Delphi 4
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Abstract
The design of Delphi is modified so that only a single source of down-converted photons is needed. This reduces the required integration time per bit by a factor of $10^6$. Delphi 4 is a superluminal optical communication system.

1. Introduction
This system uses nonlocal, two-photon interference like that described in [1].

A representation of the system is shown in Figure 1. The system is composed of a Source (Src), a Transmitter (Tx), and a Receiver (Rx).

The optical path length from the Source to the Transmitter is somewhat less than the optical path length from the Source to the Receiver. The Source, Transmitter, and Receiver are all assumed to be stationary.

To simplify the description of this system, the effects of optical filters, detector quantum efficiency and dark counts, and most other potential losses are not included in the following discussion.

2. Notation
In the following discussion, both probability amplitude and probability will be calculated. As an example:

$$P[D_1,D_7,(\Delta)] = |pa[D_1,D_7,(\Delta)]|^2$$

In the above, $pa[D_1,D_7,(\Delta)]$ is the probability amplitude for the detection of a signal photon of a down-converted pair in detector $D_1$ in the Transmitter, and the detection of the idler photon of the pair in detector $D_7$ in the Receiver. The time parameter $(\Delta)$ is the time between the detection of the signal photon in the Transmitter and the detection of the idler photon in the Receiver.

$P[D_1,D_7,(\Delta)]$ is the probability for the same detection events.
The variable designation “pa” is used, rather than “Ψ”, to emphasize that the probability amplitude is a mathematical function (only).

Both intensity and amplitude variables are used in the following. As an example, for amplitude beam splitter ABS1:

\[ R_1 = |r_1|^2, \quad T_1 = |t_1|^2 \quad \text{and} \quad R_1 + T_1 = 1 \]

In the above, \( R_1 \) is the intensity reflectance, \( T_1 \) is the intensity transmittance, \( r_1 \) is the amplitude reflection coefficient, and \( t_1 \) is the amplitude transmission coefficient of ABS1.

3a. Source

The Source (Src) contains a single-mode, continuous wave (cw) pump laser (LSR), a periodically-poled lithium niobate crystal (PPLN), a dichroic mirror (DM), a polarizing beam splitter (PBS1), and a beam stop (Stp).

Laser LSR has a stable output, and the coherence length of the pump photons from LSR is greater than 100 meters.

The PPLN crystal is temperature-controlled, and is set to allow collinear, degenerate, type II spontaneous parametric down-conversion (SPDC). On average, one of every \( 10^6 \) of the photons from pump laser LSR is annihilated in a SPDC event that creates a signal and idler pair of photons. The signal photon is horizontally (H) polarized, and the idler photon is vertically (V) polarized.

Photons from pump laser LSR that are not down-converted in the PPLN are reflected at long pass dichroic mirror DM and are incident on beam stop Stp.

Polarizing beam splitter PBS1 is set to transmit incident H polarized photons and to reflect incident V polarized photons.

The longer wavelength signal and idler photons exit from the PPLN and are transmitted through DM. The H polarized signal photons are then transmitted through PBS1 and travel to the Transmitter (Tx). The V polarized idler photons reflect at PBS1 and travel to the Receiver (Rx).

3b. Transmitter

The Transmitter (Tx) contains a Pockels cell (PC), three polarizing beam splitters (PBS2-PBS4), four amplitude beam splitters (ABS1-ABS4), two special beam splitters (PABS1 and PABS2), three half-wave plates (HWP1-HWP3), nineteen mirrors (m), and five detectors (D1-D5). The fast detectors are capable of photon counting.
The Pockels cell may be used to rotate the polarization direction of a signal photon. If the PC is turned off, an H polarized photon will remain H polarized when it exits from the PC. If the PC is turned on, an H polarized photon will be V polarized when it exits from the PC.

Amplitude beam splitters ABS1-ABS4 may be partially-silvered plate beam splitters. Beam splitters ABS1-ABS4 are all “50/50” amplitude beam splitters. For example, the characteristics of amplitude beam splitters ABS1 and ABS4 are:

\[ R_1 = |r_1|^2 = 0.5; \quad T_1 = |t_1|^2 = 0.5 \]
\[ R_4 = |r_4|^2 = 0.5; \quad T_4 = |t_4|^2 = 0.5 \]

Special beam splitters PABS1 and PABS2 are actually “lossy” polarizing beam splitter plates. The special beam splitters act like polarizing beam splitters for H polarized photons (transmit all H polarized photons), but act like amplitude beam splitters for V polarized photons. This is because the wavelength of the down-converted photons is somewhat longer than the design wavelength of the polarizing plates. The characteristics of PABS1 and PABS2 are:

\[ R_{P1H} = |r_{P1H}|^2 = 0; \quad T_{P1H} = |t_{P1H}|^2 = 1.0 \]
\[ R_{P1V} = |r_{P1V}|^2 = 0.5; \quad T_{P1V} = |t_{P1V}|^2 = 0.5 \]
\[ R_{P2H} = |r_{P2H}|^2 = 0; \quad T_{P2H} = |t_{P2H}|^2 = 1.0 \]
\[ R_{P2V} = |r_{P2V}|^2 = 0.5; \quad T_{P2V} = |t_{P2V}|^2 = 0.5 \]

Polarizing beam splitters PBS2-PBS4 are set to transmit incident H polarized photons and to reflect incident V polarized photons.

Half-wave plates HWP1-HWP3 are all set with their “fast” axes 45 degrees above horizontal. A +H (+V) polarized photon that passes through the half-wave plate has its polarization direction rotated, and the photon exits from the wave plate +V (+H) polarized.

Special amplitude beam splitter PABS1 and three mirrors (m) form optical circulator OC1. Half-wave plate HWP1 is placed in the path through OC1. The time required for a photon to make one cycle around through OC1 is equal to X.

The fixed time X should be much longer than the coherence time of a signal (or idler) photon but should also be much shorter than the coherence time of a photon from pump laser LSR in the Source.

The path lengths through OC1 are adjusted so that the net phase difference from input to output through OC1 depends on the
reflections at the mirrors and the reflections (or transmissions) at the beam splitter and wave plate [2].

