The Information Loss Paradox and Dirac's Sea of Negative Energy

Joseph Palazzo

In this paper, it will be proposed that

1. The production of a pair of particles requires a source of energy.

2. This source of energy comes from Einstein's equation, $E=mc^2$.

3. In this context Hawking radiation is unlikely to be realized in nature even if one appeals to Dirac's sea of negative energy to the rescue.

Preliminary

Some say that the Information Paradox Loss is insolvable. And even if one day someone does have a solution, no one will ever be able to verify it. Others say that not only it is solvable but that there are many solutions – the implication being that if one gets a solution, someone else can propose a different one. Hence the endless list of papers that could be published on this topic. In this paper, we are saying the following: There is no information loss, as there are two major flaws in the argument claiming that Hawking radiation can ever exist.

A. Two Massless Particles

Our first goal is to show that pair production of two massless particles requires a source of energy. This will be done with the derivation of Einstein's famous equation.

Note: It is through this equation that nature allows the decay of matter. Similarly, it is also with this equation that we can accelerate particles to very high speed, and convert their kinetic energy into the creation of new massive particles and anti-particles. In a universe without the conversion of mass into energy, and vice versa, these two phenomena would be impossible.

In Fig. 1 a particle emits two gamma rays, that is, the creation of two massless particles. Einstein knew from experiments previously done that a particle could decay and release gamma rays. He reasoned that the creation of the gamma particles could only be explained by the particle's loss of mass.

So how did he come to that conclusion? He did it by analyzing the situation both in a rest frame and in a moving frame.

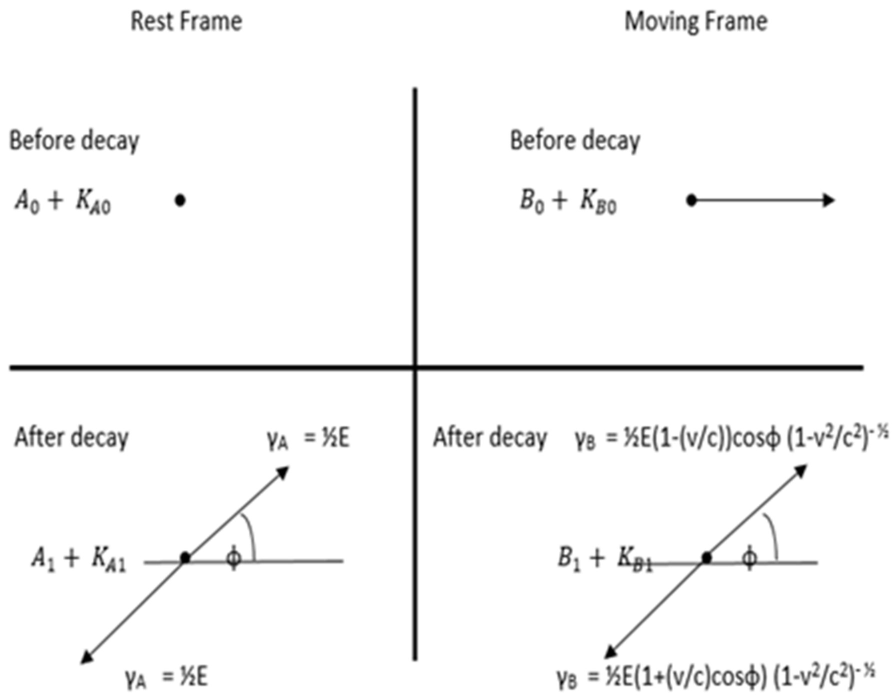

Fig. 1

The symbol γ refers to the energy of a photon .The value of γ in Fig. 1 was already known by Einstein.

The particle has some internal property, which is not needed to be identified, and we labelled it as A in the rest frame, and B in the moving frame. The symbol K is for kinetic energy. We use the symbol "0" for the event taking place before the decay, and "1" for after the decay.

