Arguing Against Lorentz Transformation is Illogical

Jian-zhong Zhao

Geophysics Department, Yunnan University, Kunming, Yunnan, China
E-mail: jzhzhao@ynu.edu.cn

Keywords: special relativity, Lorentz transformation, simple derivation, illogical and invalid refuting

Abstract:

A counterfeit and illogical simple derivation of the Lorentz transformation is criticized. A wrong simple derivation of composition of Lorentz transformations is revealed. It is concluded that it is impossible and invalid to refute the Lorentz transformation, then the special theory of relativity, by illogical argument and wrong mathematical inference.

1. Introduction

A research paper “Electromagnetic phenomena in a system moving with any velocity less than that of light” was published by Lorentz in 1904, in which a transformation between inertia systems was suggested.[1]. The Lorentz transformation was deduced respectively by Larmor (1900) and Poincaré (1906).[2,3] Albert Einstein published his famous paper “ On the electrodynamics of moving bodies” in 1905, suggesting the two principles of special relativity and deriving the Lorentz transformation .[4,5] The Lorentz transformation was derived with variant postulates or assumptions by authors.[6-13]

Composition of Lorentz transformations was deduced respectively by B. Coll and F. S. J. Martínez,[14] K. S. Karplyuk and O.O .Zhmudskyy,[15]J. Wilson and

M. Visser[16] .

A simple derivation of the Lorentz transformation was published by Einstein. [17,18] Some authors gave other simple derivations of the Lorentz transformation.[19-27]

In this paper we comment on and criticize a counterfeit simple derivation of the transformation and wrong inference of composition of the Lorentz transformations.[28] It is concluded that it is impossible and invalid to refute the Lorentz transformation, then the special theory of relativity, by illogical argument and wrong mathematical inference.

2.Our Criticism of Li's Simple Derivation of the Lorentz Transformation

Z.-f. Li published so-called "the derivation of the Lorentz transformation"[28]. Our criticism is as follows:

A. Li's derivation is confusing:

Both x in

$$x = k(x' + vt') \tag{1}$$

and x' in

$$x' = k(x - vt) \tag{2}$$

are permanently zeros, but x in

$$x = ct \tag{3}$$

changes with t ,

x' in

$$x' = ct' \tag{4}$$

changes with t'. Therefore, it is confusing to substitute x in Equation (3) for x in Equation (1) and to substitute x' in Equation (4) for x' in Equation (2). That is, it is confusing, in the article of Ref 28, to substitute Equation (3) and Equation (4) into the product of Equation (1) and Equation (2) to have k. Confusing the constants in Equation (1) and (2) with the variables in Equation (3) and (4) proves that the derivation of "the Lorentz transformation" in the article of Ref. 28 is wrong and invalid.

B. It is impossible to obtain the Lorentz transformation, following Li's deduction:

Both x in Equation(1) and x' in Equation(2) are permanently zeros. Substituting

$$k = \frac{1}{\sqrt{1-(\frac{v}{c})^2}} \tag{5}$$

of Li's article [28] into Equation(1) and (2) should result in

$$0= \frac{x'+vt'}{\sqrt{1-(\frac{v}{c})^2}} \tag{6}$$

and

$$0= \frac{x-vt}{\sqrt{1-(\frac{v}{c})^2}} \quad . \tag{7}$$

Definitely, Equation(6) and (7) are not of the Lorentz transformation.

C. Furthermore, Equation (5) by Li is invalid:

The correct expressions of Equations (1) and (2) should be

$$k(x' + vt') = 0 \tag{8}$$

and

$$k(x - vt) = 0, \tag{9}$$

because x in Equation (1) and x' in Equation (2) are permanently zeros.

Following the approach of Li's article [28], substituting Equation (3) and (4) into the product of Equation (8) and (9) leads to

$$k^2(c+v)t'(c-v)t = 0\ , \qquad (10)$$

which results in

$$k = \begin{cases} 0, & t \neq 0 \text{ and } t' \neq 0 \\ \text{indeterminate}, & t = t' = 0 \end{cases}\ , \qquad (11)$$

where $v < c$.

D.From A- C, Li's deduction of the Lorentz transformation is wrong and invalid.

E. By the way, the statement in the article [28] that "The principle of relativity requires that K is equal to K' " is grammatically wrong and physically confusing.

