Cosmological Special and General Theory of Relativity and Quantum Physics

Sangwha-Yi

Department of Math , Taejon University 300-716, South Korea

ABSTRACT

According to Newton and Einstein, space is static if the space is full of gravity. But today it is a known fact that universe's space is expanding. In other words, empty space without gravity expands.

Cosmological Special Relativity Theory is special relativity theory that treats in non-gravity empty space. We can treat quantum mechanics and electromagnetism of early universe that is not yet star gravity, for example, Cosmic Microwave Background (CMB). Of course, Cosmological Special Relativity Theory should be dealt with in the time that early universe had a flat space after the epoch of inflation.

In the Cosmological Special Relativity Theory, we study Maxwell equations, electromagnetic wave equation and function. In the Cosmological Special Theory of Relativity, we study energy-momentum relations, Klein-Gordon equation and wave function. We study Yukawa potential dependent about time in cosmological inertial frame. If we solve Klein-Gordon equation, we obtain Yukawa potential dependent about time in cosmological inertial frame. Schrodinger equation is a wave equation. Wave function uses as a probability amplitude in quantum mechanics. We make Schrodinger equation from Klein-Gordon free particle's wave function in cosmological special theory of relativity. Dirac equation is a one order-wave equation. Wave function uses as a probability amplitude in quantum mechanics. We make Dirac Equation from wave function, Type A in cosmological inertial frame. The Dirac equation satisfy Klein-Gordon equation in cosmological inertial frame. We found equations of complex scalar fields and electromagnetic fields on interaction of complex scalar fields and electromagnetic fields in Klein-Gordon-Maxwell theory from Type A of wave function and Type B of expanded distance in cosmological inertial frame. In the Cosmological Special Theory of Relativity, we quantized Klein-Gordon scalar field. We treat Lagrangian density and Hamiltonian in quantized Klein-Gordon scalar field.

PACS Number:**03.30, 41.20,03.65,04,04.90.+e**

Key words:Cosmological special relativity theory; Klein-Gordon equation;Yukawa potential; Schrodinger equation;Dirac equation;Klein-Gordon-Maxwell Theory;

Klein-Gordon scalar field;Quantization; Cosmological General Theory of Relativity; Newtonian Gravity; Schwarzschild solution;Kerr-Newman solution; Robertson-Walker solution

e-mail address:sangwha1**@nate.com**

Tel:010-2496-3953

1. Introduction

Cosmological Special Relativity Theory is special relativity theory that treats in non- gravity empty space. We can treat quantum mechanics and electromagnetism of early universe that is not yet star gravity, for example, Cosmic Microwave Background (CMB). Of course, Cosmological Special Relativity Theory should be dealt with in the time that early universe had a flat space after the epoch of inflation

Our article's aim is that we make cosmological special theory of relativity and apply it to quantum mechanics.

At first, Robertson-Walker metric is

$$d\tau^2 = dt^2 - \frac{1}{c^2}\Omega^2(t)\,[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2] \tag{1}$$

According to ΛCDM model, our universe's k is zero. In this time, if t_0 is cosmological time[6],

$$k = 0, t = t_0 >> \Delta t\,,\ \ \Delta t \text{ is period of matter's motion} \tag{2}$$

Hence, the proper time is in cosmological time,

$$\begin{aligned} d\tau^2 &= dt^2 - \frac{1}{c^2}\Omega^2(t_0)\,[dr^2 + r^2 d\Omega^2] \\ &= dt^2 - \frac{1}{c^2}\Omega^2(t_0)\,[dx^2 + dy^2 + dz^2] \\ &= dt^2(1 - \frac{1}{c^2}\Omega^2(t_0)V^2)\,, \quad V^2 = \frac{dx^2 + dy^2 + dz^2}{dt^2} \end{aligned} \tag{3}$$

In this time,

$$d\bar{t} = dt, d\bar{x} = \Omega(t_0)dx, d\bar{y} = \Omega(t_0)dy, d\bar{z} = \Omega(t_0)dz \tag{4}$$

Cosmological special theory of relativity's coordinate transformations are

$$c\bar{t} = ct = \gamma(c\bar{t}\,' + \frac{v_0}{c}\Omega(t_0)\bar{x}\,') = \gamma(ct + \frac{v_0}{c}\Omega(t_0)x\,'\Omega(t_0))$$

$$\bar{x} = x\Omega(t_0) = \gamma(\bar{x}\,' + v_0\Omega(t_0)\bar{t}\,') = \gamma(\Omega(t_0)x' + v_0\Omega(t_0)t\,')$$

$$\begin{aligned}\bar{y} &= \Omega(t_0)y = \bar{y}' = \Omega(t_0)y', \\ \bar{z} &= \Omega(t_0)z = \bar{z}' = \Omega(t_0)z'\end{aligned} \quad , \ \gamma = 1/\sqrt{1-\frac{v_0^{\ 2}}{c^2}\Omega^2(t_0)} \tag{5}$$

