}$$

Here D^p_s and D_{sp} are two matrices representing the components of some tensor field $\mathbf{D}$. Using (4.7), we derive the following formula for D^p_s in (4.11):

$$D^p_s = \frac{R'}{R}\begin{Vmatrix} 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{Vmatrix}. \tag{4.12}$$

The components of the second matrix (4.11) are derived by means of the formula

$$D_{sp} = \sum_{q=0}^{3} D^q_s\, g_{qp}. \tag{4.13}$$

Applying (4.4) and (4.9) to (4.12) and (4.13), we derive

$$D^p_s = \frac{R'}{R}\,(\delta^p_s - n_s\, n^p), \qquad\qquad D_{sp} = \frac{R'}{R}\,(g_{sp} - n_s\, n_p). \tag{4.14}$$

Then, substituting (4.14) into (4.11), we get

$$\nabla_{\!s} n^p = \frac{R'}{R}\,(\delta^p_s - n_s\, n^p), \qquad\qquad \nabla_{\!s} n_p = \frac{R'}{R}\,(g_{sp} - n_s\, n_p). \tag{4.15}$$

Now we can use the formulas (4.15) in place of the formulas (2.9).

Definition 4.1. The four-dimensional energy current $\mathbf{J}$ of an electromagnetic field in cosmology is defined by means of the formula

$$J^s = \sum_{p=0}^{3} c\, T^{ps}\, n_p, \tag{4.16}$$

where T^{ps} are components of the energy-momentum tensor $\mathbf{T}$ of an electromagnetic field and n_p are the components of the unit vector $\mathbf{n}$ tangent to the geodesic lines of evolution and normal to the evolution front.

Like in (2.10), one can derive the differential equation

$$\sum_{s=0}^{3} \nabla_s J^s = -w \tag{4.17}$$

for the components of the energy current $\mathbf{J}$ in (4.16). Formally the equation (4.17) coincides with the equation (2.14). However, the value of the scalar field w in the right hand side of it is different:

$$w = \sum_{p=0}^{3}\sum_{s=0}^{3} F^{ps}\, j_s\, n_p - c\sum_{p=0}^{3}\sum_{s=0}^{3} T^{ps}\,\nabla_s n_p. \tag{4.18}$$

The first term in the right hand side of (4.18) is a regular one. It reduces to $(\mathbf{E}, \mathbf{j})$ like in (1.4) if we choose a comoving frame, i.e. a coordinate system, where the vector $\mathbf{n}$ is given by the formula (4.9), see Section 1.1.3 in [8].

The second term in the right hand side of (4.18) is different. Applying (4.15) to it, we derive the following formula:

$$-c\sum_{p=0}^{3}\sum_{s=0}^{3} T^{ps}\,\nabla_s n_p = -\frac{c\,R'}{R}\sum_{p=0}^{3}\sum_{s=0}^{3} T^{ps}\, g_{sp} + \frac{c\,R'}{R}\sum_{p=0}^{3}\sum_{s=0}^{3} T^{ps}\, n_s\, n_p. \tag{4.19}$$

The energy-momentum tensor $\mathbf{T}$ of an electromagnetic field is traceless. Therefore the first term in the right hand side of (4.19) does vanish. As a result we get

$$-c\sum_{p=0}^{3}\sum_{s=0}^{3} T^{ps}\,\nabla_s n_p = \frac{c\,R'}{R}\sum_{p=0}^{3}\sum_{s=0}^{3} T^{ps}\, n_s\, n_p. \tag{4.20}$$

The variable x^0 in (4.1) is associated with the time variable t through the formula $x^0 = c\,t$. This time variable corresponds to the *cosmic time* (see [10]). Applying $x^0 = c\,t$ to the function $R = R(x^0)$, we get $\dot{R} = c\,R'$. Therefore

$$\frac{c\,R'}{R} = \frac{\dot{R}}{R} = H = H(t). \tag{4.21}$$

The value in the right hand side of (4.21) is known as the *Hubble parameter*. Its value at present time is known as Hubble constant (see (4.10) and [11]):

$$H_0 = H(t_0) \approx 73.4\,\frac{\mathrm{km}}{\mathrm{s}\cdot\mathrm{Mpc}} \approx 2.38\cdot 10^{-12}\,\mathrm{s}^{-1}. \tag{4.22}$$

With the use of (4.21) the equality (4.20) is transformed as

$$-c\sum_{p=0}^{3}\sum_{s=0}^{3} T^{ps}\,\nabla_s n_p = H\sum_{p=0}^{3}\sum_{s=0}^{3} T^{ps}\, n_s\, n_p. \tag{4.23}$$

Now we can summarize the results obtained in the form of the following theorems.

Theorem 4.1. *The scalar field w in the right hand side of the equation (4.17) is presented as the sum of two terms*

$$w = w_{\text{reg}} + w_{\text{Hub}}, \tag{4.24}$$

the regular term and the Hubble term, which are given by the formulas

$$\begin{aligned} w_{\text{reg}} &= \sum_{p=0}^{3}\sum_{s=0}^{3} F^{ps}\, j_s\, n_p,\\ w_{\text{Hub}} &= -c\sum_{p=0}^{3}\sum_{s=0}^{3} T^{ps}\, \nabla_{\!s} n_p. \end{aligned} \tag{4.25}$$

Theorem 4.2. *In a comoving frame the regular term w_{reg} from (4.24) and (4.25) reduces to $w_{\text{reg}} = (\mathbf{E}, \mathbf{j})$, where $\mathbf{E}$ is the electric field and $\mathbf{j}$ is the three-dimensional density of electric current.*

Theorem 4.3. *In a comoving frame the Hubble term w_{Hub} from (4.24) and (4.25) reduces to $w_{\text{Hub}} = H\,\varepsilon$, where ε is the density of electromagnetic energy from (1.4) and H is the Hubble parameter from (4.21).*

Theorems 4.2 and 4.3 are proved by means of direct calculations using (4.21), (4.20), (4.9), (2.6), (2.2), (2.1), and (1.7).

5. Conclusions.

Definition 4.1 and Theorems 4.1, 4.2, 4.3 along with the equation (4.17) constitute the main result of the present paper. They are derived under the assumption of a homogeneous and isotropic universe. This assumption is valid at large scales. At smaller scales galaxies, nebulas, stars and black holes can substantially disturb the smooth shape of the evolution front and evolution lines (see Fig. 4.1). Nevertheless Definition 4.1, the equation (4.17) and Theorems 4.1 and 4.2 remain valid in the vicinity of these massive objects. As for Theorem 4.3, it should be properly changed in each particular case.

Peculiar velocities of the Solar system as a whole and of most regular objects on the Earth are much smaller than the speed of light. Therefore our results do not affect engineering applications of electrodynamics which use the formulas (1.4). The Hubble term from (4.24) and (4.23) also does not affect engineering applications since the Hubble parameter H is very small, see (4.22). Nevertheless the above results can be useful from the conceptual point of view, e. g. for defining the coordinate presentation of the quantum wave function of an individual photon.

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