# What's important

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*Abstract*

Generating number systems reveals most of the structure and behavior of our universe. A system of separable Hilbert spaces that all share the same underlying vector space describes all aspects of the dynamic field that our universe represents. One of these Hilbert spaces acts as the background and reference platform. Via its non-separable companion, this separable Hilbert space resents the dynamic field that physicists consider the dynamic universe. All other members of the system represent moving particles.

# 1 Set theory

Importantly, the set theory makes no sense without considering the container of the set and is meaningless without considering the type of elements of the set. A finite set is fundamentally different from an infinite set. An infinite set cannot be achieved by gradually expanding a finite set. The set must be redefined to get an infinite set. A countable set is fundamentally different from an innumerable set. Countable means that each member of the set can be labeled with a natural number. The set of natural numbers is infinite. A countable set can be infinite. A countable set must be redefined to make it an innumerable set. An innumerable set is always an infinite set. The elements of sets that cannot otherwise be distinguished can be identified by elements of a number system.

A simple space can act as a container of locations. A vector is a combination of a base point and a pointer connected by a directional line. A scalar measures the distance between these points. A shift parallel to the direction line does not change the integrity of the vector. Using vectors instead of locations as elements of a set located in space has the advantage that vectors obey simple arithmetic. This vector arithmetic allows the vectors to reach any location in simple space. The vector arithmetic can be used to generate number systems whose elements can be applied to identify the locations in space. It turns out that this can be done in many ways.

The arithmetic of number systems does not regulate all the freedom of choice that remains in the container. This means that numbering systems exist in many versions that differ in the included choice freedoms. Freedom of selection is called symmetry. Most of these symmetries relate to the geometry of the locations. Some selection exemptions concern choices which are left open in the rules of the arithmetic.

## 2 Number systems

A treatise on number systems usually emphasizes the arithmetic of the number system, but most of the time the treatise ignores the symmetries of the number system. In physical reality, both arithmetic and symmetries play an essential role.

It seems that two base number systems can be mixed into three associative division rings.

### 2.1 Real numbers

The rational numbers for which the square is a zero or a positive rational number already form an associative division ring. The arithmetic of this set is taught in primary schools. All elements fit on a directional line and occupy a single dimension. If all irrational numbers take part, the set becomes countless. This combination is the set of real numbers. In real numbers, all converging series of elements end in a limit that is also a real number. The resulting set is a continuum and shows a special behavior when deformed.

#### 2.1.1 The arithmetic of real numbers

We will indicate the real numbers with the suffix $_r$.

For real numbers, addition and multiplication are commutative, associative, and distributive.

$$\begin{aligned} b_r + a_r &= a_r + b_r \\ (a_r + b_r) + c_r &= a_r + (b_r + c_r) \end{aligned} \tag{2.1.1}$$

$$\begin{aligned} b_r a_r &= a_r b_r \\ (a_r b_r) c_r &= a_r (b_r c_r) \end{aligned} \tag{2.1.2}$$

$$a_r (b_r + c_r) = a_r b_r + a_r c_r \tag{2.1.3}$$

For real numbers, the square is zero or it is positive

$$a_r a_r \geq 0 \tag{2.1.4}$$

## 2.2 Spatial numbers and mixed numbers

There is another number system where the squares are equal to zero or a negative real number. This is the spatial number system that is often called the system of imaginary numbers. It exists in a one-dimensional and a three-dimensional version. Together with the real numbers, the one-dimensional version of the spatial numbers forms the two-dimensional associative division ring of the complex numbers. Together with the real numbers, the three-dimensional version of the spatial numbers forms the four-dimensional associative division ring of the quaternions. The arithmetic of the spatial number system contains a commutative and associative addition. The product splits into an inner product that has a scalar value and is responsible for the negative square and an outer product that occurs only in the three-dimensional version. The outer product may be chiral right-handed, or it may be chiral left-handed.

The elements of a continuum obey the arithmetic of the corresponding number system. The geometry of a continuum can change in a well-ordered way. This change is regulated by special change arithmetic that mathematicians call differential calculus.

The dimension of associative division rings is always less than five. The spatial number system is not considered an associative division ring. Three-dimensional associative division rings do not exist.

Each dimension in a number system corresponds to a directional line and can be ordered by a countable or non-countable coordinate system based on real numbers.

Quaternions are commonly not introduced as combinations of spatial numbers and real numbers. The spatial part of a quaternion is often named a vector, but that is a mistake. Vectors obey arithmetic that differs from the arithmetic of spatial numbers.