Special amplitude beam splitter PABS2 and five mirrors (m) form optical circulator OC2. Half-wave plate HWP2 is placed in the path through OC2. The time required for a photon to make one cycle around through OC2 is equal to X.

Amplitude beam splitters ABS3 and ABS4 are components in optical circulator OC3. Optical circulator OC3 is composed of a balanced Mach-Zehnder interferometer and a feedback path. The balanced MZ is formed by ABS3, ABS4, PBS4, and one mirror. The feedback path is from ABS4 via five mirrors back to ABS3. The time required for a photon to make one cycle from ABS3 through the balanced MZ and the feedback path back to ABS3 is equal to (2X).

3c. Receiver

The Receiver (Rx) contains two amplitude beam splitters (ABS5 and ABS6), two mirrors (m), and two detectors (D6 and D7). The fast detectors are capable of photon counting.

Amplitude beam splitters ABS5 and ABS6 may be partially-silvered plate beam splitters. The beam splitters are both “50/50” amplitude beam splitters. The characteristics of the amplitude beam splitters are:

\[ R_5 = |r_5|^2 = 0.5 \quad T_5 = |t_5|^2 = 0.5 \]
\[ R_6 = |r_6|^2 = 0.5 \quad T_6 = |t_6|^2 = 0.5 \]

The two amplitude beam splitters and two mirrors are arranged to form an unbalanced Mach-Zehnder interferometer (MZ). Unbalanced MZ provides a short path and a long path between ABS5 and ABS6 for the idler photons.

The path lengths through the MZ are adjusted so that the net phase difference from input to output for a given path depends on the reflections at the mirrors and the reflections (or transmissions) at the beam splitters [2].

The time difference between the time an idler photon may be incident on detector D6 (D7) via the short path, and the time the photon may be incident on detector D6 (D7) via the long path through the MZ is equal to (2X).

The fixed time (2X) should be of sufficient duration to allow the short path and the long path to be temporally distinct. Time (2X) should be much longer than the coherence time of an idler (or signal) photon but should also be much shorter than the coherence time of a photon from pump laser LSR in the Source.
Time X is the same as the time required for a photon to make one cycle around through OC1 or OC2 in the Transmitter.

Note: To facilitate the following descriptions, it is assumed that there are an integer number of wavelengths between the Source and the Transmitter, and also an integer number of wavelengths between the Source and the Receiver.

4a. Binary Zero

To send a binary zero from the Transmitter to the Receiver, Pockels cell PC in the Transmitter is turned off.

A down-converted pair of photons that is created in the PPLN exits the Source. The H polarized signal photon travels to the PC in the Transmitter, and the V polarized idler photon travels to MZ in the Receiver.

At the Receiver, the idler photon travels via either the short path or the long path through MZ to either detector D6 or D7.

At the Transmitter, the H polarized signal photon of the pair passes through the PC. Since the PC is off, the signal photon remains H polarized when it exits from the PC. The signal photon travels to and passes through PBS2 and is then incident on detector D1. In the binary zero case, signal photons do not reach amplitude beam splitter ABS1 in the Transmitter.

If the signal photon of a down-converted pair travels from the Source to the Transmitter and is detected in detector D1, and the idler photon of the pair travels to the Receiver and passes through the short path through MZ and is detected in either detector D6 or D7, then the time between the detection of the signal photon in the Transmitter and the idler photon in the Receiver is equal to τ. Note that τ >> X.

If the time difference between the detection of a signal photon in the Transmitter in detector D1, and the detection in the Receiver in detector D6 of the idler photon of the down-converted pair is equal to τ, then there is no ambiguity as to which paths the photons travelled.

The signal photon travelled directly to detector D1 in the Transmitter, and the idler photon travelled via the short path through MZ to detector D6 in the Receiver. The probability amplitude and probability for this are:

\[ p_{0[D1,D6,(τ)]} = [1][i^5t_6] = i/2 \]

\[ P_{0[D1,D6,(τ)]} = |p_{0[D1,D6,(τ)]}|^2 = 1/4 \]
If the time difference between the detection of a signal photon in the Transmitter in detector D1, and the detection of the idler photon of the pair in the Receiver in detector D6 is equal to \((\tau+2X)\), then there is also no ambiguity as to which paths the photons travelled.

The signal photon travelled directly to detector D1 in the Transmitter, and the idler photon travelled via the long path through MZ to detector D6 in the Receiver. The probability amplitude and probability for this are:

\[
p_{0[D1,D6, (\tau+2X)]} = [1][ir_5r_6] = 1/2
\]

\[
P_0[D1,D6, (\tau+2X)] = 1/4
\]

If the time difference between the detection of a signal photon in the Transmitter in detector D1, and the detection of the idler photon of the pair in the Receiver in detector D7 is equal to \(\tau\), then there is no ambiguity as to which paths the photons travelled.

The signal photon travelled directly to detector D1 in the Transmitter, and the idler photon travelled via the short path through MZ to detector D7 in the Receiver. The probability amplitude and probability for this are:

\[
p_{0[D1,D7, (\tau)]} = [1][-r_5r_6] = -1/2
\]

\[
P_0[D1,D7, (\tau)] = |p_{0[D1,D7, (\tau)]}|^2 = 1/4
\]

If the time difference between the detection of a signal photon in the Transmitter in detector D1, and the detection of the idler photon of the pair in the Receiver in detector D7 is equal to \((\tau+2X)\), then, again, there is no ambiguity as to which paths the photons travelled.

The signal photon travelled directly to detector D1 in the Transmitter, and the idler photon travelled via the long path through MZ to detector D7 in the Receiver. The probability amplitude and probability for this are:

\[
p_{0[D1,D7, (\tau+2X)]} = [1][r_5t_6] = 1/2
\]

\[
P_0[D1,D7, (\tau+2X)] = 1/4
\]

In the binary zero case, the probabilities for the detection of idler photons in detectors D6 and D7 in the Receiver are:

\[
P_0[D6] = (1/4) + (1/4) = (1/2) = 0.500
\]

\[
P_0[D7] = (1/4) + (1/4) = (1/2) = 0.500
\]
4b. Binary One

To send a binary one from the Transmitter to the Receiver, Pockels cell PC in the Transmitter is turned on.

