Division by zero
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An algebraic theory of extended complex numbers.

\[
\frac{x^x}{x} = x^{x-1} \Rightarrow \frac{0^0}{0} = 0^{-1} = \frac{1}{0} = \infty \iff \frac{0}{0} = 0^0 = \frac{1}{0} \times 0 = \infty \times 0 = 1 \tag{1}
\]

\[
\frac{x}{\infty} = \frac{1}{\infty} \times x = 0 \times x = 0 \iff \frac{\infty}{x} = \frac{1}{0} \times x = \frac{1}{0} = \infty, \quad x \neq 0 \neq 0, \quad -\infty < x < \infty \tag{2}
\]

\[
\ln (1^\infty) = \infty \times \ln (1) = \infty \times 0 = 1 \iff 1^\infty = e \tag{3}
\]

\[
\ln ((-1)^\infty) = \infty \times \ln (-1) = \infty \times \pi i \iff (-1)^\infty = e^{\pi i \infty} \tag{4}
\]

An extended number is in the form \(\tau = x^{\infty^n} + y\) where the infinite part \(\infty^n (n > 0)\) is a number with the property \((\pm \infty)^n \times 0^n = (\pm 1)^n (1)\), and a result of the indivisibility of infinite elements \(2\) is \(f(x) = x^{\infty} \Rightarrow f|_\mathbb{Q} : \mathbb{Q} \rightarrow \mathbb{Z}_\infty\); the sign is located in the dividend.

The theory is noncommutative to keep compatibility with multiplication by zero: \(x^{\infty}0^n = x^{(\infty)0^n} = x\) and \(4\) are only true with the retention of finite elements; i.e. with multiplication from the right if both sides are raised to the power of 0 as only then Euler’s identity holds, this also solves the problem
“which \(y\) is the solution to \(y \times 0 = x^?\)”: \(y = x^{\infty}\). It is also nondistributive in this case:
1 = \(\infty \times (0 \pm 0) \neq \infty \times 0 + \infty \times 0 = 2 \neq \infty \times 0 - \infty \times 0 = 0\); zeros are cancelled out inside parentheses first.

“The Seven Spirits of God”: \(\infty - \infty, 0 \times \infty, 0 \div 0, \infty \div \infty, 0^0, \infty^0\) and \(1^\infty\) which equals 1, 1, 1, 1, 1 and \(e\) \((3)\). Fallacies are corrected by first multiplying by 0 if also dividing by 0 in this case: \(0 \times a \div 0 = 0 \times b \div 0 \Rightarrow (0 \times a) \div 0 = (0 \times b) \div 0 = 0 \div 0 = 1\).