Lecture Notes on Symmetry Optics

Lecture 4:
Critical Planes of the Beam

To accompany https://youtu.be/3Ph0w_Vvla0
v4, April 13, 2022
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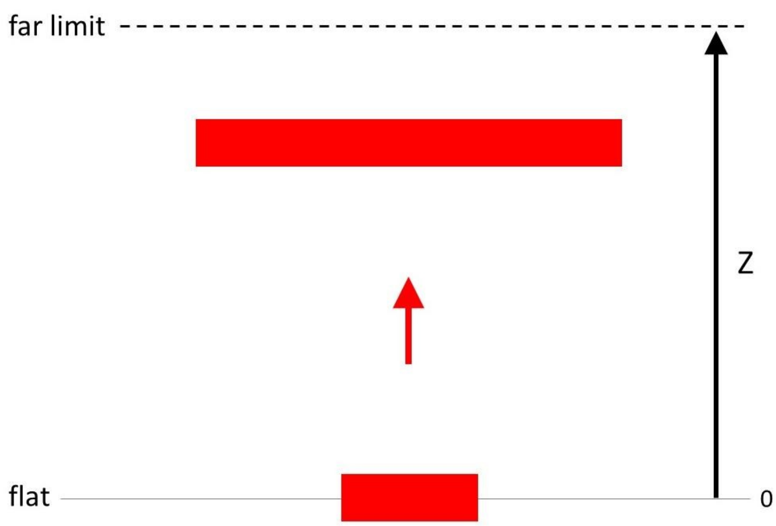
1 Overview

1.1 The propagating beam

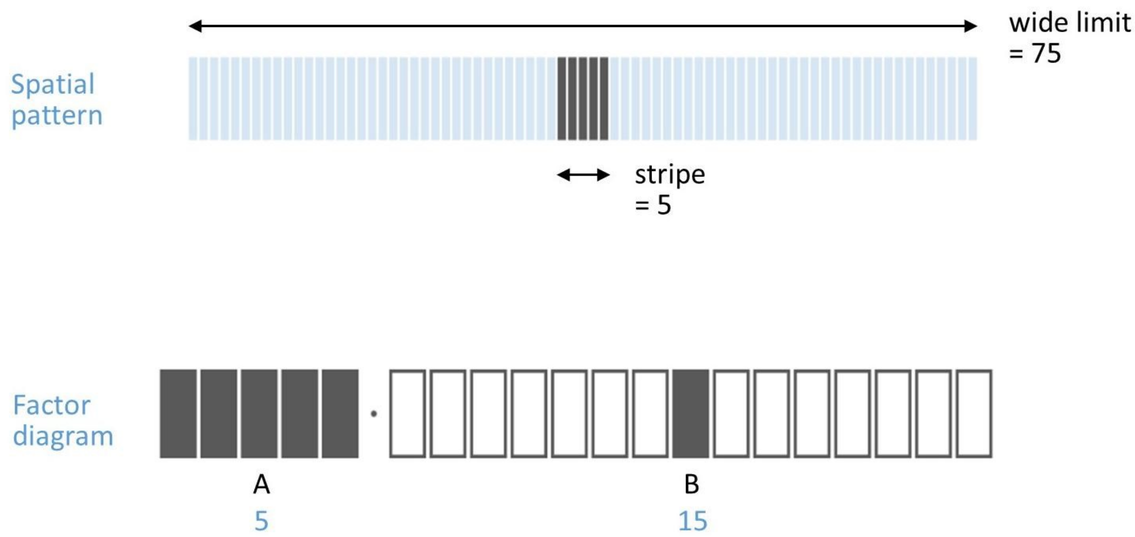


Welcome to Lectures on Symmetry Optics. I'm Paul Mirsky. This is Lecture 4 of the series, and the topic is: Critical Planes of the Beam.

Consider what happens when a flat wavefront passes through a single slit, and then continues propagating. When it leaves the slit, the beam is a single stripe of bright space, and the wavefront is flat. As it propagates, the wavefront becomes curved, and the stripe eventually grows wider because of diffraction.

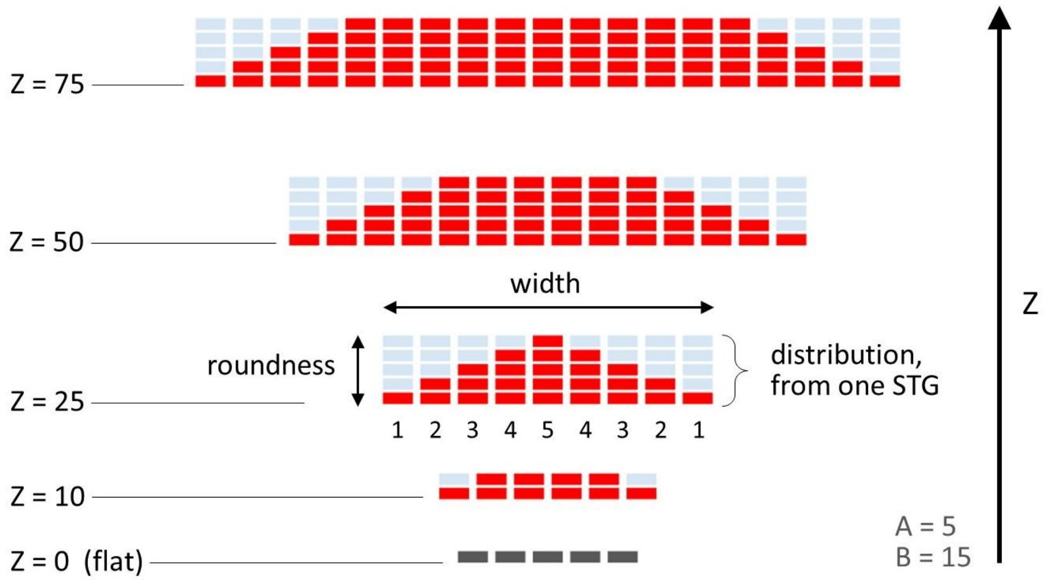
In this lecture, we'll study the distributions in all planes, and we'll talk about the overall trends.