A down-converted pair of photons that is created in the PPLN exits the Source. The H polarized signal photon travels to the PC in the Transmitter, and the V polarized idler photon travels to MZ in the Receiver.

The optical path length from the Source to the Transmitter is somewhat less than the optical path length from the Source to the Receiver. This ensures that almost every signal photon of a down-converted pair will be detected in the Transmitter before the idler photon of the pair reaches the Receiver.

At the Receiver, the idler photon travels via either the short path or the long path through the MZ to either detector D6 or D7.

At the Transmitter, the H polarized signal photon of the pair passes through the PC. Since the PC is on, the polarization direction of the signal photon is rotated and the photon is V polarized when it exits from the PC. The signal photon travels to and reflects at PBS2 and then travels to amplitude beam splitter ABS1.

With probability equal to one-half, the signal photon reflects at ABS1 and travels via reflection from three mirrors (m) to ABS2. With probability equal to one-half, the signal photon passes through ABS1 and travels to PABS1, the first optical component of OC1.

I) If the signal photon travels to PABS1, then, with probability equal to one-half, the V polarized signal photon may reflect from PABS1 and travel to PABS2.

If the photon reflects at PABS1, then, with probability equal to one-half, the V polarized photon may then reflect at PABS2 without entering either OC1 or OC2. The probability amplitude for this is:

$$p_{a0,1} = -i/(2\sqrt{2})$$

With probability equal to one-half, the V polarized photon may pass through PABS2 and enter OC2. The photon then travels via reflection from one mirror to HWP2. When it passes through HWP2, the polarization direction of the photon is rotated to H polarized. The photon then travels via reflection from four mirrors back to PABS2. The now H polarized photon passes through PABS2. The probability amplitude for this is:
\[ \text{pa0,2} = -i/(2\sqrt{2}) \]

Note that component \text{pa02} exits from PABS2 after component \text{pa01}. The time separation between \text{pa01} and \text{pa02} is equal to time \(X\).

Rather than initially reflecting at PABS1, if the V polarized signal photon passes through PABS1, it enters OC1. The photon travels via reflection from one mirror to HWP1. When it passes through HWP1, the polarization direction of the photon is rotated to H polarized. The photon then travels via reflection from two mirrors back to PABS1. The now H polarized photon passes through PABS1. The photon then travels to and passes through PABS2 and enters OC2.

The H polarized photon travels via reflection from one mirror to HWP2. When it passes through HWP2, the polarization direction of the photon is rotated to V polarized. The photon then travels via reflection from four mirrors back to PABS2. With probability equal to one-half, the now V polarized photon passes through PABS2. The probability amplitude for this is:

\[ \text{pa0,3} = +i/(2\sqrt{2}) \]

Note that component \text{pa03} exits from PABS2 after component \text{pa02}. The time separation between \text{pa02} and \text{pa03} is equal to time \(X\).

Rather than passing through PABS2, with probability equal to one-half, the V polarized photon reflects at PABS2 and travels back into OC2. The V polarized photon travels via reflection from one mirror to HWP2. When it passes through HWP2, the polarization direction of the photon is rotated to H polarized. The photon then travels via reflection from four mirrors back to PABS2. The now H polarized photon passes through PABS2. The probability amplitude for this is:

\[ \text{pa0,4} = -i/(2\sqrt{2}) \]

Note that component \text{pa04} exits from PABS2 after component \text{pa03}. The time separation between \text{pa03} and \text{pa04} is equal to time \(X\).

The two H polarized probability amplitude components \text{pa0,2} and \text{pa0,4} travel to and pass through polarizing beam splitter PBS3. The two components then travel via reflection from two mirrors (m) to ABS3, the first optical component of OC3. Component \text{pa0,2} is incident on ABS3 at a time equal to \((2X)\) before component \text{pa0,4} reaches ABS3.

The two V polarized probability amplitude components \text{pa0,1} and \text{pa0,3} travel to and reflect at PBS3. The two components are then incident on detector D2. Component \text{pa0,1} is incident on detector D2 at a time equal to \((2X)\) before component \text{pa0,3} reaches detector D2.
II) With probability equal to one-half, the signal photon reflects at ABS1 and travels via reflection from three mirrors (m) to ABS2. With probability equal to one-half, the photon may reflect at ABS2 and be incident on detector D3. The probability amplitude incident on detector D3 in this case is:

\[ pa_{0,5} = -\frac{1}{2} \]

Rather than reflecting at ABS2, with probability equal to one-half, the V polarized signal photon may pass through ABS2 and travel to HWP3. When it passes through HWP3, the polarization direction of the photon is rotated to H polarized. The now H polarized photon then travels to and is incident on PBS4 of OC3. The probability amplitude incident on PBS4 in this case is:

\[ pa_{0,6} = +\frac{i}{2} \]

III) Three H polarized probability amplitude components of the signal photon are incident on OC3:

\[ pa_{0,6} = +\frac{i}{2} \text{ incident at PBS4} \]
\[ pa_{0,2} = +\frac{i}{(2\sqrt{2})} \text{ and } pa_{0,4} = +\frac{i}{(2\sqrt{2})} \text{ incident at ABS3 (after reflection at two mirrors)} \]

Path lengths through the system are adjusted so that component \( pa_{0,6} \) arrives at OC3 first. The H polarized component passes through PBS4 and travels to ABS4. With probability equal to one-half, the component reflects at ABS4 and is incident on detector D5. The probability amplitude incident on detector D5 in this case is:

\[ pa_{3,1} = -\frac{1}{(2\sqrt{2})} \]

With probability equal to one-half, the component from PBS4 passes through ABS4 and travels via reflection at five mirrors (m) to ABS3. Path lengths are adjusted so that the reduced component from ABS4 arrives at ABS3 at the same time as component \( pa_{0,2} \). One-photon interference occurs at ABS3 (both \( pa_{0,6} \) and \( pa_{0,2} \) are components of the same signal photon). Since the components are now equal in magnitude, total constructive interference occurs in the output from ABS3 to PBS4. The combined H polarized component passes through PBS4 and is incident on detector D4. The probability amplitude incident on detector D4 in this case is:
\[ pa_{3,2} = -\frac{1}{2} \]

