New Schwarzschild black hole solution for Kerr-Newman-like black holes

Wen-Xiang Chen^a

Department of Astronomy, School of Physics and Materials Science, GuangZhou University, Guangzhou 510006, China*

When Λ is the cosmological constant about $(g^{\theta\theta})^2[3]$. We assume that both the macroscopic system and the microscopic system are closed systems, the entropy of the system increases to 0, the entropy of the macroscopic open system increases, and the entropy of the microscopic system decreases. We know that Ads space can constitute Ads/CFT theory, and ds space has serious difficulties (experimentally proves that the universe is ds spacetime). Assuming that there is a spontaneous entropy reduction process in the microscopic system, the Ads space can evolve into a ds space. We got that new Schwarzschild black hole. At that time we saw a similar situation with the new Schwarzschild black hole and the Kerr-Newman-like black hole.

Keywords: Ads space, ds space , new Schwarzschild, Kerr-Newman-like

1. INTRODUCTION

As we all know, Einstein's general theory of relativity is considered to be the most beautiful of all existing physical theories, and it has successfully passed a series of observations and experimental tests. In the general theory of relativity, since the geodesic equations of particles are separable, the dynamical system is integrable, so in the usual Kerr-Newman-like black hole space-time, the geodesic motion of particles will not exhibit chaotic behavior. However, in some space-times with complex geometric structures or some additional interaction terms are introduced, the dynamical systems of particles often have non-integrability. Therefore, the chaotic motion of particles in the background of disturbed Schwarzschild space-time, or multi-black hole space-time, or black hole space-time in a magnetic field, or accelerated rotating black hole space-time, or non-standard rotating black hole has been widely studied by scholars. In addition, the chaotic behavior of ring strings is found in Schwarzschild black hole spacetime and AdS black hole spacetime. Chaos also occurs in string-ring dynamics around black holes, in the motion of charged particles in the magnetosphere, and in the motion of particles very close to the event horizon.

With the rapid development of cosmological observations in recent years, from the emission of gravitational waves from distant binary stars to the event horizon telescope, as well as the observation of supermassive compact celestial bodies, there is a lot of convincing evidence that black holes exist and are rotating. In general relativity (GR), a stationary axisymmetric rotating black hole is described by Kerr metric. The Kerr metric is the only vacuum black hole in GR with the above symmetry. Although it is a fairly complex metric for solving partial differential equations, it has many hidden symmetries and mathematical properties that make it tractable in physical applications. In view of this, one can solve the partial differential equation analytically, especially in the case of containing the cosmological constant, which is already a remarkable achievement. In fact, it is only in four-dimensional space that we can analytically find the relevant band electrolysis, and a large number of high-dimensional rotational solutions have not been fully computed until recently. In addition, it may be due to the existence of the additional Killing tensor that the related Hamilton-Jacobi functions are found to be separable, so that the geodesics in the Kerr background can be calculated analytically. Using the Newman-Penrose form, the linear perturbation of Kerr space-time can be written as a The separable form, the Teukolsky equation. These mathematical properties lay the foundation for understanding linear stability and quasi-normal patterns (which are important for the ringing phase of binary stars) and the geodesics of black hole shadows. In addition, due to the existence of the energy layer, the research on the phenomenon of superradiance, the Penrose process of extracting energy from rotating black holes, and even the quantum laser effect of black holes have achieved considerable results. Therefore, whether it is considered from the perspective of phenomenology or theoretical research, it can be said that Kerr's solution may be the most important solution of Einstein's equation.

When Λ is the cosmological constant about $(g^{\theta\theta})^2[3, 10]$. We assume that the macro system and the micro system are closed systems, the system entropy increases to 0, the macro open system increases in entropy, and the micro system decreases in entropy. We know that Ads space can constitute Ads/CFT theory, while ds space has serious difficulties (experiments prove that the universe is ds space-time). Assuming that there is a spontaneous entropy reduction process in the microscopic system, the Ads space can evolve into a ds space. We get that new Schwarzschild black hole.

^{*}Electronic address: wxchen4277@qq.com

At that time we saw that the new Schwarzschild black hole had a similar situation to the Kerr-Newman-like black hole.

2. NEW CLASS OF ACTION AND FIELD EQUATIONS

The purpose of this theory is that we find that the modified Einstein gravitational equation has a Reissner-Nodstrom solution in vacuum. First, we can consider the following equation (modified Einstein's gravitational equation). The proper time of spherical coordinates is [1, 2, 4–6] (The metric in which is in exponential form)

$$ds^{2} = -e^{G(t,r)}dt^{2} + e^{-G(t,r)}dr^{2} + \left[r^{2}d\theta^{2} + r^{2}\sin^{2}\theta d\varphi^{2}\right]$$
(1)

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda((g^{\theta\theta})^2)g_{\mu\nu} = -\frac{8\pi G}{C^4}T_{\mu\nu}$$
(2)

In this work, the action (we set 8G = c = 1) is given by the following relation which in the special case, reduces to the Einstein-Maxwell dilaton gravity:[10]