1.2 Beam pattern at the flat



Let's assume that we're given the starting pattern at the flat. Here's an example, which we'll use in this lecture. The stripe is 5 patches wide, and the wide limit is 75 patches wide. In terms of factors, we have a plenary factor, size 5, and a singular factor, size 15.

1.3 Distribution in many planes

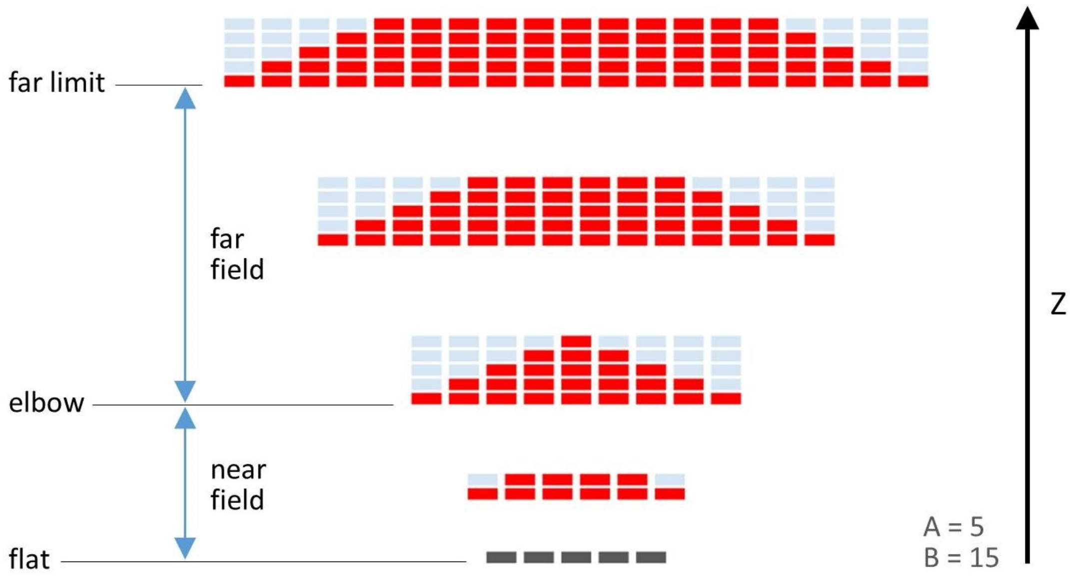


For any other plane, we need to follow the algorithm from the previous lecture. For that, we calculate the live, then tabulate the source-target grid, and then collapse the grid. That algorithm gives us the distribution. It might look something like this, which is the distribution for Z = 25. It shows us which target patches are bright, which tells us the width, and it shows the roundness at each patch.

We'll calculate some of these in a few minutes. But first, let's look at an overview of how the distribution evolves. Note that we are now showing many different planes, all together on a single slide. Each of these distributions is in a different plane, and each one is collapsed from a different source-target grid.

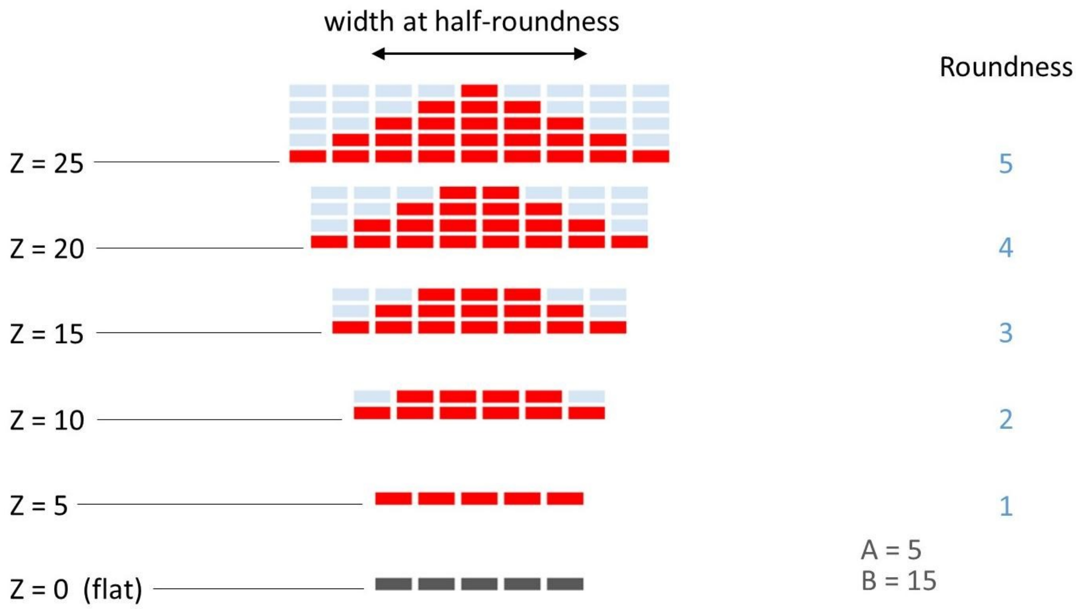
There is some potential for confusion in this type of diagram, because the vertical axis actually represents two different things. On the larger scale, the vertical axis is the direction of propagation Z, so that for example, this is the distribution at Z = 10, and this is the distribution at Z = 75. But *within* each distribution, the vertical axis shows the roundness, so for example all the rows of this distribution are in the same plane, Z = 75.