At a time equal to \((2X)\) after the time that the \(pa_{0,2}\) component arrived at \(ABS_3\), the \(pa_{0,4}\) component of the signal photon arrives at \(ABS_3\). With probability equal to one-half, the H polarized component reflects at \(ABS_3\) and travels to and passes through \(PBS_4\) and is incident on detector \(D_4\). The probability amplitude incident on detector \(D_4\) in this case is:

\[ pa_{3,3} = -\frac{1}{4} \]

Rather than reflecting at \(ABS_3\), with probability equal to one-half, the H polarized component may pass through \(ABS_3\) and travel via reflection from one mirror to \(ABS_4\). With probability equal to one-half, the component may pass through \(ABS_4\) and be incident on detector \(D_5\). The probability amplitude incident on detector \(D_5\) in this case is:

\[ pa_{3,4} = -\frac{1}{4\sqrt{2}} \]

With probability equal to one-half, the reduced component that began as \(pa_{0,4}\) may reflect at \(ABS_4\) and travel via reflection from five mirrors back to \(ABS_3\). With probability equal to one-half, the reduced component may pass through \(ABS_3\), travel to and pass through \(PBS_4\), and be incident on detector \(D_4\). The probability amplitude incident on detector \(D_4\) in this case is:

\[ pa_{3,5} = +\frac{1}{8} \]

Rather than passing through \(ABS_3\), with probability equal to one-half, the reduced component of the original \(pa_{0,4}\) may reflect at \(ABS_3\) and travel via reflection from one mirror to \(ABS_4\). With probability equal to one-half, the reduced component may pass through \(ABS_4\) and be incident on detector \(D_5\). The probability amplitude incident on detector \(D_5\) in this case is:

\[ pa_{3,6} = -\frac{1}{8\sqrt{2}} \]

This process repeats, with successive probability amplitude components incident on detector \(D_4\) in the form:

\[ pa_{3,Y} = +\frac{1}{(2^M)} , \text{ for integer } M > 3 ; \ Y = (2M) - 1 \]

and with successive probability amplitude components incident on detector \(D_5\) in the form:

\[ pa_{3,Z} = -\frac{1}{(2^N \cdot \sqrt{2})} , \text{ for integer } N > 3 ; \ Z = (2N) \]
IV) If the signal photon of a down-converted pair travels from the Source to the Transmitter and is detected as component $pa_{3,1}$ in detector $D_5$ at the output from OC3, and the idler photon of the pair travels to the Receiver and passes through the short path through MZ and is detected in either detector $D_6$ or $D_7$, then the time between the detection of the signal photon in the Transmitter and the idler photon in the Receiver is equal to $\partial_1$. Time $\partial_1$ is somewhat less than time $\tau$. Note that $\partial_1 >> X$.

If the time difference between the detection of a signal photon in the Transmitter in detector $D_5$, and the detection in the Receiver in detector $D_6$ of the idler photon of the down-converted pair is equal to $(\partial_1+2X)$, then there is no ambiguity as to which paths the photons travelled.

The signal photon reached detector $D_5$ as component $pa_{3,1}$ and the idler photon travelled via the long path through MZ to detector $D_6$ in the Receiver. The probability amplitude and probability in this case are:

$$pa_{1}[D_5,D_6,(\partial_1+2X)] = [pa_{3,1}][ir_{5r_{6}}]$$

$$= [-1/(2\sqrt{2})][i/2] = -i/(4\sqrt{2})$$

$$P_1[D_5,D_6,(\partial_1+2X)] = |pa_{1}[D_5,D_6,(\partial_1+2X)]|^2 = 1/32$$

If the time difference between the detection of a signal photon in the Transmitter in detector $D_5$, and the detection in the Receiver in detector $D_7$ of the idler photon of the down-converted pair is equal to $(\partial_1+2X)$, then there is no ambiguity as to which paths the photons travelled.

The signal photon reached detector $D_5$ as component $pa_{3,1}$ and the idler photon travelled via the long path through MZ to detector $D_7$ in the Receiver. The probability amplitude and probability in this case are:

$$pa_{1}[D_5,D_7,(\partial_1+2X)] = [pa_{3,1}][r_{st_{6}}]$$

$$= [-1/(2\sqrt{2})][1/2] = -1/(4\sqrt{2})$$

$$P_1[D_5,D_7,(\partial_1+2X)] = |pa_{1}[D_5,D_7,(\partial_1+2X)]|^2 = 1/32$$

If the time difference between the detection of a signal photon in the Transmitter in detector $D_5$, and the detection in the Receiver in detector $D_6$ of the idler photon of the down-converted
pair is equal to \( \partial 1 \), then there is an ambiguity as to which paths the photons travelled.

The signal photon may have reached detector D5 as component \( pa3,1 \) and the idler photon may have travelled via the short path through MZ to detector D6 in the Receiver.

Alternately, the signal photon may have reached detector D5 as component \( pa3,4 \) and the idler photon may have travelled via the long path through MZ to detector D6 in the Receiver.

Because of this ambiguity, non-local, two-photon interference occurs [1]. The probability amplitudes of the two possibilities must be combined. The probability amplitude and probability in this case are:

\[
p_{a1}[D5, D6, (\partial 1)] = |pa1[D5, D6, (\partial 1)]|^2 = 9/128
\]

If the time difference between the detection of a signal photon in the Transmitter in detector D5, and the detection in the Receiver in detector D7 of the idler photon of the down-converted pair is equal to \( \partial 1 \), then there is an ambiguity as to which paths the photons travelled.

The signal photon may have reached detector D5 as component \( pa3,1 \) and the idler photon may have travelled via the short path through MZ to detector D7 in the Receiver.

Alternately, the signal photon may have reached detector D5 as component \( pa3,4 \) and the idler photon may have travelled via the long path through MZ to detector D7 in the Receiver.

Because of this ambiguity, non-local, two-photon interference occurs [1]. The probability amplitudes of the two possibilities must be combined. The probability amplitude and probability in this case are:

\[
p_{a1}[D5, D7, (\partial 1)] = |pa1[D5, D7, (\partial 1)]|^2 = 1/128
\]

If the time difference between the detection of a signal photon in the Transmitter in detector D5, and the detection in the Receiver in detector D6 of the idler photon of the down-converted pair is equal to \( [\partial 1 -(2X)] \), then there is an ambiguity as to which paths the photons travelled.
The signal photon may have reached detector D5 as component pa3,4 and the idler photon may have travelled via the short path through MZ to detector D6 in the Receiver.