### 2.2.1 The arithmetic of spatial numbers

For spatial numbers, addition and multiplication are commutative and associative.

$$\begin{aligned}&\vec{b}+\vec{a}=\vec{a}+\vec{b}\\&\left(\vec{a}+\vec{b}\right)+\vec{c}=\vec{a}+\left(\vec{b}+\vec{c}\right)\end{aligned} \tag{2.2.1}$$

The product $d$ of two spatial numbers $\vec{a}$ and $\vec{b}$ results in a real scalar part $d_r$ and a new spatial part $\vec{d}$

$$d=d_r+\vec{d}=\vec{a}\vec{b} \tag{2.2.2}$$

$d_r=-\left\langle\vec{a},\vec{b}\right\rangle$ is the inner product of $\vec{a}$ and $\vec{b}$

For the inner product and the norm $\|\vec{a}\|$ holds $\langle\vec{a},\vec{a}\rangle=\|\vec{a}\|^2$

$$\left\langle\vec{a},\vec{b}\right\rangle=\|\vec{a}\|\|\vec{b}\|\cos(\alpha) \tag{2.2.3}$$

The angle $\alpha$ between the spatial numbers $\vec{a}$ and $\vec{b}$ is measured in radians.

The square of a spatial number equals zero or it is a negative real number.

$$\vec{a}\vec{a}=-\langle\vec{a},\vec{a}\rangle\leq 0 \tag{2.2.4}$$

$\vec{d}=\vec{a}\times\vec{b}$ is the outer product of $\vec{a}$ and $\vec{b}$

The spatial part $\vec{d}$ is independent of $\vec{a}$ and independent of $\vec{b}$. This means that $\left\langle\vec{a},\vec{d}\right\rangle=0$ and $\left\langle\vec{b},\vec{d}\right\rangle=0$

$$\left\|\vec{a}\times\vec{b}\right\| = \left\|\vec{a}\right\|\left\|\vec{b}\right\|\left|\sin(\alpha)\right|$$
$$\vec{a}\times\vec{b} = -\vec{b}\times\vec{a} \tag{2.2.5}$$

It is possible to write spatial numbers as superpositions of base numbers. For the three-dimensional spatial numbers, this means.

$$\vec{a} = a_i\vec{i} + a_j\vec{j} + a_k\vec{k}$$
$$\vec{i}\ \vec{j} = \pm\vec{k} \tag{2.2.6}$$

The $\pm$ sign indicates the chiral choice of the handedness of the outer product.

## 3 Mixed arithmetic

The addition and multiplication of real numbers with spatial numbers are commutative.

$$\begin{aligned} a_r + \vec{b} &= \vec{b} + a_r \\ a_r\vec{b} &= \vec{b}a_r \end{aligned} \tag{3.1.1}$$

Mixed numbers are indicated without suffixes and caps. In the next formula $c$ is a mixed number.

$$c = c_r + \vec{c} \tag{3.1.2}$$

Quaternionic multiplication obeys the equation

$$\begin{aligned} c = c_r + \vec{c} = ab &= \left(a_r + \vec{a}\right)\left(b_r + \vec{b}\right) \\ &= a_r b_r - \left\langle \vec{a}, \vec{b} \right\rangle + a_r\vec{b} + \vec{a}b_r \pm \vec{a} \times \vec{b} \end{aligned} \tag{3.1.3}$$

The $\pm$ sign indicates the freedom of choice of the handedness of the product rule that exists when selecting a version of the quaternionic number system. In this way, the handedness of the product rule is treated as a special kind of symmetry. The version must be selected before it can be used in calculations.

Two quaternions that are each other's inverse can rotate the spatial part of another quaternion.

$$c = ab / a \tag{3.1.4}$$

The construct rotates the spatial part of $b$ that is perpendicular to $\vec{a}$ over an angle that is twice the angular phase $\theta$ of $a = \|a\| e^{\vec{i}\theta}$ where $\vec{i} = \vec{a} / \|\vec{a}\|$ .

Cartesian quaternionic functions apply a quaternionic parameter space that is sequenced by a Cartesian coordinate system. In the parameter space, the real parts of quaternions are often interpreted as instances of (proper) time, and the spatial parts are often interpreted as spatial locations. With these interpretations, the real parts of quaternionic functions represent dynamic scalar fields. The spatial parts of quaternionic functions represent dynamic spatial fields. These fields are often called vector fields. This is a misleading name. Vectors obey different arithmetic.

### 3.1 The arithmetic of change

In continuums, all convergent series of numbers end in a limit that is a member of that continuum. This fact enables the differentiation of the continuum. Differential calculus shows that a continuum can change. The continuum shows astonishing behavior. It has the habit to remove deformations via spherical shock fronts. Without disturbing actuators, the continuum stays flat.