1.4 Regions of the beam



Here's the big picture: The beam starts at the flat, and goes up to the far limit. In between, there is a plane called the *elbow*. It's also known as the *Rayleigh range*. It divides the beam into two main regions: The *near field* region starts at the flat and it goes up to the elbow. The *far field* region starts at the elbow, and goes to the far limit. Now we'll look at each of these regions more carefully.

1.5 Distribution in the near field

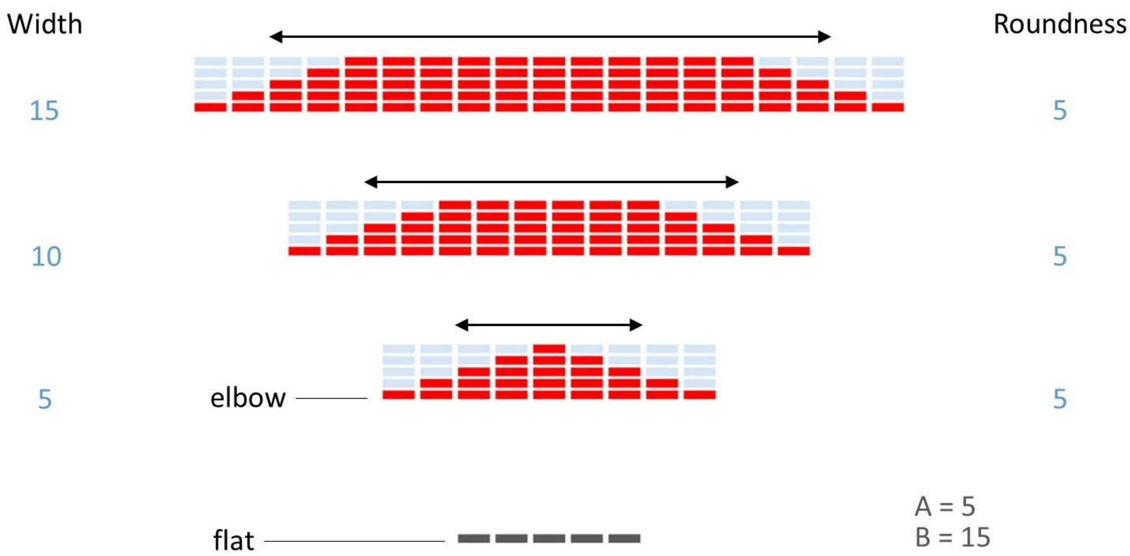


This shows the near field. The trend is this region is that the roundness increases, proportionally to the distance from the flat. So at 10, the roundness is 2, but at 20, which is twice is far, the roundness is 4.

On the other hand, the width increases only by a small amount. In fact, if you measure the width at the point of half the maximum roundness, it essentially doesn't increase at all.

This is what most people imagine when they think of a laser beam: it propagates through space, but the width stays more-or-less constant. In that typical beam the elbow is very far away from the flat, so that all you notice is the near field.

1.6 Distribution in the far field



Now let's look at the far field. In this region, the trends are the exact opposite of the trends in the near field. In other words, the roundness stays the same; in this case, it's 5 from the elbow onwards. But meanwhile, the width increases, and it's roughly proportional to Z, so the shape is like a cone.

The elbow is an abrupt turning point between the near-field trend and the far-field trend, which is why it's called the elbow.