Alternately, the signal photon may have reached detector D5 as component pa3,6 and the idler photon may have travelled via the long path through MZ to detector D6 in the Receiver.

Because of this ambiguity, non-local, two-photon interference occurs [1]. The probability amplitudes of the two possibilities must be combined. The probability amplitude and probability in this case are:

\[ p_{1[D5,D6,(\partial_1-2X)]} = [pa_{3,4}][it_{56}] + [pa_{3,6}][ir_{56}] \]

\[ = [-1/(4\sqrt(2))] [i/2] + [-1/(8\sqrt(2))] [i/2] = -i/3/(16\sqrt(2)) \]

\[ P_{1[D5,D6,(\partial_1-2X)]} = |p_{1[D5,D6,(\partial_1-2X)]}|^2 = 9/512 \]

If the time difference between the detection of a signal photon in the Transmitter in detector D5, and the detection in the Receiver in detector D7 of the idler photon of the down-converted pair is equal to \( \partial_1-(2X) \), then there is an ambiguity as to which paths the photons travelled.

The signal photon may have reached detector D5 as component pa3,4 and the idler photon may have travelled via the short path through MZ to detector D7 in the Receiver.

Alternately, the signal photon may have reached detector D5 as component pa3,6 and the idler photon may have travelled via the long path through MZ to detector D7 in the Receiver.

Because of this ambiguity, non-local, two-photon interference occurs [1]. The probability amplitudes of the two possibilities must be combined. The probability amplitude and probability in this case are:

\[ p_{1[D5,D7,(\partial_1-2X)]} = [pa_{3,4}][-t_{56}] + [pa_{3,6}][r_{56}] \]

\[ = [-1/(4\sqrt(2))] [-1/2] + [-1/(8\sqrt(2))] [1/2] = 1/(16\sqrt(2)) \]

\[ P_{1[D5,D7,(\partial_1-2X)]} = |p_{1[D5,D7,(\partial_1-2X)]}|^2 = 1/512 \]

In general:

\[ P_{1[D5,D6,(\partial_1-2NX)]} = 9/(4^N \cdot 128) \], for integer N > 0

\[ P_{1[D5,D7,(\partial_1-2NX)]} = 1/(4^N \cdot 128) \], for integer N > 0

Summation:
\( P_1[D5,D6] = (1/32) + \{(9/128) \cdot [1 + (1/4) + (1/16) + \ldots]\} \)
\( = (1/32) + [(9/128) \cdot (1/(1-(1/4)))] \)
\( = (1/32) + (9/128)(4/3) = 4/32 = 1/8 \)

\( P_1[D5,D7] = (1/32) + \{(1/128) \cdot [1 + (1/4) + (1/16) + \ldots]\} \)
\( = (1/32) + [(1/128) \cdot (4/3)] = 4/96 = 1/24 \)

V) If the signal photon of a down-converted pair travels from the Source to the Transmitter and is detected as component \( pa_{3,2} \) in detector \( D4 \) at the output from OC3, and the idler photon of the pair travels to the Receiver and passes through the short path through MZ and is detected in either detector \( D6 \) or \( D7 \), then the time between the detection of the signal photon in the Transmitter and the idler photon in the Receiver is equal to \( \partial_2 \). Time \( \partial_2 \) is somewhat less than time \( \tau \). Note that \( \partial_2 >> X \).

If the time difference between the detection of a signal photon in the Transmitter in detector \( D4 \), and the detection in the Receiver in detector \( D6 \) of the idler photon of the down-converted pair is equal to \( (\partial_2+2X) \), then there is no ambiguity as to which paths the photons travelled.

The signal photon reached detector \( D4 \) as component \( pa_{3,2} \) and the idler photon travelled via the long path through MZ to detector \( D6 \) in the Receiver. The probability amplitude and probability in this case are:

\[ pa_{1}[D4,D6, (\partial_2+2X)] = [pa_{3,2}][ir_5r_6] \]
\[ = [-1/2][i/2] = -i/4 \]

\[ P_1[D4,D6, (\partial_2+2X)] = |pa_{1}[D4,D6, (\partial_2+2X)]|^2 = 1/16 \]

If the time difference between the detection of a signal photon in the Transmitter in detector \( D4 \), and the detection in the Receiver in detector \( D7 \) of the idler photon of the down-converted pair is equal to \( (\partial_2+2X) \), then there is no ambiguity as to which paths the photons travelled.

The signal photon reached detector \( D4 \) as component \( pa_{3,2} \) and the idler photon travelled via the long path through MZ to detector \( D7 \) in the Receiver. The probability amplitude and probability in this case are:

\[ pa_{1}[D4,D7, (\partial_2+2X)] = [pa_{3,2}][r_5t_6] \]
\[
\mathbf{P}[D_4, D_7, (\partial_2+2X)] = \left| \mathbf{p}_a[D_4, D_7, (\partial_2+2X)] \right|^2 = 1/16
\]

If the time difference between the detection of a signal photon in the Transmitter in detector D4, and the detection in the Receiver in detector D6 of the idler photon of the down-converted pair is equal to \(\partial_2\), then there is an ambiguity as to which paths the photons travelled.

The signal photon may have reached detector D4 as component \(\mathbf{p}_a 3, 2\) and the idler photon may have travelled via the short path through MZ to detector D6 in the Receiver.

Alternately, the signal photon may have reached detector D4 as component \(\mathbf{p}_a 3, 3\) and the idler photon may have travelled via the long path through MZ to detector D6 in the Receiver.

Because of this ambiguity, non-local, two-photon interference occurs [1]. The probability amplitudes of the two possibilities must be combined. The probability amplitude and probability in this case are:

\[
\mathbf{p}_a[D_4, D_6, (\partial_2)] = \left[ \mathbf{p}_a 3, 2 \right] [\text{it}_{56}] + \left[ \mathbf{p}_a 3, 3 \right] [\text{ir}_{56}]
\]

\[
= [-1/2][i/2] + [-1/4][i/2] = -i3/8
\]

\[
\mathbf{P}_1[D_4, D_6, (\partial_2)] = \left| \mathbf{p}_a[D_4, D_6, (\partial_2)] \right|^2 = 9/64
\]

If the time difference between the detection of a signal photon in the Transmitter in detector D4, and the detection in the Receiver in detector D7 of the idler photon of the down-converted pair is equal to \(\partial_2\), then there is an ambiguity as to which paths the photons travelled.

