The Gravitational Wave of the Crab Pulsar in the O3b series from LIGO.

Herbert Weidner, Am Stutz 3, D-63864 Glattbach, Germany
eMail: herbertweidner@gmx.de

Identification of the crab pulsar spectral line in the records of LIGO and measurement of the frequency drift. After removing the known frequency drift of the Crab pulsar, sufficiently long data segments from the LIGO interferometers can be narrow-band filtered in order to reduce the noise. The spectral line at 59.23 Hz is clearly visible in 84 records of L1, H1 and V1. The spectral lines of other pulsars of neighboring frequency can be separated well due to different values of the frequency drift.

1 Introduction

In the data series of the LIGO interferometers, several unsuccessful searches have been made in recent years for signs of continuous gravitational wave (GW). The many failures suggest that the methods used so far are not very suitable for detecting extremely weak signals in the noise. In communications engineering, very weak signals from GPS satellites or even more distant space probes are always received according to the same principle: the interfering noise is removed with extremely narrowband filters.

Besides a good signal-to-noise ratio (S/N), this also requires a constant signal frequency. No binary star system or pulsar fulfills this requirement, because the emission of GW means a loss of energy. To detect a GW, one must first remove all modulations and the frequency drift from the source. This can be done with the procedure described below.

2 The sensitivity of the LIGO interferometers

The S/N determines the quality of the signal reception. The average noise amplitude A_{noise} depends on the inherent noise of the receiver, described by the PSD value, and the bandwidth BW of the receive channel.

$$A_{noise} = \sqrt{PSD \cdot BW} \tag{1}$$

The LIGO interferometers have PSD values of around $10^{-45}\ \frac{1}{Hz}$ [1] [2]. One cannot narrow the bandwidth of the signal processing arbitrarily in order to eliminate the disturbing noise. Because then the necessary recording period T_{min}, which the filter needs to settle down, increases. This relationship was first formulated by Küpfmüller and is reminiscent of the Heisenberg uncertainty principle.

$$T_{min} \cdot BW \geq 0.5 \tag{2}$$

Each record of LIGO [3] lasts 4096 seconds and limits the spectral resolution to 122 μHz. Previous measurements have shown that such a wide range usually contains several conspicuous spectral lines that could be GWs. It makes little sense to study the GW of different sources together. Separation can only be achieved by improved frequency resolution of the signal processing, which cannot be achieved by zero padding.

The only way to achieve this goal is to concatenate as many records as possible without gaps. A review of the LIGO records shows that as of 2019-11-06, the V1 and H1 interferometers have recorded data almost simultaneously. The data gaps are short enough for a promising evaluation. Unfortunately, this is not true for L1.

According to formula (2), the recording duration of about 130 hours allows a resolution of 1.1 μHz – sufficient to separate adjacent GWs. Filtering the received data with this bandwidth, the noise floor has the dimensionless value 3.3×10^{-26} according to formula (1).

Since rotating neutron stars are expected to have strains around 10^{-26} [4], the GW of strong sources should be detectable in the records of the LIGO interferometers. Therefore, the frequency range around 59.2 Hz will be searched for signals from the neighboring Crab pulsar.

3 Preparation of the data

For 40 years, astronomers at the Jodrell Bank Centre for Astrophysics have been observing the Crab pulsar, which rotates about 30 times per second [5]. In the following investigation, we assume that $f_{GW} = 2 \cdot f_{rot}$ holds. Because the frequency drift also doubles, the pulsar decreases its transmit frequency by 345 μHz during a measurement period of 130 hours. This value exceeds by far the spectral resolution of about 1.1 μHz, whereby the spectrum loses any significance. All investigations therefore require a careful compensation of the frequency drift.

In the historical records of the three LIGO interferometers [3] one finds few periods, in which several days were measured almost continuously. Starting from GPS time 1257119744 one connects the 120 and 111 files respectively to separate data chains {H1dat} and {V1dat} – these are the basis of all investigations. In this period, the L1 files have too many gaps and are therefore ignored.

