Formula for number of primes less than a given number.

Juan Elias Millas Vera

juanmillaszgz@gmail.com

Zaragoza (Spain)

June 2022

0- Abstract.

In this paper I want to expose the possibility of making a formula which counts the exact number of primes less than a given number, using some tools in analysis of functions, some tools in series of functions and some new tools.

1- Introduction.

We should first take a look to my development in the serial tool named divisory: the serial operator of divisions.

Theorem 1.0: The division operation can be translated into a serial operator with delta notation, it has some different variations of notation, we can see two of them:

\[
\begin{align*}
(1) \quad & \delta_{n=a}^{b} f(n) = f(a) \div f(a+1) \div f(a+2) \div \ldots \div f(b-2) \div f(b-1) \div f(b) \\
(2) \quad & \delta_{n=i}^{k} a_i = a_1 \div a_2 \div a_3 \div \ldots \div a_{k-1} \div a_k
\end{align*}
\]

Remark 1.1: For example, if we want to do the serial divisions between the interval (2,5) we can start in the smaller number and apply the serial division:
\[ (3) \quad \Delta_{n=2}^{5} a_i = 2/(3/(4/5)) = 1/30 \]

2- The divisory of sets.

**Theorem 2.0:** We can use the tool of divisory but instead a variable we introduce a constant, we should apply one-to-one division operations to obtain a set as result:

\[
(4) \quad \{ \Delta \}_{n=i}^k a_i = \{ a/1, a/2, a/3, ..., a/(k-1), a/k \}
\]

As you can see, we do not obtain a single number, we obtain a set of numbers.

**Remark 2.1:** For example, if you do the set divisory of 4 at all range, you will get the following set:

\[
(5) \quad \{ \Delta \}_{n=1}^4 4 = \{ 4/1, 4/2, 4/3, 4/4 \} = \{ 4, 2, 4/3, 1 \}
\]

3- Definition of our f(x) function.

**Theorem 3.0:** We define our f(x) function as 1 if in the process of analyze the set we have, \( s = 2 \) and we define f(x) function a 0 if we have \( s < 2 \) or \( s > 2 \). Where \( s \) is the number of Natural numbers.

\[
(6) \quad f(x) = \begin{cases} 
0 & \text{if } \{ \Delta \}_{n=i}^k \text{ has } s < 2 \\
1 & \text{if } \{ \Delta \}_{n=i}^k \text{ has } s = 2 \\
0 & \text{if } \{ \Delta \}_{n=i}^k \text{ has } s > 2 
\end{cases}
\]
\[ s = \#n \text{ for } n \in \mathbb{N} \]

With the option of \( s < 2 \) we exclude the option of number 1 as prime later on.

**Remark 3.1:** For example \( f(x) \) is equal to 1 for the number 3:

\[
\begin{align*}
\Delta_{n=1}^3 &= \{3/1, 3/2, 3/3\} = \{3, 3/2, 1\} \\
f(x) &= 1 \text{ because in set } \{3, 3/2, 1\} \text{ there are } 2 \text{ natural numbers.}
\end{align*}
\]

**Remark 3.2:** For example \( f(x) \) is equal to 1 for the number 7:

\[
\begin{align*}
\Delta_{n=1}^7 &= \{7/1, 7/2, 7/3, 7/4, 7/5, 7/6, 7/7\} = \{7, 7/2, 7/3, 7/4, 7/5, 7/6, 1\} \\
f(x) &= 1 \text{ because } s=2.
\end{align*}
\]

**Remark 3.3:** For example \( f(x) \) is equal to 0 for the number 4:

\[
\begin{align*}
\Delta_{n=1}^4 &= \{4/1, 4/2, 4/3, 4/4\} = \{4, 2, 4/3, 1\} \\
f(x) &= 0 \text{ because } s=3 > 2.
\end{align*}
\]

**Remark 3.4:** For example \( f(x) \) is equal to 0 for the number 1:

\[
\begin{align*}
\Delta_{n=1}^1 &= \{1/1\} = \{1\} \\
f(x) &= 0 \text{ because } s=1 < 2.
\end{align*}
\]
Lemma 3.5: A number prime always give an \( f(x) = 1 \) and a composite number always give an \( f(x) = 0 \).

4- A formula for the number of prime numbers less than a given number.

Theorem 4.0: The formula for the exact number of prime less than a given number implies a summation for the accumulation of the results and a set serial division which gives us the partial results, and it is the next formula:

\[
\sum_{m=j}^{t} \sum_{n=i}^{k} a_j \{ \Delta \} a_i = \# \text{ primes less than } a_i
\]

Where \( f(x) = \{ \Delta \} a_i \) and \( a_j \) ranges from 1 to k.

Note: the variable \( t \) is a hypothetical result as maximum of the summation, the result of number of primes will always less than \( t \).

Remark 4.1: For example the number of primes less than 4 are given by:

\[
\sum_{m=1}^{4} \sum_{n=1}^{4} a_j \{ \Delta \} a_i = \sum_{m=1}^{4} f(x_1) + f(x_3) + f(x_5) =
\]

\[
= \sum_{m=1}^{4} (\{ \Delta \} 1) + (\{ \Delta \} 2) + (\{ \Delta \} 3) + (\{ \Delta \} 4) = 1 + 2 + 3 + 4 = 10
\]

5- Number primes in an interval.

Finally, I will exemplify the use of this formula as an interval, in example if you want to get the number of primes between 10 and 15 (there are 2 it is obvious), you can put this numbers on the formula:
\[ \sum_{m=10}^{15} a_j \Delta a_i = \sum_{n=1}^{10} f(x_{10}) + f(x_{11}) + f(x_{12}) + f(x_{13}) + f(x_{14}) + f(x_{15}) = \]
\[ = \sum_{m=10}^{15} (\Delta_{10}^{10} + \Delta_{11}^{11} + \Delta_{12}^{12} + \Delta_{13}^{13} + \Delta_{14}^{14} + \Delta_{15}^{15}) = 0 + 1 + 0 + 1 + 0 + 0 = 2 \]

6- Conclusions:

This is my perception of a truly formula for this old problem, I imagine that it is not the most efficient way to compute in a machine a result of this formula (it will take a long process to compute this with large numbers), but in my opinion this is an exact method and number theory needs the two ways of solving the problems: the approximation way methods and the methods of accuracy.