The signal photon may have reached detector D4 as component \(\mathbf{p}_a 3, 2\) and the idler photon may have travelled via the short path through MZ to detector D7 in the Receiver.

Alternately, the signal photon may have reached detector D4 as component \(\mathbf{p}_a 3, 3\) and the idler photon may have travelled via the long path through MZ to detector D7 in the Receiver.

Because of this ambiguity, non-local, two-photon interference occurs [1]. The probability amplitudes of the two possibilities must be combined. The probability amplitude and probability in this case are:

\[
\mathbf{p}_a[D_4, D_7, (\partial_2)] = \left[ \mathbf{p}_a 3, 2 \right] [-t_{56}] + \left[ \mathbf{p}_a 3, 3 \right] [r_{56}]
\]

\[
= [-1/2][-1/2] + [-1/4][1/2] = 1/8
\]

\[
\mathbf{P}_1[D_4, D_7, (\partial_2)] = \left| \mathbf{p}_a[D_4, D_7, (\partial_2)] \right|^2 = 1/64
\]
If the time difference between the detection of a signal photon in the Transmitter in detector D4, and the detection in the Receiver in detector D6 of the idler photon of the down-converted pair is equal to [\(\partial z - (2X)\)], then there is an ambiguity as to which paths the photons travelled.

The signal photon may have reached detector D4 as component \(p_{a3,3}\) and the idler photon may have travelled via the short path through MZ to detector D6 in the Receiver.

Alternately, the signal photon may have reached detector D4 as component \(p_{a3,5}\) and the idler photon may have travelled via the long path through MZ to detector D6 in the Receiver.

Because of this ambiguity, non-local, two-photon interference occurs [1]. The probability amplitudes of the two possibilities must be combined. The probability amplitude and probability in this case are:

\[
p_{a1[D4,D6,(\partial z - 2X)]} = [p_{a3,3}][i t_5 s_6] + [p_{a3,5}][i r_5 r_6]
\]
\[
= [-1/4][i/2] + [1/8][i/2] = -i/16
\]
\[
P_{1[D4,D6,(\partial z - 2X)]} = |p_{a1[D4,D6,(\partial z - 2X)]}|^2 = 1/256
\]

If the time difference between the detection of a signal photon in the Transmitter in detector D4, and the detection in the Receiver in detector D7 of the idler photon of the down-converted pair is equal to [\(\partial z - (2X)\)], then there is an ambiguity as to which paths the photons travelled.

The signal photon may have reached detector D4 as component \(p_{a3,3}\) and the idler photon may have travelled via the short path through MZ to detector D7 in the Receiver.

Alternately, the signal photon may have reached detector D4 as component \(p_{a3,5}\) and the idler photon may have travelled via the long path through MZ to detector D7 in the Receiver.

Because of this ambiguity, non-local, two-photon interference occurs [1]. The probability amplitudes of the two possibilities must be combined. The probability amplitude and probability in this case are:

\[
p_{a1[D4,D7,(\partial z - 2X)]} = [p_{a3,3}][-t_5 r_6] + [p_{a3,5}][r_5 t_6]
\]
\[
= [-1/4][-1/2] + [1/8][1/2] = 3/16
\]
\[
P_{1[D4,D7,(\partial z - 2X)]} = |p_{a1[D4,D7,(\partial z - 2X)]}|^2 = 9/256
\]
Receiver in detector D6 of the idler photon of the down-converted pair is equal to $[\partial_2 - (4X)]$, then there is an ambiguity as to which paths the photons travelled.

The signal photon may have reached detector D4 as component $\text{pa3,5}$ and the idler photon may have travelled via the short path through MZ to detector D6 in the Receiver.

Alternately, the signal photon may have reached detector D4 as component $\text{pa3,7}$ and the idler photon may have travelled via the long path through MZ to detector D6 in the Receiver.

Because of this ambiguity, non-local, two-photon interference occurs [1]. The probability amplitudes of the two possibilities must be combined. The probability amplitude and probability in this case are:

$$\text{pa}_1[D_4,D_6,(\partial_2-4X)] = [\text{pa3,5}] [\text{it}_5 \text{t}_6] + [\text{pa3,7}] [\text{ir}_5 \text{r}_6]$$

$$= [1/8][i/2] + [1/16][i/2] = +i3/32$$

$$P_1[D_4,D_6,(\partial_2-4X)] = |\text{pa}_1[D_4,D_6,(\partial_2-4X)]|^2 = 9/(1024)$$

If the time difference between the detection of a signal photon in the Transmitter in detector D4, and the detection in the Receiver in detector D7 of the idler photon of the down-converted pair is equal to $[\partial_2 - (4X)]$, then there is an ambiguity as to which paths the photons travelled.

The signal photon may have reached detector D4 as component $\text{pa3,5}$ and the idler photon may have travelled via the short path through MZ to detector D7 in the Receiver.

Alternately, the signal photon may have reached detector D4 as component $\text{pa3,7}$ and the idler photon may have travelled via the long path through MZ to detector D7 in the Receiver.