1.7 Table of critical planes

far field

near field

Plane	Z
far limit	AB *or* $A^2 \cdot E$
elbow	A^2
flat	0

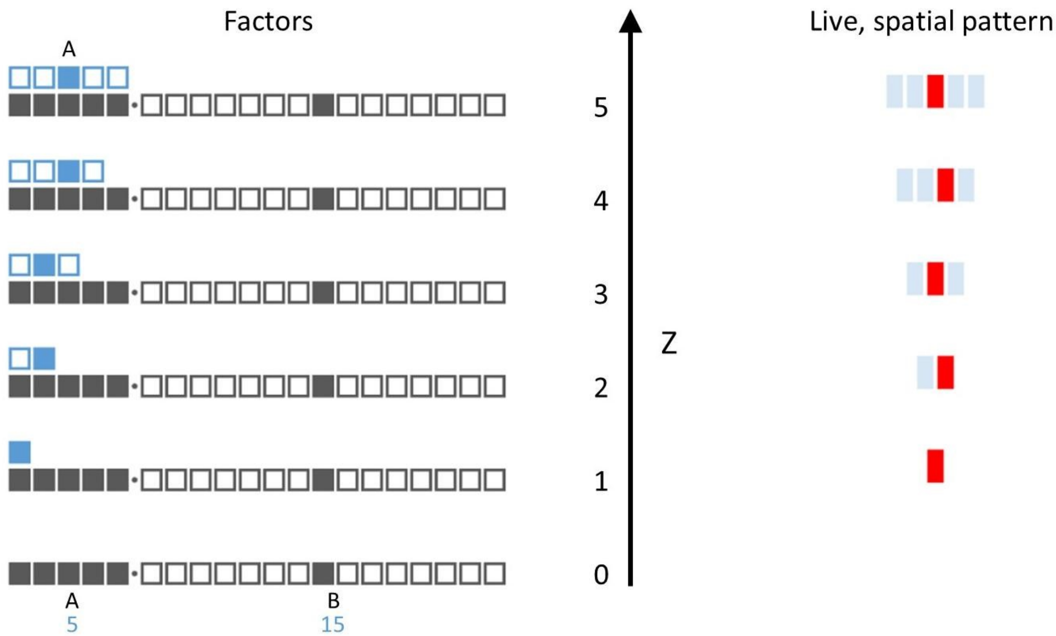
The elbow is an example of what we call a *critical plane*. These are especially important in many-slit interference, which we discuss in the next lecture. The critical planes delineate important transitions, such as between the near field and the far field. Their locations are determined by the factor sizes, such as A and B.

The critical planes of the beam are listed in this table. The first is the flat, which is located at Z = 0. The next is the elbow, which is located at A^2. The last is the far limit, which is located at AB. Alternatively, we can express it as $A^2 \cdot E$, where E is simply the ratio B/A. Most planes are not critical planes, and instead they lie somewhere in between two critical planes.

Up to now, we've been looking at distributions, without showing how they were calculated. Next, we'll dive into the details of those calculations.

2 Near field

2.1 Live factor A

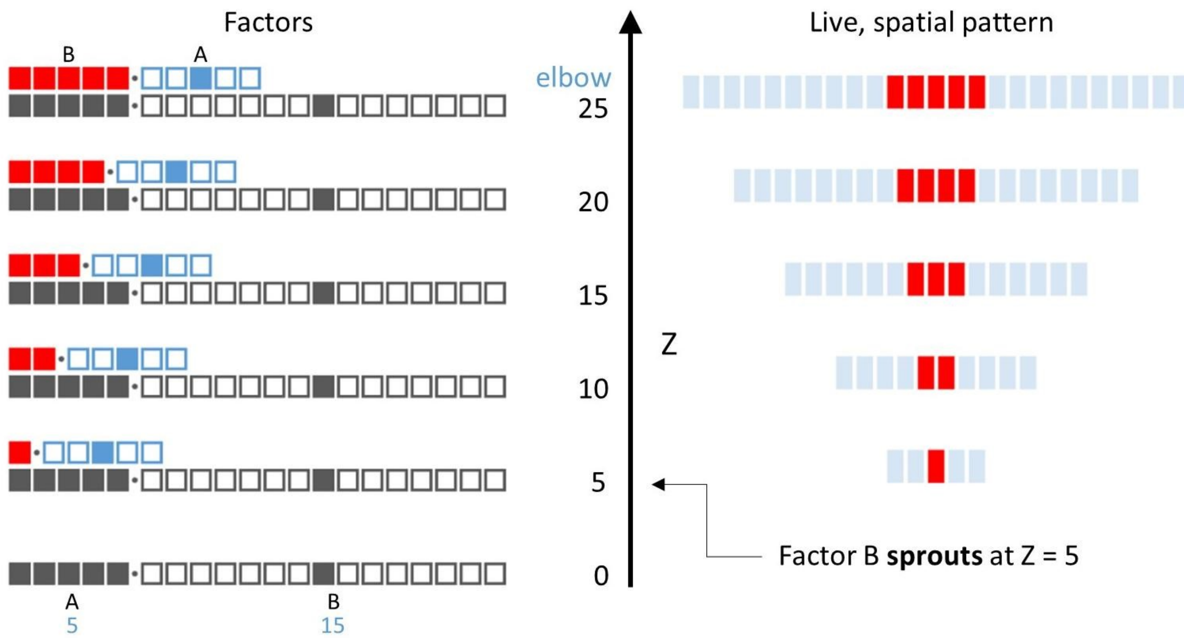


We'll start at the beginning of the near field, from 0 to 5. The left side of the slide shows the static and live factor diagrams in a series of different planes. The right side shows the live only, not the static. It also shows the live *not* as a factor diagram, but rather as a spatial pattern.

In this very first region, the live consists only of singular factor A. In this region – but only this region – the live spatial pattern looks just like the live factor chain. So for instance, let's look at Z = 1, a single wavelength from the flat. Here, the live consists of a singular factor, size 1. In spatial terms, this corresponds to one bright patch. As the singular factor grows to sizes 2, 3, 4, and 5, we see that in spatial terms, the live grows by adding dark patches without adding any bright patches.