By the law of conservation of energy: The energy before decay = the energy after decay.

(1A) in the rest frame:

$$A_0 + K_{A0} = A_1 + K_{A1} + ½E + ½E = A_1 + K_{A1} + E \qquad \text{A.1}$$

(1B) in the moving frame:

$$B_0 + K_{B0} = B_1 + K_{B1} + ½\, E\left(1 - \left(\frac{v}{c}\right) cos\Phi\right)\, (1 - \frac{v^2}{c^2})^{-½}$$
$$+ ½\, E\left(1 + \left(\frac{v}{c}\right) cos\Phi\right)\, (1 - \frac{v^2}{c^2})^{-½}$$
$$= B_1 + K_{B1} + E(1 - \frac{v^2}{c^2})^{-½} \qquad \text{A.2}$$

Now taking a look at the energy difference of the particle in each of the frame:

(2A) in the rest frame:

$$A_1 - A_0 = -(K_{A1} - K_{A0}) - E = -\Delta K_A - E$$

$$B_1 - B_0 = -(K_{B1} - K_{B0}) - E(1 - \frac{v^2}{c^2})^{-½}$$

A.3

(2B) in the moving frame:

$$= -\Delta K_B - E(1 - \frac{v^2}{c^2})^{-½} \qquad \text{A.4}$$

Whether the observer is at rest or moving with respect to the particle, the energy difference (in 3 and 4) should be the same, independent of the internal properties of the decaying particle.

$$-\Delta K_A - E = -\Delta K_B - E(1 - \frac{v^2}{c^2})^{-½} \qquad \text{A.5}$$

Calculating the difference in kinetic energy

$$\Delta K = \Delta K_A - \Delta K_B$$

$$= E\left(1 - \frac{v^2}{c^2}\right)^{-½} - E$$

$$= E\left[\left(1 - \frac{v^2}{c^2}\right)^{-½} - 1\right]$$

$$\approx \mathrm{E}(1 + ½ \frac{v^2}{c^2} - 1) = ½E\frac{v^2}{c^2} \qquad \text{A.6}$$

By definition the kinetic energy is,

$$K = ½\, m\, v^2 \qquad \text{A.7}$$

If the particle was initially at rest, the change in kinetic energy is simply,

$$\Delta K = K = ½\, m\, v^2 \qquad \text{A.8}$$

Equating 6 and 8, we get

$$½E\,\frac{v^2}{c^2} = ½\, mv^2 \qquad \text{A.9}$$

Therefore,

$$E = mc^2 \qquad \text{A.10}$$

Einstein had reasoned that if the kinetic energy of the particle is smaller by $½E\ \frac{v^2}{c^2}$, the only way this can happen is that the particle must lose mass when emitting radiation.

What can we draw from this? What's important to retain is that the two gamma rays were produced because there was a source of energy provided by the decaying particle's mass, and the conversion takes place according to the formula $E = mc^2$.

B. Two Massive Particles

Our second goal is to show that the source of energy for the production of massive particles is also the conversion of mass according to E = mc^2. In doing so, we will also emphasize the argument that kinetic energy can never be negative.

Before we look at a body decaying into two massive particles, we will introduce the Minkowski space-time coordinates.

We set c = 1

We will measure the velocity of a free particle with respect to the proper time τ, and not the ordinary time t.

$$d\tau = dt/\gamma \qquad \text{B.1}$$

And where γ is now the Lorenz factor (with c = 1),

$$\gamma = \frac{1}{\sqrt{1-v^2}} \qquad \text{B.2}$$

The velocity of a free particle with respect to the proper time is defined as,

$$u^\beta = \frac{dx^\beta}{d\tau} \qquad \text{B.3}$$

Using the chain rule and B.1,

$$u^\beta = \frac{dt}{d\tau}\frac{dx^\beta}{dt} = \gamma\frac{dx^\beta}{dt} \qquad \text{B.4}$$