Because of this ambiguity, non-local, two-photon interference occurs [1]. The probability amplitudes of the two possibilities must be combined. The probability amplitude and probability in this case are:

$$\text{pa}_1[D_4,D_7,(\partial_2-4X)] = [\text{pa3,5}] [-\text{t}_5 \text{r}_6] + [\text{pa3,7}] [\text{r}_5 \text{t}_6]$$

$$= [1/8][-1/2] + [1/16][1/2] = -1/32$$

$$P_1[D_4,D_7,(\partial_2-4X)] = |\text{pa}_1[D_4,D_7,(\partial_2-4X)]|^2 = 1/(1024)$$

In general:

$$P_1[D_4,D_6,(\partial_2-2NX)] = 9/(4^N \cdot 64) \text{, for integer } N > 1$$

$$P_1[D_4,D_7,(\partial_2-2NX)] = 1/(4^N \cdot 64) \text{, for integer } N > 1$$
Summation:

\[ P_1[D4, D6] = \frac{1}{16} + \frac{9}{64} + \frac{1}{256} + \left[(\frac{9}{1024}) \cdot (\frac{4}{3})\right] = \frac{7}{32} \]

\[ P_1[D4, D7] = \frac{1}{16} + \frac{1}{64} + \frac{9}{256} + \left[(\frac{1}{1024}) \cdot (\frac{4}{3})\right] = \frac{11}{96} \]

VI) If the signal photon of a down-converted pair travels from the Source to the Transmitter and is detected as component \(pa_0,1\) in detector \(D_2\), and the idler photon of the pair travels to the Receiver and passes through the short path through the MZ and is detected in either detector \(D_6\) or \(D_7\), then the time between the detection of the signal photon in the Transmitter and the idler photon in the Receiver is equal to \(\sigma\). Time \(\sigma\) is somewhat less than time \(\tau\). Note that \(\sigma \gg X\).

If the time difference between the detection of a signal photon in the Transmitter in detector \(D_2\), and the detection in the Receiver in detector \(D_6\) of the idler photon of the down-converted pair is equal to \((\sigma+2X)\), then there is no ambiguity as to which paths the photons travelled.

The signal photon reached detector \(D_2\) as component \(pa_0,1\) (after reflection at PBS3) and the idler photon travelled via the long path through MZ to detector \(D_6\) in the Receiver. The probability amplitude and probability in this case are:

\[ pa_1[D2, D6, (\sigma+2X)] = [pa_0,1][ir_5r_6] \]

\[ = \left[\frac{1}{(2\sqrt{2})}\right][i/2] = i/(4\sqrt{2}) \]

\[ P_1[D2, D6, (\sigma+2X)] = |pa_1[D2, D6, (\sigma+2X)]|^2 = 1/32 \]

If the time difference between the detection of a signal photon in the Transmitter in detector \(D_2\), and the detection in the Receiver in detector \(D_7\) of the idler photon of the down-converted pair is equal to \((\sigma+2X)\), then there is no ambiguity as to which paths the photons travelled.

The signal photon reached detector \(D_2\) as component \(pa_0,1\) (after reflection at PBS3) and the idler photon travelled via the long path through MZ to detector \(D_7\) in the Receiver. The probability amplitude and probability in this case are:

\[ pa_1[D2, D7, (\sigma+2X)] = [pa_0,1][r_5t_6] \]

\[ = \left[\frac{1}{(2\sqrt{2})}\right][1/2] = 1/(4\sqrt{2}) \]

\[ P_1[D2, D7, (\sigma+2X)] = |pa_1[D2, D7, (\sigma+2X)]|^2 = 1/32 \]
If the time difference between the detection of a signal photon in the Transmitter in detector D2, and the detection in the Receiver in detector D6 of the idler photon of the down-converted pair was equal to $\sigma$, then there would be an ambiguity as to which paths the photons travelled.

The signal photon might have reached detector D2 as component $p_{a0,1}$ (after reflection at PBS3) and the idler photon might have travelled via the short path through MZ to detector D6 in the Receiver.

Alternately, the signal photon might have reached detector D2 as component $p_{a0,3}$ (after reflection at PBS3) and the idler photon might have travelled via the long path through MZ to detector D6 in the Receiver.

Because of this ambiguity, non-local, two-photon interference would occur [1]. The probability amplitudes of the two possibilities must be combined. The probability amplitude and probability in this case are:

$$p_{a1[D2,D6,(\sigma)]} = [p_{a0,1}[it_{5}t_{6}] + [p_{a0,3}[ir_{5}r_{6}]$$

$$= [1/(2\sqrt{2})]i/2 + [-1/(2\sqrt{2})]i/2 = 0$$

$$P_{1[D2,D6,(\sigma)]} = |p_{a1[D2,D6,(\sigma)]}|^2 = 0$$

If the time difference between the detection of a signal photon in the Transmitter in detector D2, and the detection in the Receiver in detector D7 of the idler photon of the down-converted pair is equal to $\sigma$, then there is an ambiguity as to which paths the photons travelled.

The signal photon may have reached detector D2 as component $p_{a0,1}$ (after reflection at PBS3) and the idler photon may have travelled via the short path through MZ to detector D7 in the Receiver.

Alternately, the signal photon may have reached detector D2 as component $p_{a0,3}$ (after reflection at PBS3) and the idler photon may have travelled via the long path through MZ to detector D7 in the Receiver.

Because of this ambiguity, non-local, two-photon interference occurs [1]. The probability amplitudes of the two possibilities must be combined. The probability amplitude and probability in this case are:

$$p_{a1[D2,D7,(\sigma)]} = [p_{a0,1}[-t_{5}r_{6}] + [p_{a0,3}[r_{5}t_{6}]$$

$$= [1/(2\sqrt{2})][-1/2] + [-1/(2\sqrt{2})][1/2] = -1/(2\sqrt{2})$$
\[ P_1[D2,D7,(\sigma)] = |p_{a1}[D2,D7,(\sigma)]|^2 = \frac{1}{8} \]

If the time difference between the detection of a signal photon in the Transmitter in detector D2, and the detection in the Receiver in detector D6 of the idler photon of the down-converted pair is equal to \([\sigma-(2X)]\), then there is no ambiguity as to which paths the photons travelled.

The signal photon reached detector D2 as component \(p_{a0,3}\) (after reflection at PBS3) and the idler photon travelled via the short path through MZ to detector D6 in the Receiver. The probability amplitude and probability in this case are:

\[ \text{pa}_{1}[D2,D6,(\sigma-2X)] = [p_{a0,3}] [i_{5}t_{6}] \]
\[ = [-1/(2\sqrt{(2)})][i/2] = -i/(4\sqrt{(2)}) \]

\[ P_1[D2,D6,(\sigma-2X)] = |\text{pa}_{1}[D2,D6,(\sigma-2X)]|^2 = \frac{1}{32} \]

If the time difference between the detection of a signal photon in the Transmitter in detector D2, and the detection in the Receiver in detector D7 of the idler photon of the down-converted pair is equal to \([\sigma-(2X)]\), then there is no ambiguity as to which paths the photons travelled.