At Z = 5, live factor A is now fully-formed at size 5, and it can not grow any further. Instead, a transition occurs in which live factor A stops growing, and live factor B begins growing.

2.2 Live factor B



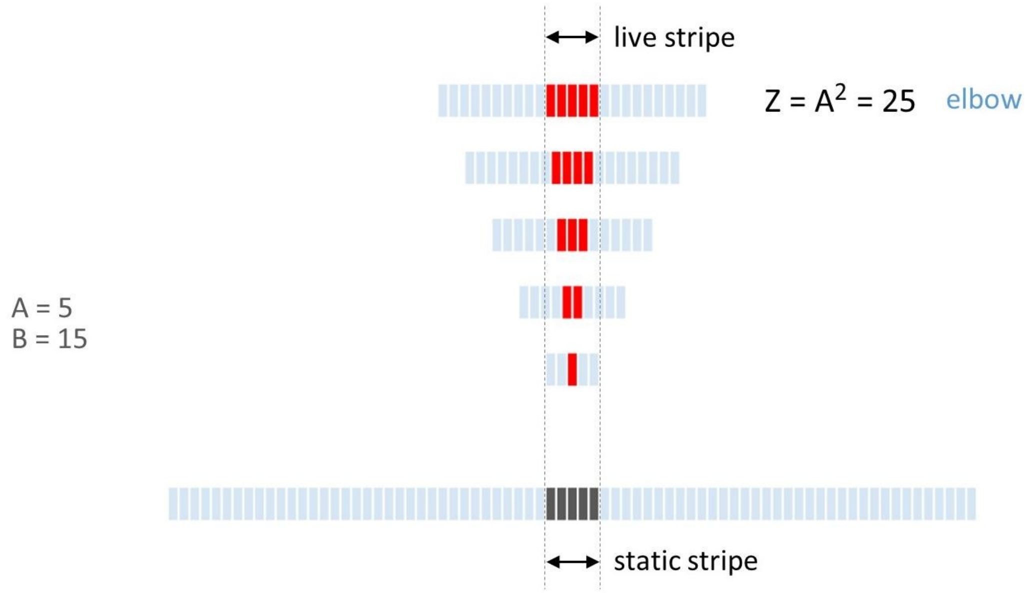
This slide shows the next stage, which picks up at 5, and goes up to the elbow at 25. Factor A, in blue, doesn't change at all. Instead, factor B grows in linear proportion to Z. But, the constant of proportionality is different from before. In the previous slide, each new point of factor A corresponded to a step of 1 wavelength, so the planes were at 0, 1, 2, 3, 4, and 5.

But in this slide, each new point of factor B corresponds to a step of 5 wavelengths, so the planes are at 0, 5, 10, 15, 20, and 25. The transition between these two different stages occurs at Z = 5. We say that factor B *sprouts* in this plane. A sprouting factor is exactly size 1, so it's just on the threshold of coming into being.

In later planes, the stripe is many patches wide, corresponding to the size of plenary factor B. But at the sprouting plane, the stripe is only 1 patch wide and so factor B is trivial – it's almost as if it weren't even there, and only factor A even existed. That's why the previous slide didn't even draw factor B, even though it was the exact same plane and the exact same pattern.

Whenever one factor reaches its full size, the next factor sprouts. Also, the distance at which one factor reaches its full size (5 in this case) is also the distance for *each* additional step of the next factor.

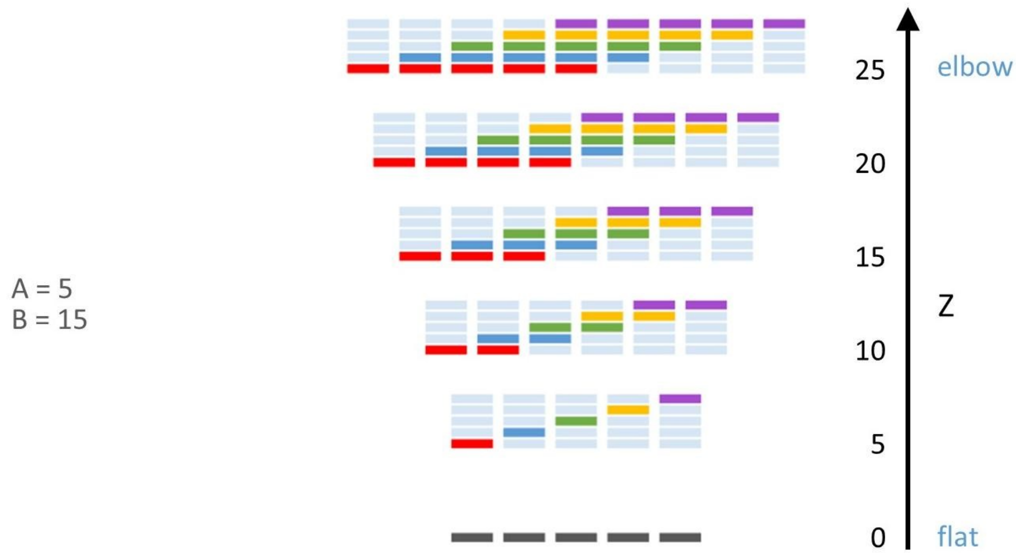
2.3 Live, spatial



The elbow occurs at Z = 25. We can best understand it in spatial terms. This slide shows the static, and it shows the live at many different planes. Note that the live stripe expands linearly with distance, like a cone.