Therefore, proper time is

$$\begin{aligned} d\tau^2 &= d\bar{t}^{\,2} - \frac{1}{c^2}[d\bar{x}^2 + d\bar{y}^2 + d\bar{z}^2] \\ &= dt^2 - \frac{1}{c^2}\Omega^2(t_0)\,[dx^2 + dy^2 + dz^2] \\ &= dt'^2 - \frac{1}{c^2}\Omega^2(t_0)\,[dx'^2 + dy'^2 + dz'^2] \\ &= d\bar{t}'^2 - \frac{1}{c^2}[d\bar{x}'^2 + d\bar{y}'^2 + d\bar{z}'^2] \end{aligned} \tag{6}$$

Hence, velocities are

$$\begin{aligned} \frac{dx}{dt} &= V_x = \frac{V_x' + v_0}{1 + \frac{\Omega^2(t_0)}{c^2}V_x' \cdot v_0}, V_x' = \frac{dx'}{dt'} \\ \frac{dy}{dt} &= V_y = \frac{V_y'}{\gamma(1 + \frac{\Omega^2(t_0)}{c^2}V_x' \cdot v_0)}, V_y' = \frac{dy'}{dt'} \\ \frac{dz}{dt} &= V_z = \frac{V_z'}{\gamma(1 + \frac{\Omega^2(t_0)}{c^2}V_x' \cdot v_0)}, V_z' = \frac{dz'}{dt'} \end{aligned} \tag{7}$$

In cosmological special theory of relativity(CSTR)'s differential operators are

$$\begin{aligned} \frac{1}{c}\frac{\partial}{\partial \bar{t}} = \frac{1}{c}\frac{\partial}{\partial t} &= \gamma\left(\frac{1}{c}\frac{\partial}{\partial \bar{t}'} - \frac{v_0}{c}\Omega(t_0)\frac{\partial}{\partial \bar{x}'}\right) \\ &= \gamma\left(\frac{1}{c}\frac{\partial}{\partial t'} - \frac{v_0}{c}\frac{\partial}{\partial x'}\right) \end{aligned}$$

$$\begin{aligned} \frac{\partial}{\partial \bar{x}} = \frac{\partial}{\partial x}\frac{1}{\Omega(t_0)} &= \gamma\left(\frac{\partial}{\partial \bar{x}'} - \frac{v_0}{c}\Omega(t_0)\frac{1}{c}\frac{\partial}{\partial \bar{t}'}\right) \\ &= \gamma\left(\frac{\partial}{\partial x'}\frac{1}{\Omega(t_0)} - \frac{v_0}{c}\Omega(t_0)\frac{1}{c}\frac{\partial}{\partial t'}\right) \end{aligned}$$

$$\frac{\partial}{\partial \bar{y}} = \frac{\partial}{\partial y}\frac{1}{\Omega(t_0)} = \frac{\partial}{\partial \bar{y}'} = \frac{\partial}{\partial y'}\frac{1}{\Omega(t_0)}$$

$$\frac{\partial}{\partial \bar{z}} = \frac{\partial}{\partial z}\frac{1}{\Omega(t_0)} = \frac{\partial}{\partial \bar{z}'} = \frac{\partial}{\partial z'}\frac{1}{\Omega(t_0)}, \gamma = 1/\sqrt{1-\frac{v_0^{\ 2}}{c^2}\Omega^2(t_0)} \tag{8}$$

Hence,

$$\frac{1}{c^2}\frac{\partial^2}{\partial \bar{t}^2} - \bar{\nabla}^2 = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{1}{\Omega^2(t_0)}\{(\frac{\partial}{\partial x})^2 + (\frac{\partial}{\partial y})^2 + (\frac{\partial}{\partial z})^2\}$$

$$= \frac{1}{c^2}\frac{\partial^2}{\partial t'^2} - \frac{1}{\Omega^2(t_0)}\{(\frac{\partial}{\partial x'})^2 + (\frac{\partial}{\partial y'})^2 + (\frac{\partial}{\partial z'})^2\} \tag{9}$$

The electric charge density ρ and the electric current density $\vec{j}$ are

$$j^\mu = \rho_0 \frac{dx^\mu}{d\tau}, j^0 = c\rho = c\gamma\rho_0, j^i = \vec{j} = \rho\vec{u}, \ i = 1,2,3 \tag{10}$$

