Is Special Relativity compatible with General Relativity?

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Abstract

Is special relativity theory applicable to the matter universe, as this theory was developed for places with no acceleration or gravity? Is Special Relativity compatible with General Relativity?

Yes. General Relativity is a generalization of Special Relativity. Special Relativity applies locally in inertial frames, while General Relativity describes gravity and curved spacetime globally.

There is a difference between Special Relativity (SR) and General Relativity (GR). Special Relativity describes the laws of physics in inertial frames and explains how measurements of space, time, energy, and momentum depend on the relative motion of observers. It was developed by Einstein in 1905 and was based on works done by Newton, Lorentz, Maxwell, Michelson-Morley, and others. General Relativity, published in 1915, generalized Special Relativity to include gravity as the manifestation of curved spacetime. According to the equivalence principle, acceleration and gravity are equivalent. Although gravity exists everywhere in the universe, spacetime can always be approximated as locally flat within a sufficiently small freely falling region.

Einstein recognized that Special Relativity alone could not describe gravity. This realization led him to General Relativity, in which SR emerges as the local limit in freely falling frames. A quote from the collected papers of Albert Einstein Volume 7: The Berlin Years: Writings, 1918-1921

https://einsteinpapers.press.princeton.edu/vol7-trans/156?highlightText=%22spatially%20variable%22

"Second, this consequence shows that the law of the constancy of the speed of light no longer holds, according to the general theory of relativity, in spaces that have gravitational fields. As a simple geometric consideration shows, the curvature of light rays occurs only in spaces where the speed of light is spatially variable. **From this, it follows that the entire conceptual system of the theory of special relativity can claim rigorous validity only for those space-time domains where gravitational fields (under appropriately chosen coordinate systems) are absent. The theory of special relativity, therefore, applies only to a limiting case that is nowhere precisely realized in the real world**. Nevertheless, this limiting case ⟨also⟩ is of fundamental significance for the theory of general relativity; because of the fact from which we started, namely that no gravitational field exists near a free-falling observer, this very fact shows that in the vicinity of every world point the results of the theory of special relativity are valid (in the infinitesimal) for a suitably chosen local coordinate system."

Beginning in 1907, Einstein realized that a freely falling observer experiences no local gravitational field. This insight—the equivalence principle—eventually led to General Relativity, in which Special Relativity appears as the local limit.

Another formulation is: Special Relativity is not globally applicable in curved spacetime, but it remains locally valid at every spacetime event in a freely falling inertial frame. General Relativity reduces to Special Relativity in the limit of negligible curvature and over sufficiently small regions.

SR predicts several counterintuitive phenomena, including time dilation, length contraction, and the relativity of simultaneity, all of which have been confirmed experimentally.

I will bring here the known example of the global positioning system (GPS), which is used many million times a day by users all over the world. The GPS includes a relativistic correction factor that is comprised of both GR gravitational time dilation and SR kinematic time dilation. Not using the relativistic correction factor of GR and SR renders the GPS useless.

I will explain with the example of GPS, as shown in Fig. 1, why SR is valid in the gravitational field of Earth. Figure 1 provides an intuitive geometric illustration of Einstein's concept of local inertial frames. It shows schematically the structure of the GPS. The GPS is comprised of three different segments: the space segment, the control segment, and the user segment. The space segment includes 30+ navigation satellites circling Earth at an altitude of 20,000 km and a speed of 14,000km/h. The control segment is a ground station and the user segment e.g., cellular phone.

At the start of measurement, the reference frames of ground station G_0 and satellite frame S_0 are parallel. If the measurement time is big then the frames G_f and S_f are no longer parallel and are rotated by angle Φ. For example, if the measurement duration is one hour, then Φ1=~30deg and Φ2=~15deg. Thus, the relative rotation Φ =Φ1- Φ2=~15deg. The smaller the measurement time, the smaller Φ, and the reference frames stay nearly parallel.

Fig. 1 illustrates that the satellite's reference frame changes continuously as it orbits the Earth. Therefore, a single inertial frame cannot describe the entire orbit. Instead, the orbit is treated as a succession of infinitesimal local inertial frames. Over each infinitesimal interval, the satellite occupies a tiny region of spacetime that is effectively flat, so the equations of Special Relativity apply. The complete relativistic correction is then obtained by combining the local Special Relativity contribution with the General Relativity contribution over the entire orbit.

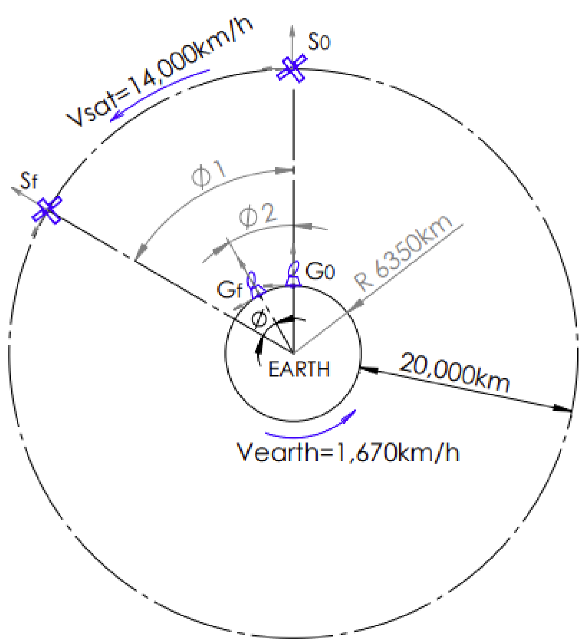

Fig 1. – GPS - Global Positioning System

According to Einstein, the rate of advance of two identical clocks placed one on the satellite and the other on the ground station, will differ due to the difference in the gravitational potential (GR) and the relative speed between them (SR). From GR, it can be calculated that the orbiting clocks in the satellites tick slightly faster than the clock at the ground station, by about 45 microseconds per day. On the other hand, from SR calculations the orbiting clock ticks slower, about 7 microseconds per day. The net result is that time on a GPS satellite clock advances faster than a clock on the ground by about 38 microseconds per day. If relativity compensation is not taken into account, it would cause navigational errors that accumulate to 10 km per day! **Note:** a detailed calculation is given at https://www.scienceofgadgets.com/post/how-relativistic-time-dilation-and-gps-are-related

Thus, SR is applicable because each infinitesimal segment of the satellite's orbit can be regarded as a local inertial frame occupying a tiny region of nearly flat spacetime. The complete relativistic correction is obtained by integrating these local SR and GR contributions over the entire orbit. In practice, the infinitesimal intervals are replaced by sufficiently small finite intervals, making the calculation an excellent engineering approximation.

Can Hafele and Keating experiment be explained by GR and SR?

In Hafele and Keating's experiment, two airplanes with atomic clocks on board started from the same place on Earth. An atomic clock was left on the ground. The airplanes were flown around Earth one eastwards and the second westwards. Finally, the airplanes were brought near the clock that stayed on Earth. Their measurements showed that there is a time dilation that can be calculated by both contributions of SR and GR. The calculation does **not** assume that one inertial frame extends over the entire flight around the Earth. Instead, physicists calculate the proper time by following the clock continuously as it moves through curved spacetime. Each infinitesimal step is treated locally, where SR applies, while the global geometry is described by GR.

Conclusion

Special Relativity and General Relativity are complementary rather than competing theories. General Relativity extends the principles of Special Relativity to curved spacetime. Whenever spacetime can be regarded as locally flat, the laws of Special Relativity apply. This relationship is confirmed by experiments such as the Hafele–Keating flight and by the successful operation of the GPS system, which relies on both theories simultaneously.