The elbow occurs when the live stripe grows to be as large as the static stripe. Actually, critical planes in general occur when some feature of the live grows to be as wide as some feature of the static.

2.4 Source-target grids



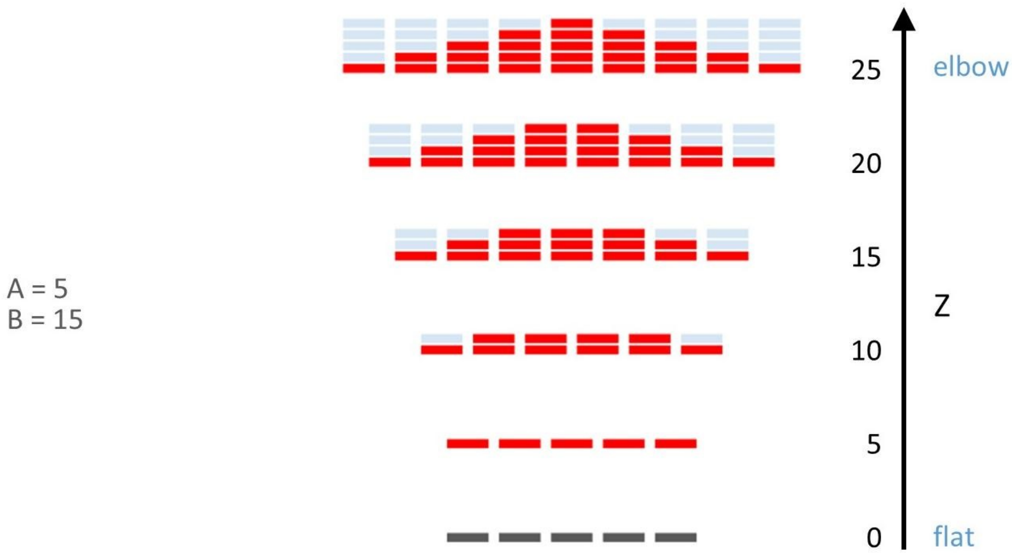
Now, let's tabulate the live into source-target grids. Remember again that we are looking at a whole series of planes, and each of these blocks is an entire source-target grid.

Each source patch emits an instance of live. These red patches are the live emitted by the first source patch. These purple patches are the live emitted by the last source patch, etc.

This picture gives us an intuitive feel for how roundness is created. As Z increases, each instance of live reaches more and more target patches, while the number of target patches only increases by a small amount. The result is that each target patch receives light from more and more source patches, or in other words, the roundness increases.

But at the elbow, the central target patch receives light from all of the available source patches. So, the maximum roundness is 5 (or size A), and it's impossible for roundness to increase any further.

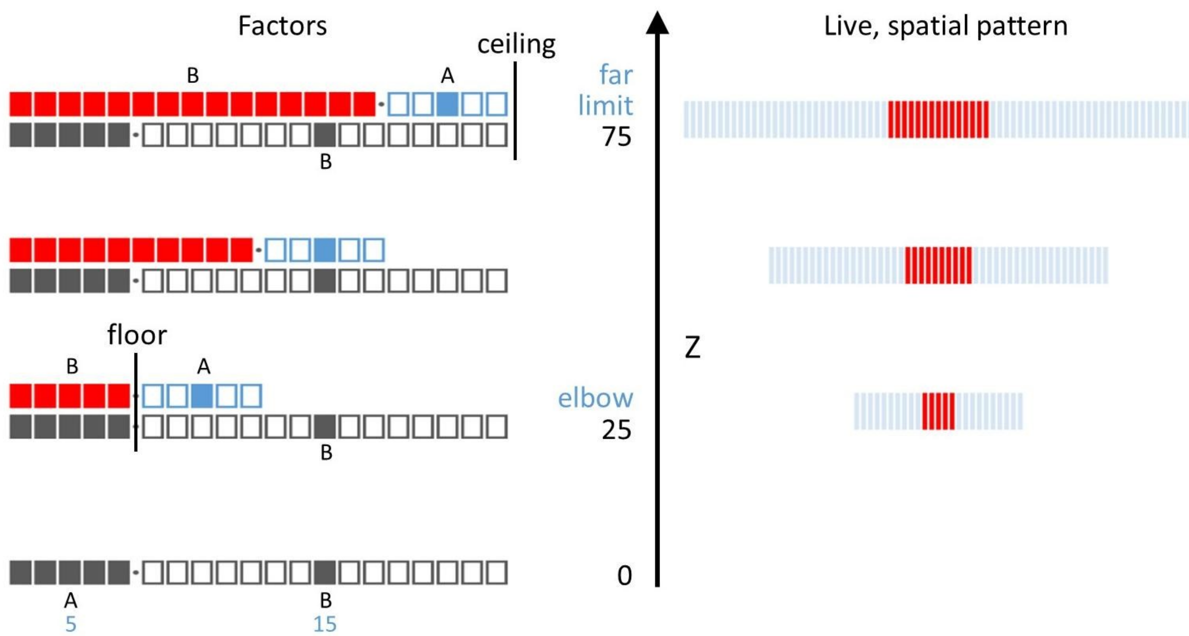
2.5 Distributions



When we collapse each grid, we get the distribution that we saw earlier in the overview. This completes our analysis of the near field. We'll now move on to the far field.