Expanding the velocity vector into its 4 components,

$$u^\beta = (u^0, u^\mathrm{k}) = \gamma\frac{dx^\beta}{dt}$$

$$= \gamma\left(\frac{dt}{dt}, \frac{dx^\mathrm{k}}{dt}\right) \; where \; \mathrm{k} = 1,2,3$$

$$= \gamma(1, v) = (\gamma, \gamma v) \qquad \text{B.5}$$

The dot product between two vectors is now defined with the metric tensor,

$$u^2 = \mathbf{u} \bullet \mathbf{u} = \eta_{\alpha\beta}\, u^\alpha u^\beta \qquad \text{B.6}$$

Note that in u^2 the 2 means squaring, not the 2nd component of u.

Expanding the above into the temporal and spatial components,

$$u^2 = \eta_{00}\, u^0 u^0 + \eta_{ij} u^\mathrm{i} u^\mathrm{j} \qquad \text{B.7}$$

Using equation B.5,

$$u^2 = (-1)\gamma^2 + \gamma^2 v^2$$
$$= -1\gamma^2(1 - v^2) \quad \text{B.8}$$

But squaring equation B.2 yields

$$\gamma^2 = \frac{1}{1 - v^2} \quad \text{B.9}$$

Therefore,

$$u^2 = \mathrm{u} \bullet \mathrm{u} = -1 \quad \text{B.10}$$

The 4-vector momentum is defined as mass times velocity, that is,

$$p^\alpha = mu^\alpha \quad \text{B.11}$$

Similarly, the 4-vector momentum has components,

$$p^\alpha = (p^0, p^k) \quad \text{B.12}$$

An important result is to calculate p^2, where again the 2 means squaring, not the component 2. First using equation B.11,

$$p^2 = mu^\alpha mu^\alpha$$
$$= m^2 u^2 = -m^2 \quad \text{B.13}$$

Again expanding the above into the temporal and spatial components

$$p^2 = \eta_{00} p^0 p^0 + \eta_{ij} p^{\mathrm{i}} p^{\mathrm{j}}$$
$$= -1(p^0)^2 + 1(p^1)^2 + 1(p^2)^2 + 1(p^3)^2$$
$$-m^2 = -1(p^0)^2 + (\boldsymbol{p})^2 \quad \text{B.14}$$

Where we used equation B.13 in the last line, and ***p*** is the 3-vector momentum = (p^1,p^2,p^3).

We define $p^0 \equiv \mathrm{E}/\mathrm{c} = \mathrm{E}$, (using the convention c=1) as the energy of the free particle.

Putting this altogether, we get,

$$E^2 = (\boldsymbol{p})^2 + m^2 \quad \text{B.15}$$

Putting the factor c back into the equation,

$$E^2 = (\boldsymbol{p}c)^2 + m^2c^4 \quad \text{B.16}$$

For a particle at rest, the momentum is ***p*** = 0. We get back the equation $E = mc^2$.

What is the implication for a particle decaying spontaneously into two massive particles? Consider a particle at rest that decays into two particles, $M_0 \rightarrow m_1 + m_2$.

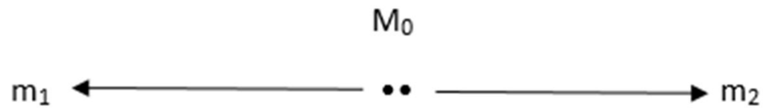

Fig. 2

Initially, the total energy is zero. After decaying, the two particles will fly away from each other with equal and opposite momentum (conservation of momentum) and with equal and opposite spin (conservation of angular momentum). But here's the crucial point: in the absence of a field, they both have only kinetic energy, and by the conservation of energy,

$$0 = KE_1 + KE_2 \qquad \text{B.17}$$

This would necessitate that one of the particles would carry NEGATIVE kinetic energy?! However this is forbidden as ½ mv^2 can only be positive. The only way this can happen is that some positive energy is given to these two particles, and Einstein provided the answer in E= mc^2, that is, there is a mechanism by which this process can take place. We will make this statement stronger: It would be impossible for decay to occur in nature if mass could not be converted to energy. And so with Einstein's equation, the above equation B.17 can be revised to read as,