Along a direction line, change can be described by a partial differential. If in a region of the space coverage inside this direction line all converging series of coordinate markers result in a limit that is a coordinate marker, then the partial change of the space coverage along the direction of $r$ is defined as the limit

$$\frac{\partial \psi}{\partial r} = \lim_{\delta r \to 0} \frac{\psi\left(r + \delta r\right) - \psi(r)}{\delta r} \tag{3.1.5}$$

If the region is covered by all its irrational numbers, then this limit exists.

If the spatial part of the neighborhood is isotropic and the limit also exists in the real number space, then the total differential change $df$ of field $f$ equals

$$df = \frac{\partial f}{\partial \tau} d\tau + \frac{\partial f}{\partial x} \vec{i} dx + \frac{\partial f}{\partial y} \vec{j} dy + \frac{\partial f}{\partial z} \vec{k} dz \qquad (3.1.6)$$

In this equation, the partial differentials $\frac{\partial f}{\partial \tau}, \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial y}$ behave like quaternionic differential operators.

The quaternionic nabla $\nabla$ assumes the ***special condition*** that partial differentials direct along the axes of the Cartesian coordinate system in a natural parameter space of a non-separable Hilbert space. Hilbert spaces are extensions of vector spaces and are treated in chapter 5. Thus, for the quaternionic nabla holds

$$\nabla = \sum_{i=0}^{4} \vec{e}_i \frac{\partial}{\partial x_i} = \frac{\partial}{\partial \tau} + \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z} \qquad (3.1.7)$$

This will be applied in the next section by splitting both the quaternionic nabla and the function in a scalar part and a spatial part.

The first-order partial differential equations divide the first-order change of a quaternionic field into five different parts that each represent a new field. We will represent the quaternionic field change operator by a quaternionic nabla operator. This operator behaves like a quaternionic multiplier.

The first order partial differential follows from

$$\nabla = \left\{ \frac{\partial}{\partial \tau}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\} = \nabla_r + \vec{\nabla} \qquad (3.1.8)$$

The spatial nabla $\vec{\nabla}$ is well-known as the del operator and is treated in detail in Wikipedia. The partial derivatives in the change operator only use parameters that are taken from the natural parameter space of the Hilbert space.

$$\begin{aligned}\phi = \nabla \psi &= \left(\frac{\partial}{\partial \tau} + \vec{\nabla}\right)(\psi_r + \vec{\psi}) \\ &= \nabla_r \psi_r - \langle \vec{\nabla}, \vec{\psi} \rangle + \nabla_r \vec{\psi} + \vec{\nabla} \psi_r \pm \vec{\nabla} \times \vec{\psi}\end{aligned} \tag{3.1.9}$$

In a selected version of the quaternionic number system, only the corresponding version of the quaternionic nabla is active. In a selected Hilbert space, this version is always and everywhere the same.

The differential $\nabla \psi$ describes the change of field $\psi$. The five separate terms in the first-order partial differential have separate physical meanings. All basic fields feature this decomposition. The terms may represent new fields.

$$\phi_r = \nabla_r \psi_r - \langle \vec{\nabla}, \vec{\psi} \rangle \tag{3.1.10}$$

$\phi_r$ is a scalar field.

$$\vec{\phi} = \nabla_r \vec{\psi} + \vec{\nabla} \psi_r \pm \vec{\nabla} \times \vec{\psi} \tag{3.1.11}$$

$\vec{\phi}$ is a spatial field.

$\vec{\nabla} f$ is the gradient of $f$ .

$\langle \vec{\nabla}, \vec{f} \rangle$is the divergence of $\vec{f}$ .

$\vec{\nabla} \times \vec{f}$ is the curl of $\vec{f}$ .

Important properties of the del operator are

$$(\vec{\nabla}, \vec{\nabla}) \psi = \Delta \psi = \nabla^2 \psi \tag{3.1.12}$$

$$(\vec{\nabla}, \vec{\nabla} \times \vec{\psi}) = 0 \tag{3.1.13}$$

$$\vec{\nabla} \times (\vec{\nabla} \psi_r) = 0 \tag{3.1.14}$$

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{\psi}\right)=\vec{\nabla}\left(\vec{\nabla},\vec{\psi}\right)-\left(\vec{\nabla},\vec{\nabla}\right)\vec{\psi} \tag{3.1.15}$$

Sometimes parts of the change get new symbols

$$\vec{E}=-\nabla_r\vec{\psi}-\vec{\nabla}\psi_r \tag{3.1.16}$$

$$\vec{B}=\vec{\nabla}\times\vec{\psi} \tag{3.1.17}$$

The formula (3.1.9) does not leave room for gauges. In Maxwell equations, the equation (3.1.10) is treated as a gauge.