3. Our Criticism of Li's Simple Composition of Lorentz Transformations：

Li deduced two simple compositions of Lorentz transformations, claiming that "Direct transformation is not equal to indirect transformation"[28]. However, the inference of "the direct transformation from K to K'' "[28] is wrong because the addition of velocities of classical mechanics, $v+u$, rather than the addition of velocities of special relativity, is used for the deduction, resulting in the wrong equation[28]

$$x'' = \frac{x-(v+u)\,t}{\sqrt{1-(\frac{v+u}{c})^2}}\ . \qquad (12)$$

The correct simple composition will be derived and presented by us in Sec.4.

4. The Correct Simple Composition, Derived by Us, of the Lorentz Transformations

Three Cartesian coordinate systems are so constructed that the X-axis, the X'-axis and the X''-axis coincide permanently, other axes are parallel respectively,

$OY /\!/ O'Y' /\!/ O''Y''$, $OZ /\!/ O'Z' /\!/ O''Z''$. The coordinate system K' $(O'X'Y'Z')$ moves with speed v relative to K(OXYZ) along the X-axis. The coordinate system K'' $(O''X''Y''Z'')$ moves with speed u relative to K' along the X'-axis (and also the X-axis). The origins of the coordinate systems, O, O' and O'', coincide at the moment $(t = t' = t'' = 0)$. Our task is to establish the Lorentz transformation between the coordinate systems K and K''.

The first derivation by an approach of two steps, transformation from K' to K'' and transformation from K to K', is:

$$x''= \frac{x'-ut'}{\sqrt{1-(\frac{u}{c})^2}} = \frac{(1+\frac{uv}{c^2})x-(u+v)t}{\sqrt{1-(\frac{u}{c})^2}\sqrt{1-(\frac{v}{c})^2}} , \tag{13}$$

$$t''= \frac{t'-\frac{u}{c^2}x'}{\sqrt{1-(\frac{u}{c})^2}} = \frac{(1+\frac{uv}{c^2})t-\frac{(u+v)}{c^2}x}{\sqrt{1-(\frac{u}{c})^2}\sqrt{1-(\frac{v}{c})^2}} . \tag{14}$$

According to the Composition of Velocities [4,5], or the Theorem of the Addition of Velocities [17,18], in special relativity, the coordinate system K'' moves with velocity

$$w = \frac{u+v}{1+\frac{uv}{c^2}} \tag{15}$$

relative to the coordinate system K . Then we have the second derivation by the one-step approach:

$$x''= \frac{x- wt}{\sqrt{1-(\frac{w}{c})^2}} = \frac{(1+\frac{uv}{c^2})x-(u+v)t}{\sqrt{1-\frac{(u^2+v^2)}{c^2}+\frac{u^2v^2}{c^4}}}\left(= \frac{(1+\frac{uv}{c^2})x-(u+v)t}{\sqrt{1-(\frac{u}{c})^2}\sqrt{1-(\frac{v}{c})^2}} \right), \tag{16}$$

$$t''= \frac{t-\frac{w}{c^2}x}{\sqrt{1-(\frac{w}{c})^2}} = \frac{(1+\frac{uv}{c^2})t-\frac{(u+v)}{c^2}x}{\sqrt{1-\frac{(u^2+v^2)}{c^2}+\frac{u^2v^2}{c^4}}}\left(= \frac{(1+\frac{uv}{c^2})t-\frac{(u+v)}{c^2}x}{\sqrt{1-(\frac{u}{c})^2}\sqrt{1-(\frac{v}{c})^2}} \right). \tag{17}$$

From Equations (13), (14),(16) and (17), our two derivations of, or our two approaches to, composition of the Lorentz transformations are equivalent.

5. Impossibility of Refuting the Lorentz Transformation and the Special Theory of Relativity

Li's derivation is neither Einstein's derivation of the Lorentz transformation nor any derivation of the Lorentz transformation in the special theory of relativity.[4-13, 17-27] It is a counterfeit derivation of the transformation. Strictly, Li's derivation is related to neither derivation of the Lorentz transformation nor the Lorentz transformation itself. Logically, any criticisms of Li's derivation, including that in this paper and that in Li's own article of Ref. 28, are neither criticism of derivation of the Lorentz transformation nor criticism of the Lorentz transformation itself. Therefore, it is illogical and impossible to refute the Lorentz transformation, then the special theory of relativity, by arguing against the counterfeit derivation of the transformation in the article of Ref. 28.

6. Conclusion

It is illogical and impossible to refute the Lorentz transformation and its simple derivation, then the special theory of relativity, by arguing against the counterfeit derivation of the transformation in the article of Ref. 28.