$$(M_0 \rightarrow \text{energy}) = KE_1 + KE_2 \qquad \text{B.18}$$

For any one particle, we have from equation B.16,

$$E^2 = (\boldsymbol{p}c)^2 + m^2c^4 \qquad \text{B.19}$$

However, we need the energy E, while in B.19, we have the square of the energy. Taking the square root on both sides, we have

$$E = \pm(m^2c^4 + p^2c^2)^{½} \qquad \text{B.20}$$

Now, it was Dirac who first suggested that the negative energies were attributed to anti-particles. He postulated that there was a sea of negative energies fully occupied. This was necessary as a negative energy particle would slide into infinite negative energy. Quantum mechanics requires that there is a ground state from which no other energy state could be lower. No ground state, no quantum mechanics.

If a particle would emerge from that sea, it would appear as a hole, hence, anti-particle, with the same mass but opposite charge. Since we don't deal with absolute energies, but differences in energies, this negative sea is imperceptible. A few years later after Dirac's publication of his proposal, anti-matter was discovered. But the idea of a negative sea was due as unnecessary. We shall come back to this idea later on.

Dropping the negative sign in B.20 and transforming the equation so it can be better manipulated for our need,

$$E = mc^2(1 + \frac{p^2}{m^2c^2})^{½}$$

$$\approx mc^2(1 + ½\frac{p^2}{m^2c^2})$$

$$= mc^2 + \frac{p^2}{2m}$$

$$= mc^2 + KE \qquad \text{B.21}$$

Where KE is the kinetic energy of the particle. For the case of a particle at rest that decays into two particles, $M_0 \rightarrow m_1 + m_2$, we then have from the conservation of energy,

$$Energy\ before\ decay = energy\ after\ decay$$

$$M_0c^2 = m_1c^2 + KE_1 + m_2c^2 + KE_2 \qquad \text{B.22}$$

Rearranging,

$$\big(M_0 - (m_1 + m_2)\big)c^2 = KE_1 + KE_2 \qquad \text{B.23}$$

Taking into consideration equation B.18, then the decaying process can only take place if,

$$M_0 - (m_1 + m_2) > 0 \qquad \text{B.24}$$

A mass can decay as long as it obeys this restriction. However in spite of that restriction, a mass can still decay through different channels, an observation that has been confirmed multiple times in high energy physics.

But what else can we draw from this? Whenever new particles appear, there must be a source of energy such that no new particle can have negative kinetic energy.