$$\left(\vec{\nabla},\vec{B}\right)=0 \tag{3.1.18}$$

$$\vec{\nabla}\times\vec{E}=-\nabla_r\vec{\nabla}\times\vec{\psi}-\vec{\nabla}\times\vec{\nabla}\psi_r=-\nabla_r\vec{B} \tag{3.1.19}$$

$$\left(\vec{\nabla},\vec{E}\right)=-\nabla_r\left(\vec{\nabla},\vec{\psi}\right)-\left(\vec{\nabla},\vec{\nabla}\right)\psi_r \tag{3.1.20}$$

The conjugate of the quaternionic nabla operator defines another type of field change.

$$\nabla^*=\nabla_r-\vec{\nabla} \tag{3.1.21}$$

$$\begin{aligned}\zeta=\nabla^*\phi&=\left(\frac{\partial}{\partial\tau}-\vec{\nabla}\right)\left(\phi_r+\vec{\phi}\right)\\&=\nabla_r\phi_r+\left\langle\vec{\nabla},\vec{\phi}\right\rangle+\nabla_r\vec{\phi}-\vec{\nabla}\phi_r\mp\vec{\nabla}\times\vec{\phi}\end{aligned} \tag{3.1.22}$$

All dynamic quaternionic fields obey the same first-order partial differential equations (3.1.9) and (3.1.22).

$$\nabla^\dagger=\nabla^*=\nabla_r-\vec{\nabla}=\nabla_r+\vec{\nabla}^\dagger=\nabla_r+\vec{\nabla}^* \tag{3.1.23}$$

In the Hilbert space, the quaternionic nabla is a normal operator. The operators

$$\nabla^\dagger \nabla = \nabla \nabla^\dagger = \nabla^* \nabla = \nabla \nabla^* = \nabla_r \nabla_r + \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle \tag{3.1.24}$$

are normal operators who are also Hermitian operators.

$\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle$ is known as the Laplace operator.

One of the second-order partial differential equations results from combining the two first-order partial differential equations $\phi = \nabla \psi$ and $\zeta = \nabla^* \phi$.

$$\begin{aligned} \zeta &= \nabla^* \varphi = \nabla^* \nabla \psi = \nabla \nabla^* \psi = \left(\nabla_r + \vec{\nabla}\right)\left(\nabla_r - \vec{\nabla}\right)\left(\psi_r + \vec{\psi}\right) \\ &= \left(\nabla_r \nabla_r + \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle\right)\psi \end{aligned} \tag{3.1.25}$$

All other terms vanish. The separate operators $\nabla_r \nabla_r$ and $\left\langle \vec{\nabla}, \vec{\nabla} \right\rangle$ are Hermitian operators.

The two operators can also be combined as $\square = \nabla_r \nabla_r - \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle$. This is the d'Alembert operator.

The solutions to $\nabla_r \nabla_r + \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle = 0$ and $\nabla_r \nabla_r - \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle = 0$ differ. These two equations offer different solutions and for that reason, they deliver different dynamic behavior of the field. The equations control the behavior of the embedding field that physicists call their universe. This dynamic field exists everywhere in the reach of the parameter space of the function. Both equations also control the behavior of the symmetry-related fields. The homogeneous d'Alembert equation is known as the wave equation and offers waves and wave packages as its solutions.

Integration over the time domain results in the Poisson equation

$$\rho = \langle \vec{\nabla}, \vec{\nabla} \rangle \psi \tag{3.1.26}$$

Solutions of differential equations are treated in a paper that treats Paul Dirac's bra-ket combination. Together with the differential equations these solutions form the quaternionic field theory.

#### 3.1.1 The spherical del

In isotropic conditions, the Poisson equation can be rewritten as

$$\varphi = \langle \vec{\nabla}, \vec{\nabla} \rangle \psi = \left( \frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} \right) \psi = \frac{1}{r} \frac{\partial^2}{\partial r^2} (r\psi) \tag{3.1.27}$$

The product $\phi = (r\psi)$ is a solution of a one-dimensional equation in which $r$ plays the variable.

The same thing holds for all differential equations that contain the Laplace operator $\langle \vec{\nabla}, \vec{\nabla} \rangle$

So, spherical solutions $\psi = \phi / r$ of the second-order differential equations can be obtained from the solutions $\phi$ of one-dimensional second-order differential equations by dividing $\phi$ by the distance $r$ to the center.

It looks as if in isotropic conditions the quaternionic differential calculus can be scaled down to complex-number-based differential calculus. This already works at local scales. If on larger scales the isotropic condition is violated, then the coordinates of the complex-number-based abstraction must be adapted to the possibly deformed Cartesian coordinates of the quaternionic platform. This makes sense in the presence of moderate deformations of the quaternionic field. After adaptation, the map of each complex-number-based coordinate line becomes a geodesic.

These tricks are possible because complex-number-based Hilbert spaces can be considered subspaces of quaternionic Hilbert spaces.

