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Concerning the Apparent magnitude

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ABSTRACT

The Apparent magnitude m, as measure of the brightness of a star, is decisive, together with the Absolute magnitude M, for the correct calculation of the Distance Modulus μ.

The quantity $\mu = m - M$, indeed it is related to the Luminosity distance d by $\mu = 5\,log(d) - 5$ (d in Parsec) from which some verifications of a Cosmological model are then derived.

Contrary to what one might think, the determination of the Apparent magnitude depends on the hypothesized model used. If no Galactic Recession is foreseen, its value coincides with that observed m_o , unless are to be applied in this passage corrections due to local phenomena, close to the observed star, such as dust or other. If, on the other hand, the existence of a recession is assumed, other conversions must be applied.

In this paper we consider the Special Relativity (SR) as the context in which these calculations are to take place. If we do not assume the star is at rest with respect to us, in which case the following is irrelevant, I also think that SR should be a good starting point for analyzing any cosmological model.

THE K CORRECTION

The calculation of a correction can take place in different ways, but it is in any case necessary to deduce, starting from the observed value m_o, the quantity m [1] to be used in the subsequent procedures.

Now, we will refer to the K_{corr} described in [2], which, here, we will express in a different but equivalent form:

$$m \simeq m_o - K_{corr}$$

(m is given in reverse scale: the brighter is the star the lower is m. With $K_{corr} > 0$ the receding star appears further away than it is)

Where deemed appropriate, the last term can also contain corrections due to local phenomena, such as dust or other, close to the star.

The principal purpose of the K_{corr} is to apply the transformations to be performed between the observed and rest-frame measurements.

In addition to changing the single frequency, the redshift can affect the functioning of the photometric equipment for the detection of frequencies within a wavelength band. The correction considers all these aspects. Given the complexity and extreme specificity of the topics involved, it is advisable to rely to articles in literature.

THE K CORRECTION IN SPECIAL RELATIVITY

We will refer here, for simplicity, to a star that behaves like a monochromatic source of light and to a photometric apparatus capable of measuring the intensity of the radiation.

Let us then view the effects of the Galactic Recession on the apparent magnitude *m,* in the Special Relativity context:

An energy δE of radiation, emitted from a source C moving away, is projected through a solid angle $\delta\Omega$ on a surface δS in the time δt towards an observer O at a distance r.

What the observer will detect will be: (symbol δ stays for infinitesimal quantity)

$\delta E_o = (1+z)^{-1}\delta E_c$	for the redshift of frequency
$\delta\Omega_o = (1+z)^2\delta\Omega_c$	for the Lorentz length contraction only in the direction of motion
$r_o = (1+z)^{-1}r_c$	for the Lorentz length contraction only in the direction of motion
$\delta t_o = (1+z)\delta t_c$	for the time dilation occurred

The increase of the solid angle $\delta\Omega$ can be seen more easily starting from 2-dimension: In a circle of radius r and center C (the star) an observer O is placed at the center of an infinitesimal arc δb. An isosceles triangle has vertex in C and base δb (tangent) to the circle in O.

If now we translate δb moving O along the height $h = r$ of the triangle, squeezing it in the direction of C, the observer O will see the vertex angle increase and the height h shorten

Expressing h as the Lorentz contraction of the radius toward the observer: $h = r/(1+z)$ and returning in 3-dimension we can write:

The solid angle $\delta\Omega_o$ is given by $\delta\Omega_o = \delta S_o/h^2 = (1+z)^2\delta S_o/r^2 = (1+z)^2\delta\Omega_c$ because $\delta S_o = \delta S_c$.

Radiant intensity [3] is the power radiated in a given direction per unit solid angle, <u>it is independent by distance of the source</u>.

From this definition: $I_\Omega = \delta E * \delta\Omega^{-1} * \delta t^{-1}$we can conclude that:

$$I_{\Omega o} = (1+z)^{-4} * I_{\Omega c}$$

As regards the Intensity, the light of a star is not uniformly distributed in the solid angle subtended by the entire quasi-spherical surface. Being $I_o/I_c \propto \delta\Omega_c r^2/\delta\Omega_o h^2$ we can conclude that the decrease in the distance from the star is compensated by the increase in the solid angle, so for the Intensity it holds:

$$I_o = (1+z)^{-2} * I_c$$

as it had to be from its definition as the power $\delta E * \delta t^{-1}$ transferred per unit area A, where the area is measured on the plane perpendicular to the direction of propagation of the energy. (From our 2d paradigm $\delta S, A \propto \delta b^2$)

Note, at last, that term $\delta E/\,\delta t$ changes due to both the redshift of the single photon and the number of photons emitted in the time unit.

Then, for the apparent magnitude relation:

$$I_c/I_o = 2.512^{\Delta m} \quad \textit{where } \Delta m = m_o - m$$

we have:

$$K_{SR\ corr} = 5\ log(1+z) \ \text{ and } \ m < m_o.$$

<u>The receding star appears further away than it is</u>.

References from Wikipedia:

[1] - Apparent magnitude

[2] - K correction

[3] - Radiant intensity

[4] - Intensity