C. Hawking Radiation

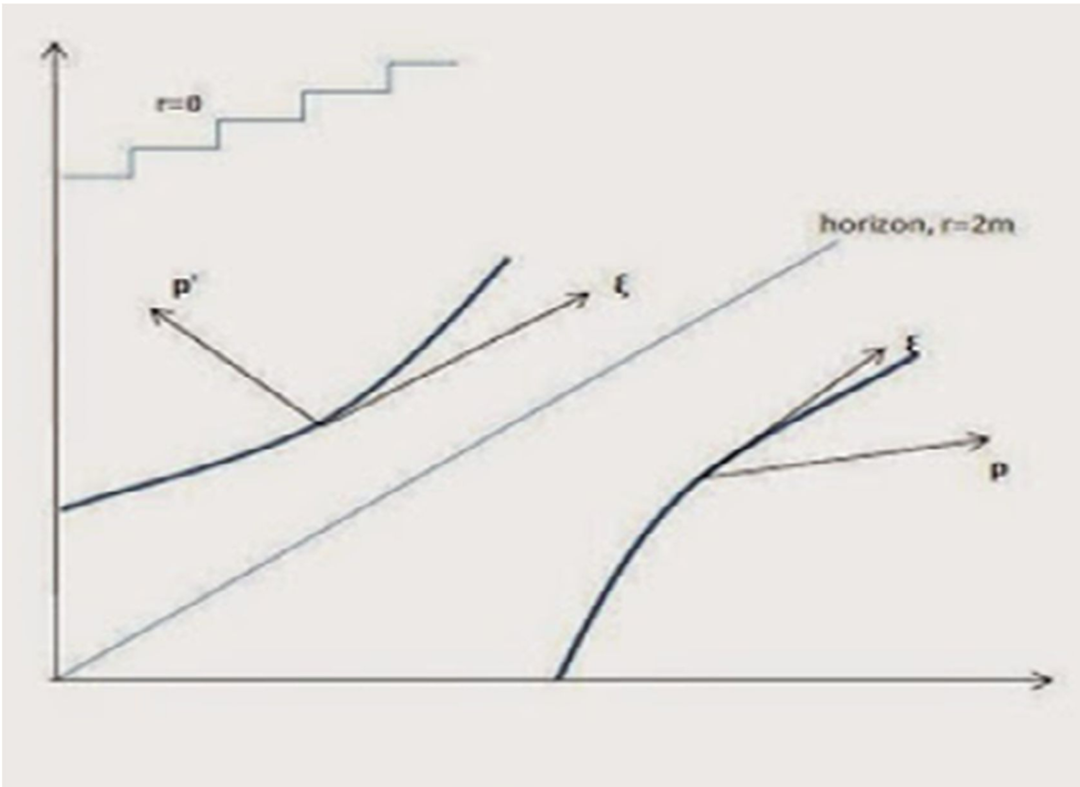

Fig. 3

We will consider Black Holes as theoretical possibilities.

Fig 3 shows a rest-mass zero particle-antiparticle pair which has been created by vacuum fluctuations in such a way that the two particles were created on opposite sides of the event horizon of a black hole. The vector **ξ** is known as a Killing vector. The argument states that the components **ξ•p** and **ξ•p'** must be equal and opposite so that

$$\boldsymbol{\xi}\bullet(\mathbf{p}+\mathbf{p'}) = 0, \text{ (value of the vacuum).} \qquad \text{C.1}$$

One particle (**ξ•p** > 0) can propagate and can be seen as radiation by an observer at infinity. This also means that the other particle, the antiparticle, (**ξ•p'** < 0) will be absorbed by the black hole, and by the conservation law of energy, it will be decreasing mass of the black hole in that process. This is the basis of Hawking's claim that black holes radiate, and in time, will evaporate.

But there are some important issues with this claim that seem to be irreconcilable with known principles:

(i) Negative energy is counter to the very basic notion in QM that a ground state – a state of minimum positive energy – must exist otherwise a particle with negative energy would slide irreversibly into negative infinite energy. In other words, no ground state means no QM. One can borrow from the vacuum as long as it doesn't violate the Uncertainty Principle. In Hawking radiation, there is no borrowing as one particle goes to infinity, the other is absorbed by the black hole.

(ii) As it was mentioned in section B, there must be a source to give energy to particles which are being created in such a process. If that energy comes from the vacuum, then the equation C.1, which is an equation about momentum, should read as an energy equation,

$$m_1c^2 + KE_1 + m_2c^2 + KE_2 \rightarrow \Delta E_{vacuum} \qquad \text{C.2}$$

In other words, as energy would be taken away from the vacuum, if that were a possibility, the vacuum itself would be losing energy! And then as the vacuum loses energy, the black hole would be gaining in mass, which is contrary to what has been proposed.

D. Negative Sea

To remedy this situation, one would have to postulate that the vacuum (by definition, it has no particles), nearby the event horizon of a black hole has positive energy due to gravity. However this doesn't fix the situation as the creation of both particle and anti-particle would still have positive energy, and the black hole would gain mass.

E. Conclusion

As it stands, Hawking radiation is problematic. Notwithstanding this puts a hole in the Information Paradox Loss: no Hawking radiation, no information loss.