If the dimension of the quaternionic Hilbert space is reduced to the dimension of a subspace that contains a complex-number-based Hilbert space, then it might become important whether the selected direction involves a polar angle or an azimuth angle. In mathematics, the range of the polar angle is twice the range of the azimuth angle. In physics, the two ranges are exchanged.

The correspondence between isotropic conditions and one-dimensional conditions is important for the mass-energy equivalence. The spherical shock fronts cause deformation and the one-dimensional shock fronts transfer energy. These shock fronts appear to be intimately related. Spherical shock fronts are dark matter objects and one-dimensional shock fronts are dark energy objects. Shock fronts are generated by corresponding pulses that trigger the shock front.

The one-dimensional solution to

$$\left(\nabla_r \nabla_r + \left\langle \vec{\nabla}, \vec{\nabla} \right\rangle\right)\psi = 4\pi\delta\left(\vec{q} - \vec{q'}\right)\theta\left(\tau \pm \tau'\right) \tag{3.1.28}$$

Along the line $\vec{q} - \vec{q'}$ is

$$\phi = f\left(\left|\vec{q} - \vec{q'}\right| \pm c\left(\tau - \tau'\right)\vec{n}\right) \tag{3.1.29}$$

$\theta(\tau)$ is a temporal step function and $\delta(\vec{q})$ is a spatial Dirac pulse response.

After the instant $\tau'$, the equation turns into a homogeneous equation.

During travel the shape of front $f$ stays the same.

For the spherical pulse response, the pulse must be isotropic.

$$\psi = \phi / r = \frac{f\left(\left|\vec{q} - \vec{q}'\right| \pm c\left(\tau - \tau'\right)\vec{n}\right)}{r} \tag{3.1.30}$$

The spherical shock front keeps its shape $f$, but the amplitude of the front shrinks with distance $r$.

Here $\vec{n}$ is a normed spatial quaternion. This spatial quaternion has an arbitrary direction that does not vary in time. In the one-dimensional shock front, the normalized spatial number $\vec{n}$ can be interpreted as the polarization of the solution. $\psi = \phi / r$ describes the spherical shock front solution. In that solution $\vec{n}$ is a spin vector. The solution for the quaternionic equivalent of the wave equation does not contain the normed spatial quaternion $\vec{n}$.

Both shock fronts are tiny field excitations. It will be impossible to perceive these excitations in isolation. Several occasions exist in which these shock fronts join their effect and become perceivable. This happens in the footprints of some of the elementary fermions and in the halos of galaxies.

If a fermion emits or absorbs a dark energy object, then its kinetic energy will change accordingly. A moving fermion must emit dark energy objects at one side or emit them at the other side. This works better perceivable for photons, which are strings of equidistant black energy objects.

## 4 Vector space (1.31)

In this paper, a plain space is a container that has the capability to harbor sets of point-like objects that represent locations. Empty space contains nothing that can be referred to. It has no size, no boundaries, and no center.

A vector is a combination of two point-like objects that are connected by a line. This line defines the direction of the vector. One of the points is the base of the vector and the other point is its pointer. The vector has a length that is represented by a scalar. Shifting the vector along its direction line does not change the integrity of the vector. Also shifting the vector parallel to its direction does not change its integrity. Adding a vector to an empty space turns that space into a vector space. Vectors obey vector arithmetic. Via that arithmetic, vectors can reach all locations of point-like objects that are contained in the vector space.

For example, by recurrently repeating the described shift along the direction line, the set of natural numbers can be constructed such that each new vector pointer location is identified by a corresponding natural number. This enables humans to think about these vector pointer locations.

### 4.1 Vector arithmetic

In this section vectors that reside in a vector space will be indicated with boldface and scalars will be indicated with italics.

The addition of vectors is commutative. It can be done by shifting one of the vectors in parallel until it coincides with the alternative point of the other vector. Now the two resulting points represent the vector sum. The arithmetic of scalars resembles the arithmetic of rational members of the real number systems. Vector addition is commutative. The addition creates new vectors.

$$\mathbf{v}+\mathbf{w}=\mathbf{w}+\mathbf{v} \tag{4.1.1}$$

Vector addition is also associative.

$$(\mathbf{u}+\mathbf{v})+\mathbf{w}=\mathbf{u}+(\mathbf{v}+\mathbf{w}) \tag{4.1.2}$$

Multiplication with a scalar is commutative. This multiplication may change the length and thus the integrity of the vector. It may create a new vector.

$$\mathbf{w}=a\mathbf{v}=\mathbf{v}a \tag{4.1.3}$$

Multiplication with scalars is distributive for scalars and vectors.

$$\begin{aligned}(a+b)\mathbf{v}&=a\mathbf{v}+b\mathbf{v}\\ a(\mathbf{v}+\mathbf{w})&=a\mathbf{v}+a\mathbf{w}\end{aligned} \tag{4.1.4}$$

Multiplication with negative scalars reverses the direction of the vector. In particular

$$(-1)\mathbf{v}=-\mathbf{v} \tag{4.1.5}$$

Vectors obey an inner product. However, they do not obey an outer product. Otherwise, their arithmetic would be equal to the arithmetic of the spatial numbers, and the dimension of the vector space would be limited by three.