The goal of the survey is to search a narrow frequency range in the vicinity of the suspected frequency of the Crab pulsar ($f_{GW} \pm 60$ μHz) for matching spectral lines. Before that some adjustments have to be done:

- Both {H1dat} and {V1dat} files were recorded from 2019-11-07. Interpolation of the published ephemerides [5] gives the initial frequency $f_{GW} = 59.226941$ Hz. From this point in time, the distance to the pulsar decreased at 19 km/s [6]. Because of this blueshift, the measurable frequency of the pulsar is found to be about

3.7 mHz higher than indicated in [5]. The frequency shift is not constant and slightly changes the value of the frequency drift of the pulsar. Considering the short measurement period of only five days, it is not necessary to separate them. During this period, the ephemerides show no glitch.

- The modulation index of the phase modulation caused by the rotation of the earth is very low and only measurable at very good S/N.

- The transmission frequency of the pulsar decreases by 736.6 pHz per second [5]. To obtain sharp spectral lines, one has to compensate this frequency drift completely. This can be done with the MSH method [7]. Despite the short measurement period, the assumption of a linear frequency drift is not sufficient. All modulations of the GW are considered eliminated if the frequency changes by less than ± 0.01 μHz during the entire measurement period. The elimination of the frequency drift with the MSH method is the subject of section 4.

- For the evaluation, 20000 measured values are sufficient corresponding to a sampling period of about 20 seconds. Therefore the frequency of the GW of 59.23 Hz is reduced stepwise to $f_{ZF} = 500\ \mu$Hz, the drift is roughly compensated and the sampling frequency of LIGO is decimated by the factor 8×10^4. The resulting shortening of the file lengths speeds up the calculations.

After this preprocessing, the two spectra in the range $f_{ZF} \pm\ 60\ \mu$Hz show some similarities (Figure 1), which may be GW (see also section 5). The signal of the Crab pulsar is expected at 500 μHz. At exactly this frequency, the spectra of both {H1dat} and {V1dat} files show a coincident peak. This proves that the H1 and V1 interferometers are apparently capable of receiving the GW of the Crab pulsar.

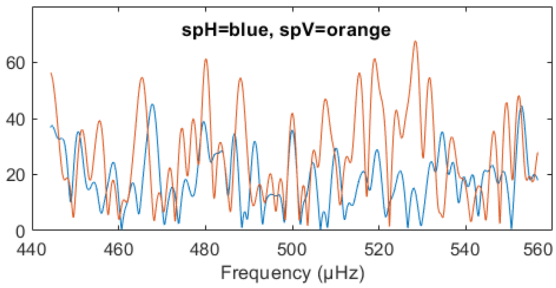

Figure 1): *The spectra of intermediate frequency of {H1dat} and {V1dat} after removal of the frequency drift* $-7.43 \times 10^{-10}\ s^{-2}$. *This value strongly influences the position and height of the spectral lines. The total duration of the records (about 120 hours each) determines the line width.*

4 Removal of frequency drift

The (linear) frequency drift $\dot{f}_x$ of the Crab pulsar measured by astronomers at Jodrell Bank [5] is a good initial value that can be made more precise with a *modified superhet* (MSH).

- A superhet of common design uses a *constant* oscillator frequency so that the frequency reduction to f_{ZF} does *not* change the modulation content. This process transfers the modulation from the signal to the intermediate frequency.
- In an MSH, one tries to control the oscillator frequency so that f_{ZF} is constant after the frequency change of the signal. After performing the MSH procedure, the oscillator has the modulation of the original signal and the intermediate frequency is unmodulated.

Measurements with electromagnetic waves suggest that the frequency of the GW of the crab pulsar decreases by about 350 μHz during the measurement period. Since no spectrum can represent a time-dependent frequency, one must fully compensate for the drift. In the preparation of the data (section 3) the mean frequency of a narrow range was reduced from f_{GW} = 59.23 Hz to f_{ZF} = 4 mHz. Now one refines the last step – the frequency change with a variable oscillator from 4 mHz to 500 μHz. One varies the estimated value of the frequency drift with the aim of minimizing the frequency fluctuations of a selectable spectral line. For the frequency of the variable oscillator applies the approach

$$f_{OSZ}(t) = f_x + \dot{f}_x(T - T_0) + \frac{1}{2}\ddot{f}_x(T - T_0)^2 \tag{3}$$