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} (-2\Lambda(\left(g^{\theta\theta}\right) + R)) \tag{3}$$

where Λ is a function of the Ricci scalar R and Φ is the representation of the dilatonic field, also similar to f(R) (We will now consider non-pathological functional forms of f(R) that can be expanded in a Taylor series of the form $f(R) = a_0 + R + a_2 R^2 + a_3 R^3 + \ldots a_n R^n + \ldots$ where we have normalized all coefficients with respect to the coefficient of the linear term). Variation of the action with respect to the metric $g_{\mu\nu}$, the gauge A_{μ} and dilaton field Φ gives the following field equations:

This leads to: In these equations we have:

$$\nabla_{\mu}\nabla^{\mu}\Lambda_{R} = \frac{1}{\sqrt{-g}}\partial_{r}\left(\sqrt{-g}\partial^{r}\right)\Lambda_{R} = \left(e^{G(t,r)}\Lambda_{R}' + e^{G(t,r)}\Lambda_{R}'' + \frac{e^{G(t,r)}}{r}\Lambda_{R}''\right)$$

$$\nabla^{t}\nabla_{t}\Lambda_{R} = g^{tt}\left[\left(\Lambda_{R}\right)_{,t,t} - \Gamma_{tt}^{m}\left(\Lambda_{R}\right)_{,m}\right] = \frac{1}{2}(e^{G(t,r)})'\Lambda_{R}'$$

$$\nabla^{r}\nabla_{r}\Lambda_{R} = g^{rr}\left[\left(\Lambda_{R}\right)_{,r,r} - \Gamma_{rr}^{m}\left(\Lambda_{R}\right)_{,m}\right] = \left(e^{G(t,r)}\Lambda_{R}'' + \frac{(e^{G(t,r)})'}{2}\Lambda_{R}'\right)$$

$$\nabla^{\theta}\nabla_{\theta}\Lambda_{R} = g^{\theta\theta}\left[\left(\Lambda_{R}\right)_{,\theta,\theta} - \Gamma_{\theta\theta}^{m}\left(\Lambda_{R}\right)_{,m}\right] = \frac{e^{G(t,r)}}{r}\Lambda_{R}'$$

$$(4)$$

From the tt and rr components of the field equations, one can easily show the following relation:

$$\nabla^r \nabla_r \Lambda_R = \nabla^t \nabla_t \Lambda_R \Longrightarrow e^{G(t,r)} \Lambda_R'' = 0 \Longrightarrow \Lambda_R'' = 0$$
(5)

This leads to:

$$\Lambda_R = z + yr \tag{6}$$

In this relation, y and z are just two integration constants and assumed to be positive from avoiding non-physical ambiguity.

If we assume that the matter representing the cosmological constant belongs to fermions, we see that it appears with a term with an electromagnetic-like potential. Its Lagrange equation is

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_i}\right) - \frac{\partial \mathcal{L}}{\partial x_i} = \frac{d}{dt}\left(\gamma m \dot{x}_i\right) + \frac{\partial V}{\partial x_i} = 0,\tag{7}$$

there is a solution with the term 0 at the end. The relativistic Lagrangian of a charged particle moving in an electromagnetic field can be written as

$$\mathcal{L} = -mc^2 \sqrt{1 - \frac{v^2}{c^2}} - q\phi(\mathbf{r}) + q\mathbf{v} \cdot \mathbf{A}(\mathbf{r}, t)$$
(8)

where q is the charge of the charged particle, ϕ is the electric potential, and A is the magnetic vector potential. Its Lagrange equation is

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_i}\right) - \frac{\partial \mathcal{L}}{\partial x_i} = \frac{d}{dt}\left(\gamma m \dot{x}_i\right) + q\frac{dA_i}{dt} - q\frac{\partial \phi}{\partial x_i} - q\sum_{j=1}^3 v_j \frac{\partial A_j}{\partial x_i} = 0.$$
(9)

 $\mathbf{so},$

$$\frac{d}{dt}\left(\gamma m \dot{x}_i\right) = -q \frac{dA_i}{dt} + q \frac{\partial \phi}{\partial x_i} + q \sum_{j=1}^3 v_j \frac{\partial A_j}{\partial x_i}.$$
(10)

Derivation by analogy:

$$\frac{1}{2}R\Lambda'(R) - \Lambda = 0 \tag{11}$$

$$R_{kl}\Lambda'(R) - \frac{1}{2}g_{kl}\Lambda = 0$$

$$\nabla_{\sigma} \left[\sqrt{-g}\Lambda'(R)g^{\mu\nu}\right] = 0$$
(12)

In this relation, y and z are just two integration constants and assumed to be positive from avoiding non-physical ambiguity.

$$\mathbf{\Lambda} = B(\mathbf{r} \times \mathbf{p})/r^4 \tag{13}$$

B is an algebraic parameter,p is momentum or momentum operator.

3. SUMMARY

When Λ is the cosmological constant about $(g^{\theta\theta})^2$ [3, 10]. We assume that both the macroscopic system and the microscopic system are closed systems, the entropy of the system increases to 0, the entropy of the macroscopic open system increases, and the entropy of the microscopic system decreases. We know that Ads space can constitute Ads/CFT theory, and ds space has serious difficulties (experimentally proves that the universe is ds spacetime). Assuming that there is a spontaneous entropy reduction process in the microscopic system, the Ads space can evolve into a ds space. We got that new Schwarzschild black hole.

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