#### 4.1.1 Base vectors

A selected base $\{\mathbf{u}_i\}$ is a subset of the vectors that is used to define another vector as a superposition of the members of the base.

$$\mathbf{v}=\sum_{i=0}^{i=N} v_i\mathbf{u}_i \tag{4.1.6}$$

An inner product $\langle\mathbf{v},\mathbf{w}\rangle$ of two vectors $\mathbf{v}$ and $\mathbf{w}$ would be defined in terms of the orthonormal base $\{\mathbf{u}_i\}$ as

$$\langle \mathbf{v}, \mathbf{w} \rangle = \sum_{i=0}^{i=N} v_i w_j \langle \mathbf{u}_i, \mathbf{u}_j \rangle \tag{4.1.7}$$

while

$$\langle \mathbf{u}_i, \mathbf{u}_j \rangle = \delta_{ij} \tag{4.1.8}$$

If the orthonormal base spans the full vector space, then the vector space contains $N$ dimensions. $N$ can be infinite.

The inner product that is taken over all dimensions generates a metric. That metric can establish the length $\ell_{\mathbf{a}}$ of the vector $\mathbf{a}$ as a scalar. The inner product can indicate the length of a vector

$$\begin{gathered} \ell_{\mathbf{a}} = \|\mathbf{a}\| \\ \langle \mathbf{a}, \mathbf{a} \rangle = \|\mathbf{a}\|^2 \end{gathered} \tag{4.1.9}$$

If the inner product equals zero, then either one of the vectors has zero length or the two vectors live in different dimensions. In that case, the vectors are independent. In a $N$ dimensional vector space precisely $N$ vectors can be mutually independent.

The inner product can be applied to construct a set of coordinate markers that together form a coordinate system.

# 5 Hilbert space

Dirac's bra-ket combination converts a vector space into a Hilbert space. This concept applies a selected version of an associative division ring to define the combination of bras and kets. The resulting Hilbert space enables archiving partial sets of elements of the selected version of the number system that the Dirac bra-ket combination has applied to construct the Hilbert space. Consequently, all Hilbert spaces manage a private parameter space in a special operator's eigenspace. This turns any separable Hilbert space into a sampled function space.

## 5.1 Position space and change space

The position space and the change space are different representations of a quaternionic Hilbert space in which the real part of the parameter space is limited to a single point. A Fourier transformation relates functions in the position space to corresponding functions in the change space. This means that in the change space, the location in the position space has no meaning. Similarly, changes in the position space have no meaning. This shows that the scalar part of the parameters can reflect the progress of change. A point in the scalar part of the parameter space represents a standstill and can often be seen as a timestamp.

# 6 System of Hilbert spaces

The definition of Hilbert space results in a system of separable Hilbert spaces that all share the same underlying vector space. The system limits its members to the Hilbert spaces which have parameter spaces that have the axes of their Cartesian coordinate systems parallel to the coordinate system of the vector space. This constraint reduces the number of types of these parameter spaces to a shortlist.

The system of separable Hilbert spaces represents all possible coverages of the vector space by sets of locations that can be identified by a member of a countable number system. One of the separable Hilbert spaces acts as a background platform. All other members of the system float with their geometric center over the private parameter space of the background platform. Only the difference in the symmetry between the background parameter space and the floating parameter spaces appears to be relevant. The shortlist is very similar to the shortlist of electric charges that appear in the Standard Model that experimental particle physicists have discovered. This indicates that there is a strong relationship between symmetry differences in the system of Hilbert spaces and the electrical charges in the Standard Model.

## 6.1 Background platform

The background platform is a separable Hilbert space. The background platform acts as a reference for the symmetries of the other platforms.

The background platform possesses a non-separable companion who embeds its separable partner. The resulting Hilbert space provides operators who manage continuous eigenspaces. These eigenspaces can be considered as continuum extensions of the background parameter space. The continuum represents a dynamic quaternionic field. The operator is defined via a corresponding quaternionic function by Dirac's bra-ket combination. It is a normal operator. The function applies the background parameter space. This parameter space does not change. Its spatial part “behaves” as a static flat field. The real part of the parameter space takes the role of progression. It is an ordered series of timestamps. For every timestamp the function produces a standstill version of the dynamic quaternionic field.