References

[1] H.A. Lorentz, "Electromagnetic Phenomena in a System Moving with any Velocity less than that of Light," Proc. Acad. Sci. Amst. **6,** 809 (1904). Re-printed in English translation in: C.W. Kilmister, *Special Theory of Relativity,* (Pergamon Press, 1970)

[2] J. Larmor, *Aether and Matter,* (Cambridge University Press, 1900), Chapter XI.

[3] H. Poincaré, “The Dynamics of the Electron,” Rend. del Circ. Mat. di Palermo **21,** 129-176 (1906).

[4] A. Einstein，“On the electrodynamics of moving bodies,” Annalen der Physik **17,** 891(1905).

[5] A. Einstein, *The Principle of Relativity* (Methuen and Company, London, UK 1923)

[6] R. Weinstock, “New Approach to Special Relativity,” Am. J. Phys. **33,** 640-645 (1965).

[7] A.R. Lee and T.M. Kalotas, “Lorentz Transformation from the First Postulate,” Am. J. Phys. **43,** 434-437 (1975).

[8] Sukhbir Mahajan, “Comments on ‘Lorentz transformation from the first postulate’,” Am. J. Phys. **44,** 998-999(1976).

[9] J.M. Lévy-Leblond, “One more Derivation of the Lorentz Transformation,” Am. J. Phys. **44,** 271-277 (1976).

[10] A. Sen, “How Galileo could have derived the Special Theory of Relativity,” Am. J. Phys. **62,**157-162 (1994).

[11] J.H. Field, “A New Kinematical Derivation of the Lorentz Transformation and the Particle Description of Light,” HELVETICA PHYSICA ACTA **70**(4) 542-564(1997).

[12] Andrea Pelissetto and Massimo Testa, “Getting the Lorentz transformations without requiring an invariant speed,” Am. J. Phys. **83** (4), 338-340(2015).

[13] Vikramaditya Mondal, “Einstein’s special relativity from a space-time duality principle,” e-print *arXiv:*2105.10481, May, 2021.

[14] B. Coll and F.S.J. Martínez, “Composition of Lorentz transformations in terms of their generators,” General Relativity and Gravitation, **34**(9),1345–1356 (2002).

[15] K.S.Karplyuk,O.O.Zhmudskyy, “Hypercomplex representation of the Lorentz’s group,”arXiv:1807.09104,2018.

[16] Joseph Wilson and Matt Visser,“Explicit Baker-Campbell–Hausdorff–Dynkin formula for Spacetime via Geometric Algebra,”arXiv:2107.00343.Jul,2021.

[17] A. Einstein, *Relativity: The Special and General Theory* (HENRY HOLT AND COMPANY, New York, USA 1921).

[18] A. Einstein, *Relativity, the Special and the General Theory (A Popular Exposition)* (METHUEN & CO. LTD, LONDON, UK 1957).

[19] H.M. Dadourian, “On a simple derivation of the Lorentz Transformations,” The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science **45:** 265, 238-240 (1923). DOI: 10.1080/14786442308634107

[20] N. M. Gibbins, “Simple Derivation of the Lorentz Transformation,”The Mathematical Gazette **12**(178) ,474-475(1925).

[21] H. M. Schwartz, “A simple new approach to the deduction of the Lorentz transformations,” Am. J. Phys. **53,** 1007 (1985). doi: 10.1119/1.13989.

[22] Laszlo Szego, “The Lorentz transformation: a simple derivation,” Eur. J. Phys. **10,** 158-159(1989).

[23] Bernhard Rothenstein, “The Lorentz transformation: a transparent derivation,” Eur. J. Phys. **16,** 218-219(1995).

[24] Bernhard Rothenstein, Stefan Popescu and George J. Spix , "A brief and transparent derivation of the Lorentz-Einstein transformations via thought experiments," a) e-print *arXiv*:physics/0602054,2006; b) Physics No2 (2006).

[25] J.-M. Lévy, "A simple derivation of the Lorentz transformation and of the accompanying velocity and acceleration changes," Am. J. Phys. **75,** 615-618 (2007). doi: 10.1119/1.2719700.

[26] Bernhard Rothenstein and Stefan Popescu, "Lorentz transformation, time dilation, length contraction and Doppler Effect – all at once," a) e-print *arXiv*:0712.0463, 2007; b) Physics No 12 (2007).

[27] Olivier Serret, "Reply to 'A Simple Derivation of the Lorentz Transformation'," Journal of Modern Physics **8,** 2167-2178(2017).

[28] Z.-f. Li, "Special relativity arising from a misunderstanding of experimental results on the constant speed of light," Physics Essays, 21, 96—102(2008).