Not allowed odd maxima of cyclic Collatz sequences.

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Abstraction: The preprint provides a calculation of the not allowed odd maxima of cyclic Collatz sequences

1. Formulation of the lemma on cyclic sequences:

If the numbers \((ka + 1) = b2^q, (kb + 1) = c2^f, (kc + 1) = a2^u\) form a Collatz conjecture cycle, then the expression (1) holds

\[
(ka + 1) \cdot (kb + 1) \cdot (kc + 1) \ldots = 2^m \cdot a \cdot b \cdot c \ldots
\]  

(1)

In the text: \(a, b, c\) - are odd integers; \(k, m, h, f, N, q, t, u\) – are integers

2. Expression (1) in another form (2)

\[
\frac{(ka+1)(kb+1)(kc+1)}{abc} = 2^m
\]  

(2)

If the numbers \(b, c, (ka + 1), (kb + 1)\) form a Collatz conjecture corresponding to expression (3), then the only condition for the formation of a cycle is expression (4)

\[
\frac{(ka+1)(kb+1)}{bc} = 2^n
\]  

(3)

\[
\frac{(kc+1)}{a} = 2^h
\]  

(4)

Thus, the lemma can be expressed by condition (4).

Odd numbers can be represented in the form (5), then the numbers \((ka + 1)\) have the form (6)

\[
a = 2f - 1
\]  

(5)

\[
(ka + 1) = 2kf - k + 1
\]  

(6)

The number \((kc + 1)\) can be represented in the form (7)

\[
(ka + 1) - (kc + 1) = 2kΔf
\]
\[ N = \Delta f = \frac{a + 1}{2} - \frac{c + 1}{2} = \frac{a - c}{2} \]

\[(k\epsilon + 1) = (k\epsilon + 1) + 2kN \quad (7)\]

Then the condition lemma can be represented by (8.1) and (8.2)

\[ \frac{(k\epsilon + 1) + 2kN}{a} = 2^h \quad (8.1) \]
\[ \frac{(k(2f - 1) + 1) + 2kN}{(2f - 1)} = 2^h \quad (8.2) \]

3. Expression (8.2) is converted to (9)

\[ \frac{(k(2fa - 1) + 1) + 2kN}{(2fa - 1)} = 2^h, \quad kf_a - \frac{(k - 1)}{2} + kN = 2^{h - 1}(2fa - 1) \quad (9) \]

Accordingly,

If \( k = 3 \), then the expression (9) has the form (10):

\[ (3f + 3N) - 1 = 2^h f - 2^{h - 1} \quad (10) \]

If \( k = 5 \), then the expression (9) has the form (11).

\[ (5f + 5N) - 2 = 2^h f - 2^{h - 1} \quad (11) \]

4. With a certain parity ratio between the numbers \( f \) and \( N \), it is possible to exclude cycles.

\[ f_a = \frac{a + 1}{2}, \quad N = \frac{c - a}{2}, \quad \frac{a + 1}{2} = \frac{c - a}{2} \]

\[ f_a \equiv N \quad (9) \]

\[ 2f = c - (2f - 1) \]

\[ 4f - 1 = c \]

Thus, the numbers \( c \) of the form \( 4x - 1 \) - (3, 7, 11, 15, 19...) they correspond to condition (9), and cannot be the maximum of cycles for the sequence Collatz conjecture \( k = 3 \), but are the only maxima of possible cycles for sequences with \( k = 5 \). The remaining numbers \( c \) of the form \( 4x + 1 \) have the inverse property with respect to the maxima of cycles.