## 7 Quantum physics

So far, the system has not exposed any sign of the uncertainty that characterizes quantum physics. Quantum physicists believe that the wave function is the carrier of this characteristic. The wave function is interpreted as a state function. The square of the modulus of the wave function is a location density distribution. The existence of the wave function can be explained by associating a private state vector with each floating member of the Hilbert space system. The state vector comes from the underlying vector space. A stochastic process generates the hopping path of the state vector in the parameter space of the floating Hilbert space. The hopping path focuses in a stochastic blurry way to the geometric center of the parameter space. The hopping path repeatedly regenerates a hop landing swarm with many landing locations. Covering this swarm with a coordinate grid shows that the swarm can be described by a location density distribution. This distribution appears a stable function. This means that the expected value of the stochastic process is the geometric center of the private parameter space.

The stochastic process can be described as the combination of a Poisson process and a binomial process. The binomial process is implemented by a point scattering function equal to the named location density distribution. This distribution has a Fourier transformation, which is the characteristic function of the stochastic process. Therefore, the wave function can simulate a wave package. Unlike mainstream quantum physics, this paper uses a quaternionic equivalent of the complex wave function.

In some Hilbert space types, the existence of the state vector has a noticeable effect because in the images of these Hilbert spaces in the dynamic universe field, the hop landings cause distortion of the dynamic universe field. The hop landing of the floating Hilbert spaces with an isotropic symmetry difference with the background platform distort the continuum that acts as the dynamic universe. The isotropic pulse response is a spherical shock front that travels away from the location of the pulse at the speed of light in all directions until the front disappears into infinity. Thus, the distortion also extends the coverage of the vector space.

This phenomenon explains the origin of gravity and contributes to the dynamics of the universe. Each isotropic pulse depicted on the dynamic universe field causes a corresponding distortion of that field. Why this happens is a mystery, but it explains the gravitational potential in the dynamic universe field.

## 7.1 The embedding process

The background platform is a combination of a separable Hilbert space and a non-separable Hilbert space. The non-separable Hilbert space embeds its separable partner. The non-separable Hilbert space provides operators who manage continuous eigenspaces. One of these eigenspaces is the continuum extension of the background parameter space that represents a dynamic field that can represent what physicists call their universe. This dynamic field is archived in the eigenspace of a normal operator. The operator is defined via a corresponding quaternionic function by Dirac's bra-ket combination. That function applies the background parameter space. This parameter space does not change. Its spatial part "behaves" as a static flat field. The real part of the parameter space takes the role of progression. It can be considered as an ordered series of timestamps. For every timestamp the function produces a standstill version of the dynamic field.

The function constructs a picture of the embedding of the hopping paths of the floating platforms into the constructed field. Some of the hop landings deform the embedding field. This happens when the symmetry of the floating platform differs isotropic from the symmetry of the background platform. The embedding process appears to behave as an imaging process. Similar imaging processes exist in reality and can be qualified by an Optical Transfer Function. The OTF is the Fourier transform of the Point Spread Function. Here, the point spread functions describe the location density distributions of the hop landing location swarms that are recurrently regenerated by the hop landing paths of the state vectors of the embedded platforms.

If the embedding process can be considered as such an imaging process, then that makes the dynamic field that is archived by the background platform and represents what physicists call their universe an ongoing picture of a huge series of possible coverages of space by locations that are identified by values of number systems.

## 7.2 Electric charges and fields

Electric charges appear to correspond to the difference between the geometric symmetry of the background platform and the geometric symmetry of the floating platforms representing the other elements of the system of separable quaternionic Hilbert spaces. Probably, the electric charge is located in the geometric center of the floating platforms. The charge generates a corresponding electric field that moves with the floating platform. The values of the symmetry differences correspond to the shortlist -3, -2, -1, 0, 1, 2, 3. The corresponding charges form the shortlist 1, 2/3, 1/3, 0, -1/3, -2/3, -1.

The charges can be interpreted as sources or sinks through which the electric field streams. Why the symmetry-related electric charges appear in the geometric center of the corresponding Hilbert spaces is a mystery. The corresponding electric fields are fundamentally different from the dynamic universe field. Yet both fields obey the change arithmetic that governs the behavior of continuums. The dynamic universe field has existed everywhere since the beginning of time. In the beginning this field was flat. The electric fields are linked to the electric charges and indirectly to the symmetries of the prevailing number systems. The fields differ in their start and boundary conditions.

If we limit ourselves to the elementary fermions of the first generation, then electric charge -1 corresponds to the electrons, and the antiparticle called positron corresponds to electric charge 1. In the system, antiparticles are represented by Hilbert spaces in which the sign of the real parts of the parameters is reversed. As a result, the antiparticle seems to move against the direction of time. Also, the sign of the geometric symmetry difference acts reversed. The geometric symmetry of electrons differs isotropic from the geometric symmetry of the background platform. This means that the hop landings of electrons distort the dynamic universe field. The positrons also appear to distort the dynamic universe field.