The MSH procedure starts at time $T_0 = 1257119744$ with initial values $f_x = 3.5$ mHz, $\dot{f}_x = -7.43 \times 10^{-10}\ s^{-2}$ and $\ddot{f}_x = 0$. These parameters are modified until the difference frequency 500 μHz remains constant during the whole interval of about 120 hours. Then the following characteristic values of the GW (including blueshift) are obtained:

{H1}: $f_{GW} = 59.230639324$ Hz; $\dot{f}_x = -7.6420 \times 10^{-10} s^{-2}$; $\ddot{f}_x = 4.3896 \times 10^{-16} s^{-3}$
{V1}: $f_{GW} = 59.230640968$ Hz; $\dot{f}_x = -7.9680 \times 10^{-10} s^{-2}$; $\ddot{f}_x = 4.3220 \times 10^{-16} s^{-3}$

The iterative determinations of frequency and drift converge very well for both files. The poor S/N of the LIGO data does not allow the determination of $\dddot{f}_x$ or the measurement of phase modulation at 24-hour intervals. Surprisingly, however, signs of amplitude modulation with a period of 5.98 hours were detected. A verification will only succeed with a better S/N.

5 Further measurements

The interferometers L1, H1 and V1 measure at irregular time intervals and for different lengths of time. Often one finds only short records with recording durations of a few hours. At the beginning of the present investigations it was checked on the basis of many data segments of the three GW antennas whether one can find matching spectral lines in the noise which can be assigned to a single source. It quickly became apparent that data segments with short recording durations are unusable because the spectral resolving power (see formula (2)) is not sufficient to separate neighboring GWs. For recording times longer than 15 hours, a spectral line can be identified in almost all records, which can be assigned to the GW of the Crab pulsar. Figure 2 shows how the frequency of this spectral line changes over time.

There are different reasons for the broad data gaps in figure 2:

- It is not always possible to find contiguous records of minimum length 15 hours.
- Obviously, there are further GW with different drift in the frequency range shown. The peaks of their spectral lines lie along trend lines with a different slope. In the crossing area with the drawn trend line it is difficult to identify clean spectral lines.

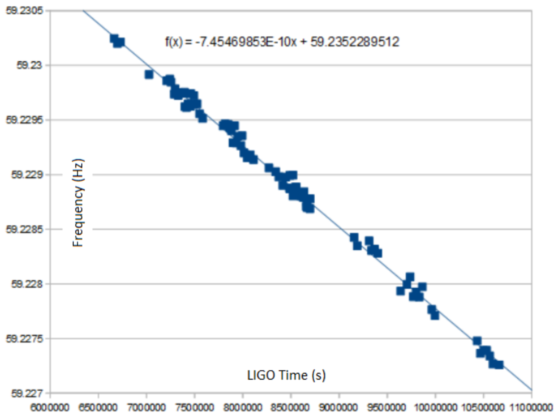

Figure 2): *The time-dependent position of the spectral line of a GW, which can be identified unambiguously in 84 of 90 investigated spectra (record duration about 15 hours each). 30 points each are from H1, L1 and V1. Frequency and drift are in good agreement with the ephemerides of the Crab pulsar [5].*

In all the previous investigations, we limited the search to GWs whose drift has the value $-7.43 \times 10^{-10} s^{-2}$. This value corresponds to the ephemerides of the Crab pulsar. Are there GWs with similar frequency and different drift? To check that, repeat the last step of the preparation (section 3) with deviating values of the drift. In a broad frequency band around $f_{ZF} \approx 500$ μHz one finds some striking matches between the spectra of H1 and V1:

- $f_{ZF} = 340$ μHz, $\dot{f}_x = -5.90 \times 10^{-10} s^{-2}$
- $f_{ZF} = 505$ μHz, $\dot{f}_x = -9.38 \times 10^{-10} s^{-2}$
- $f_{ZF} = 527$ μHz, $\dot{f}_x = -6.14 \times 10^{-10} s^{-2}$
- $f_{ZF} = 604$ μHz, $\dot{f}_x = -5.44 \times 10^{-10} s^{-2}$
- $f_{ZF} = 661$ μHz, $\dot{f}_x = -4.96 \times 10^{-10} s^{-2}$

These GW can be separated well due to their different frequency drift $\dot{f}_x$ and produce significantly higher amplitudes in the H1 and V1 interferometers than the GW of the Crab pulsar. A detailed investigation is beyond the scope of this paper.

References

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