The signal photon reached detector D2 as component \(p_{a0,3}\) (after reflection at PBS3) and the idler photon travelled via the short path through MZ to detector D6 in the Receiver. The probability amplitude and probability in this case are:

\[ \text{pa}_{1}[D2,D7,(\sigma-2X)] = [p_{a0,3}] [-t_{5}r_{6}] \]
\[ = [-1/(2\sqrt{(2)})][-1/2] = 1/(4\sqrt{(2)}) \]

\[ P_1[D2,D7,(\sigma-2X)] = |\text{pa}_{1}[D2,D7,(\sigma-2X)]|^2 = \frac{1}{32} \]

Summation:

\[ P_1[D2,D6] = (1/32) + (1/32) = 1/16 \]
\[ P_1[D2,D7] = (1/32) + (1/8) + (1/32) = 3/16 \]

VII) If the signal photon of a down-converted pair travels from the Source to the Transmitter and is detected as component \(p_{a0,5}\) in detector D3, and the idler photon of the pair travels to the Receiver and passes through the short path through the MZ and is detected in either detector D6 or D7, then the time between the detection of the signal photon in the Transmitter and the idler...
photon in the Receiver is equal to \( \gamma \). Time \( \gamma \) is somewhat less than time \( \tau \). Note that \( \gamma \gg X \).

If the time difference between the detection of a signal photon in the Transmitter in detector D3, and the detection in the Receiver in detector D6 of the idler photon of the down-converted pair is equal to \((\gamma+2X)\), then there is no ambiguity as to which paths the photons travelled.

The signal photon reached detector D3 as component \( \text{pa0,5} \) and the idler photon travelled via the long path through MZ to detector D6 in the Receiver. The probability amplitude and probability in this case are:

\[
\text{pa}_1[D3,D6,(\gamma+2X)] = [\text{pa0,5}][ir_5r_6] = [-1/2][i/2] = -i/4
\]

\[
P_1[D3,D6,(\gamma+2X)] = |\text{pa}_1[D3,D6,(\gamma+2X)]|^2 = 1/16
\]

If the time difference between the detection of a signal photon in the Transmitter in detector D3, and the detection in the Receiver in detector D7 of the idler photon of the down-converted pair is equal to \((\gamma+2X)\), then there is no ambiguity as to which paths the photons travelled.

The signal photon reached detector D3 as component \( \text{pa0,5} \) and the idler photon travelled via the long path through MZ to detector D7 in the Receiver. The probability amplitude and probability in this case are:

\[
\text{pa}_1[D3,D7,(\gamma+2X)] = [\text{pa0,5}][r_5t_6] = [-1/2][1/2] = -1/4
\]

\[
P_1[D3,D7,(\gamma+2X)] = |\text{pa}_1[D3,D7,(\gamma+2X)]|^2 = 1/16
\]

If the time difference between the detection of a signal photon in the Transmitter in detector D3, and the detection in the Receiver in detector D6 of the idler photon of the down-converted pair is equal to \( \gamma \), then there is no ambiguity as to which paths the photons travelled.

The signal photon reached detector D3 as component \( \text{pa0,5} \) and the idler photon travelled via the short path through MZ to detector D6 in the Receiver. The probability amplitude and probability in this case are:

\[
\text{pa}_1[D3,D6,(\gamma)] = [\text{pa0,5}][it_5t_6]
\]
\[ = [-1/2][i/2] = -i/4 \]

\[ P_1[D3,D6,(\gamma)] = |pa_1[D3,D6,(\gamma)]|^2 = 1/16 \]

If the time difference between the detection of a signal photon in the Transmitter in detector D3, and the detection in the Receiver in detector D7 of the idler photon of the down-converted pair is equal to \( \gamma \), then there is no ambiguity as to which paths the photons travelled.

The signal photon reached detector D3 as component \( pa_{0,5} \) and the idler photon travelled via the short path through MZ to detector D7 in the Receiver. The probability amplitude and probability in this case are:

\[ pa_1[D3,D7,(\gamma)] = [pa_{0,5}][-t_{5\rightarrow 6}] \]

\[ = [-1/2][-1/2] = 1/4 \]

\[ P_1[D3,D7,(\gamma)] = |pa_1[D3,D7,(\gamma)]|^2 = 1/16 \]

Summation:

\[ P_1[D3,D7] = (1/16) + (1/16) = 1/8 \]

\[ P_1[D3,D8] = (1/16) + (1/16) = 1/8 \]

VIII) In the binary one case, the probabilities for the detection of idler photons in detectors D6 and D7 in the Receiver are:

\[ P_1[D6] = P_1[D5,D6] + P_1[D4,D6] + P_1[D2,D6] + P_1[D3,D6] \]

\[ = (1/8) + (7/32) + (1/16) + (1/8) \]

\[ = (17/32) = 0.531 \]


\[ = (1/24) + (11/96) + (3/16) + (1/8) \]

\[ = (15/32) = 0.469 \]

Note that every down-converted signal and idler photon pair communicates information from the Transmitter to the Receiver, rather than only one in every \( 10^6 \) pairs (as in a previous version of Delphi).
However, an integration time is required. The set integration time (I) required per bit must be of adequate duration to guarantee that a sufficient number of signal and idler photon pairs will be detected at the Transmitter and Receiver to ensure that the operator at the Receiver can make a statistically sound decision as to whether a binary one or a binary zero is being transmitted. Integration time I must also take into account all system losses.

5. Conclusion

The binary zero and binary one messages produce different detection probabilities at the Receiver. The operator at the Receiver notes whether the detections in detectors D6 and D7 correspond to a binary zero or a binary one message.

Communication may begin once signal photons from the Source reach the Transmitter and idler photons reach the Receiver. The transfer of information from the Transmitter to the Receiver is almost instantaneous (independent of distance), limited only by the required integration time per bit (I).

The time required to transmit one bit of information from the Transmitter to the Receiver is equal to I. The distance (D) associated with the integration time is:

\[ D = c \cdot I \]

If the distance between the Transmitter and the Receiver is greater than D, then, using this system, the speed of transmission of information from Transmitter to Receiver will be faster than the speed of light.

Delphi 4 is an effective superluminal communication system.
References


Figure 1: System Design