3 Far field

3.1 Factors



The elbow was the *final* plane of the *near* field, but it is also the very *first* plane of the *far* field. The trend from the previous stage continues, and factor B continues to grow by 1 point for each step of 5 wavelengths.

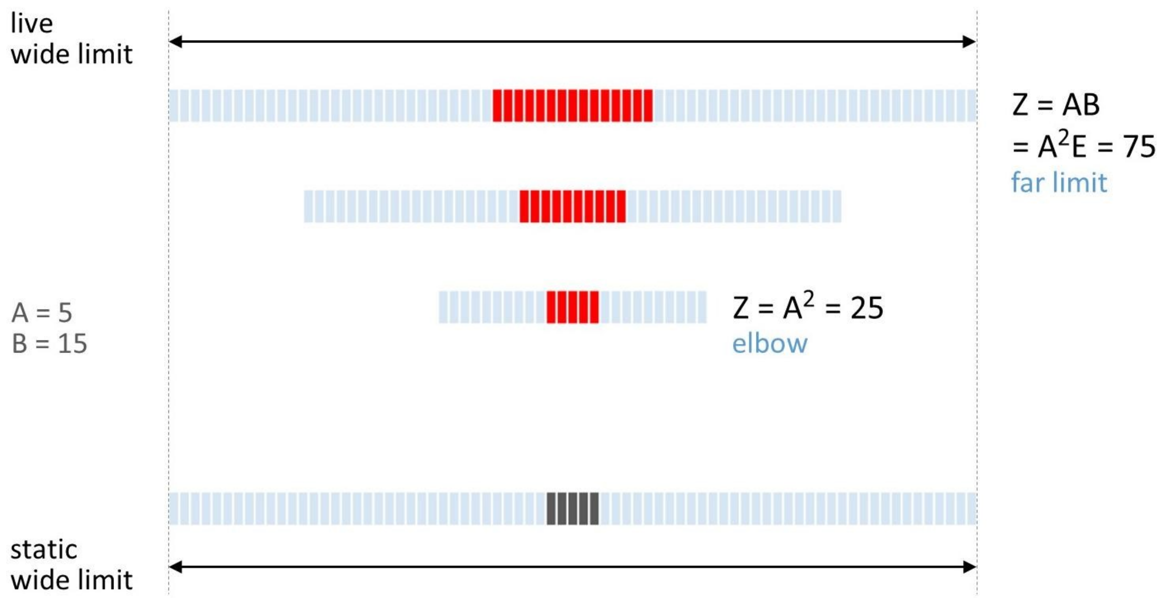
Because of space limitations, this slide doesn't show each of these steps, but in principle we could draw factor B at sizes 5, 6, 7, etc., and these would be at Z = 25, 30, 35, etc. This trend continues up until the far limit, which is the final critical plane of the beam. This occurs at Z = AB, which in this case is 75.

Here's a new term: When a static factor and a live factor are both ranked immediately *above* subchains of the same size, we say that these two factors share a *floor*. The floor is illustrated by this line. In other words, at the elbow, each point of live factor A is corresponds to 5 patches, and each point of static factor B also corresponds to 5 patches.

Two factors can also share a *ceiling*. At the far limit, live factor A shares a ceiling with static factor B. This means that the live sub-chain which *includes* factor A, is the same size as the static subchain which *includes* factor B.

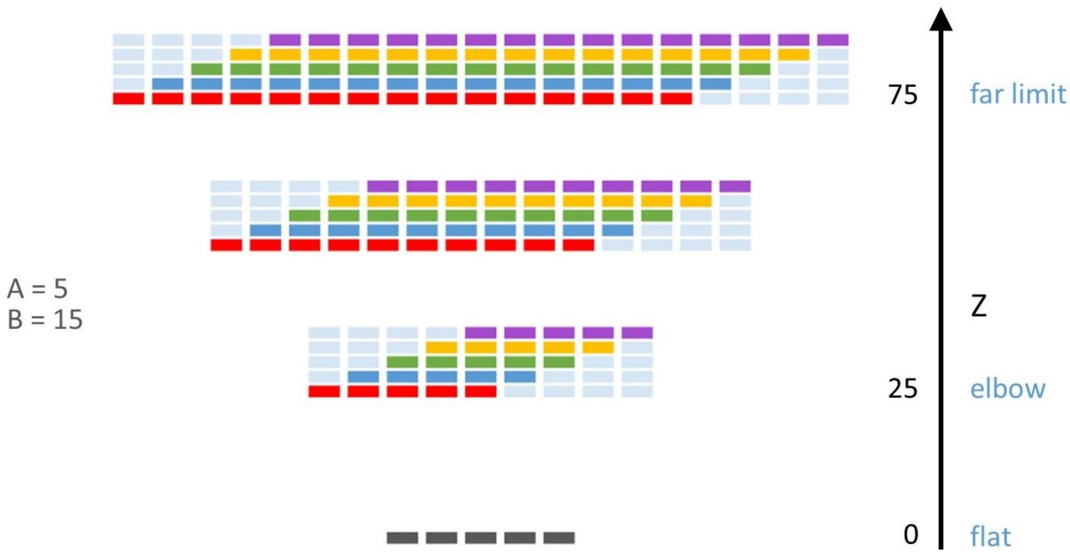
Note that the metaphor is pretty apt, in that one factor's floor is usually another factor's ceiling.