Neutrinos correspond to electric charge 0. This means that neutrinos share geometric symmetry with the background platform. It seems that neutrinos also distort the dynamic universe field. The reason is that the chiral handedness of the outer product of neutrinos differs from the chiral handedness of the outer product of the background platform.

Quarks have fractional electric charges and therefore do not differ in an isotropic way from the geometric symmetry of the background platform. The chiral handedness of the outer product also does not differ. Therefore, quarks do not distort the dynamic universe field. Certain conglomerates of quarks can form isotropic symmetry differences. These hadrons can distort the dynamic universe field. The distortion betrays the presence of the conglomerate. Isolated quarks remain undetectable. This phenomenon is called color confinement.

### 7.3 Conglomerates

Elementary fermions appear to be able to form conglomerates. These conglomerates are superpositions of elemental fermions or other conglomerates defined in the change space. In the change space, positions have no meaning.

Higher generations of elementary fermions can be interpreted as higher oscillation modes of the first generation of elementary fermions. The hop landing location swarms of higher oscillation modes contain more hop landing locations than the swarms of the lower generation fermions. More hopping landing locations mean a higher ability to distort the embedding universe field.

If the definition of the conglomerate prohibits certain oscillation modes, then this limitation applies independently of the relative location of the participating components of the conglomerate. This phenomenon is known as entanglement.

The ability to form conglomerates produces a very powerful ability to generate modular systems. Modular system generation is more economical than monolithic system generation. All modular systems in the universe are conglomerates of the elemental fermions. Since all elementary fermions have mass or can be combined into particles that have mass, all modular systems in the universe will show mass.

#### 7.3.1 Bosons

in this paper bosons are not considered to be elementary particles. Instead, they are considered to be conglomerates.

#### 7.3.2 Atoms

Atoms are conglomerates in which the components share the image of their geometric center in the dynamic universe. As a result, the compensated electric charges do not participate in the oscillations of the internal components.  Atoms that possess a resulting electric charge are ions.

#### 7.3.3 Molecules

Molecules are conglomerates of ions that share some of their electrons. Molecules archive their essential properties in the system of Hilbert spaces that share the same underlying vector space.

### 7.4 Earth

On Earth, conglomerates of molecules can form living species. Living species archive essential properties in RNA and DNA molecules.

### 7.5 Black holes

Black holes are not conglomerates. They are encapsulated regions in the continuum that represents our dynamic universe. These regions do not contain a continuum. No field excitation can leave or penetrate the area. Black holes deform their continuous surround.

## 8 Photons

Photons are not represented by a Hilbert space. A photon is not an elementary particle. Instead, a photon is a cord of equidistant energy packets. These packages consist of one-dimensional pulse reactions that act as one-dimensional shock fronts. These shock fronts are solutions of second-order partial differential equations that describe the behavior of a quaternionic continuum such as the dynamic universe. The shock fronts move at the speed of light. Photons can occur in streams called light beams. These bundles can have an energy distribution, an angle distribution, a phase distribution, or a location distribution. The location distribution may possess a Fourier transform. In that case, the light beam can behave like a wave package. The imaging properties of the light beam can be qualified by the optical transfer function. This is the Fourier transform of the point spread function. This point spread function is equal to the location density distribution of photons in the light beam.

Atoms and some interactions between elementary particles can cause photons to form or disappear. For example, the conversion of a particle into an antiparticle includes the emission or absorption of a corresponding photon containing a one-dimensional shock front for each hop landing replaced.

## 9 Conclusion

An important conclusion is that the number of Hilbert space types is one greater than the number of the first generation of fermion types. This is because the additional type represents a background platform. The other types are floating platforms. They move over the background parameter space. This suggests that the background platform represents what the Higgs object is supposed to represent. It is the object that via its non-separable companion supports the dynamic universe field, and this companion bears the origin of gravity.

Some mysteries remain unsolved. One of them is the reason for the existence of electric charges. The other mystery is why isotropic symmetry differences cause spherical shock fronts in the dynamic universe field. The most important mystery is why the embedding process is an imaging process that can be qualified by the Optical Transfer Function.

## References

More details can be found at
[https://www.researchgate.net/publication/360423479_The_quaternionic_bra-ket_combination](https://www.researchgate.net/publication/360423479_The_quaternionic_bra-ket_combination)

The referred document contains all the formulas that describe the arithmetic of number systems and the change of continuums. It also treats Dirac's bra-ket combination in detail and explains the effect of the gravitational potential.