3.2 Live, spatial



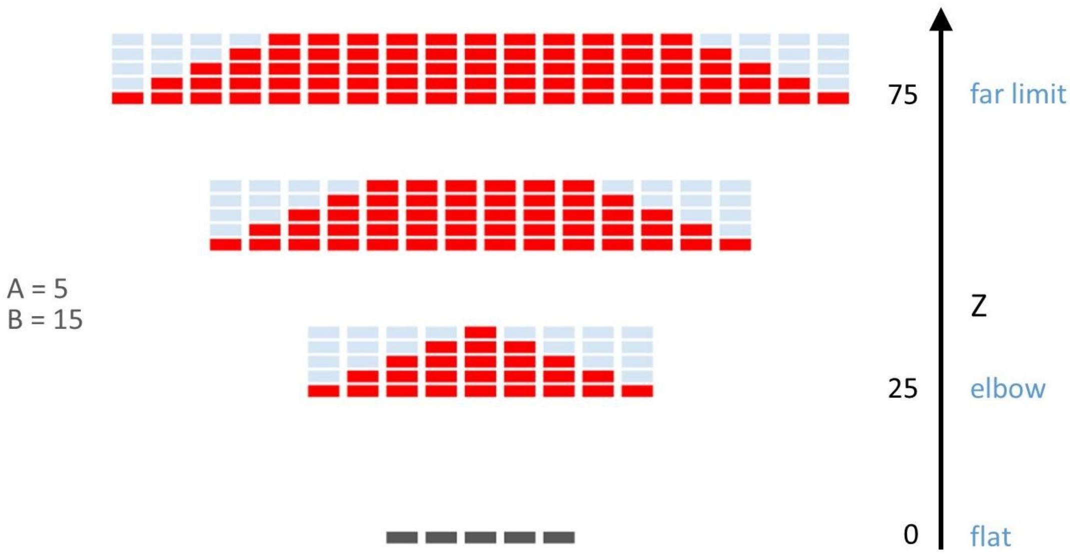
Continuing, we can view the far limit in spatial terms. As before, the critical plane occurs when two features are the same size. Here we see that at the far limit, the live wide limit is as wide as the static wide limit.

3.3 Source-target grids



When we tabulate the source-target grids, we can confirm that the roundness never grows beyond the plateau that it reaches at the elbow. The whole central portion of the beam is receiving light from every available source patch, so no further roundness is possible. On the other hand, the width has no such limitations, so it increases further and further until the model itself ends, at the far limit.

3.4 Distributions



We collapse to the distribution, and this is our answer. This completes our analysis of the far field.

4 Conclusions

4.1 Companion code

You can download the companion code for this lecture to experiment with more examples on your own. Let's look at the file *STG_manyZ_Beam.m*. This code just reuses the components from the previous lecture, so the fundamental functionality is the same. The only difference is that it plots many planes all at one time.

There are 3 different stages that you can draw, and they're defined by these code regions here. Comment out one, uncomment another, and you can draw a different stage.

Also, you can set *collapseToDistribution* to true, and it draws the distribution instead of the Source-Target Grid

Let's also see the file *factorsManyZ_Beam.m*. This is like the previous file, except that it draws factor diagrams instead of spatial ones.

There are also some other options. For example, if you set *drawCommonFloors* to true, then it draws any floor or ceiling that is shared by the chains. By the way, note that a floor can be a straight vertical line like here, but it may also be a shaped like a zig-zag. Fundamentally, this is because different factors combine multiplicatively, not additively. In a later lecture, we'll start using another type of factor diagram, where the sizes are represented logarithmically. Then, this particular quirk will disappear, and all the horizontal axis will become a 'true' representation of rank, so no more zig-zags.

One more option is to *splitLiveAtStaticFloors*. When this is true, it breaks factors up into sub-factors, so that there's always a common floor for every static factor. This will be very useful later, when we study many-slit interference.

4.2 Reviewing key points

That's all for this lecture, so let's review the key points:

- In the near field, the beam gets rounder and rounder, but stays at almost-constant width.
- At the elbow, the roundness stops increasing because all the available source patches are already contributing.
- In the far field, the beam gets wider and wider.
- Whenever one factor gets fully-formed, the next factor sprouts. The next factor grows in larger steps; every step is as large as the sprouting distance.
- At the elbow, the live stripe is as wide as the static stripe.
- At the far limit, the live wide limit is as wide as the static wide limit.

4.3 Outro

Wherever you found this video, you'll also find links to more resources for learning symmetry optics.

The lecture notes is a written version of the video.

There's also a problem set. Try it, because when you have to think the problem through on your own, you learn a lot more than when you just watch me solve it.

The next lecture will be about the critical planes of many-slit interference. You'll see that symmetry optics will reveal beautiful patterns and insights that you can't get any other way. So please join me.

I'm Paul Mirsky; thanks for listening.