In CSTR, transformations of the electric charge density and the electric current density are likely as coordinate transformations are

$$c\bar{\rho} = c\rho = \gamma(c\bar{\rho}' + \frac{v_0}{c}\Omega(t_0)\bar{j}_x{}') = \gamma(c\rho' + \frac{v_0}{c}\Omega(t_0)j_x{}'\Omega(t_0))$$

$$\bar{j}_x = j_x\Omega(t_0) = \gamma(\bar{j}_x{}' + v_0\Omega(t_0)\bar{\rho}') = \gamma(\Omega(t_0)j_x{}' + v_0\Omega(t_0)\rho')$$

$$\begin{matrix} \bar{j}_y = \Omega(t_0)j_y = \bar{j}_y{}' = \Omega(t_0)j_y{}', \\ \bar{j}_z = \Omega(t_0)j_z = \bar{j}_z{}' = \Omega(t_0)j_z{}' \end{matrix}, \ \gamma = 1/\sqrt{1-\frac{v_0^{\ 2}}{c^2}\Omega^2(t_0)} \tag{11}$$

2. Electrodynamics in CSTR

The electromagnetic potential A^μ is 4-vector potential.Hence, transformations of A^μ are

$$\bar{\phi} = \phi = \gamma(\bar{\phi}' + \frac{v_0}{c}\Omega(t_0)\bar{A}_x{}') = \gamma(\phi' + \frac{v_0}{c}\Omega(t_0)A_x{}'\Omega(t_0))$$

$$\bar{A}_x = A_x\Omega(t_0) = \gamma(\bar{A}_x{}' + \frac{v_0}{c}\Omega(t_0)\bar{\phi}') = \gamma(\Omega(t_0)A_x{}' + \frac{v_0}{c}\Omega(t_0)\phi')$$

$$\begin{matrix} \bar{A}_y = \Omega(t_0)A_y = \bar{A}_y{}' = \Omega(t_0)A_y{}', \\ \bar{A}_z = \Omega(t_0)A_z = \bar{A}_z{}' = \Omega(t_0)A_z{}' \end{matrix}, \ \gamma = 1/\sqrt{1-\frac{v_0^{\ 2}}{c^2}\Omega^2(t_0)} \tag{12}$$

In CSTR, electric field $\vec{\bar{E}}$ and magnetic field $\vec{\bar{B}}$ have to satisfy Maxwell equations of special relativity theory. Hence, in CSRT, Maxwell equations are likely as special theory of relativity,

$$\bar{\vec{\nabla}} \cdot \bar{\vec{E}} = 4\pi\bar{\rho} \tag{13-i}$$

$$\bar{\vec{\nabla}} \cdot \bar{\vec{B}} = 0 \tag{13-ii}$$

$$\bar{\vec{\nabla}} \times \bar{\vec{E}} = -\frac{1}{c}\frac{\partial \bar{\vec{B}}}{\partial \bar{t}} \tag{13-iii}$$

$$\bar{\vec{\nabla}} \times \bar{\vec{B}} = \frac{1}{c}\frac{\partial \bar{\vec{E}}}{\partial \bar{t}} + \frac{4\pi}{c}\bar{\vec{j}} \tag{13-iv}$$

In this time, Eq(13-i) is

$$\bar{\vec{\nabla}} \cdot \bar{\vec{E}} = \frac{1}{\Omega(t_0)}\vec{\nabla} \cdot \bar{\vec{E}} = 4\pi\bar{\rho} = 4\pi\rho \tag{14}$$

Hence, $\bar{\vec{E}} = \vec{E}\Omega(t_0)$ .According to special relativity, $\bar{\vec{B}} = \vec{B}\Omega(t_0)$

Eq(13-ii) is

$$\bar{\vec{\nabla}} \cdot \bar{\vec{B}} = \frac{1}{\Omega(t_0)}\vec{\nabla} \cdot \vec{B}\Omega(t_0) = \vec{\nabla} \cdot \vec{B} = 0 \tag{15}$$

Eq(13-iii) is

$$\bar{\vec{\nabla}} \times \bar{\vec{E}} = \frac{1}{\Omega(t_0)}\vec{\nabla} \times \vec{E}\Omega(t_0) = \vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \bar{\vec{B}}}{\partial \bar{t}} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}\Omega(t_0) \tag{16}$$

Eq(13-iv) is

$$\begin{aligned} \bar{\vec{\nabla}} \times \bar{\vec{B}} &= \frac{1}{\Omega(t_0)}\vec{\nabla} \times \vec{B}\Omega(t_0) = \vec{\nabla} \times \vec{B} \\ &= \frac{1}{c}\frac{\partial \bar{\vec{E}}}{\partial \bar{t}} + \frac{4\pi}{c}\bar{\vec{j}} = \Omega(t_0)\left(\frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{j}\right) \end{aligned} \tag{17}$$

Hence, in CSTR, Maxwell equations are

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho \tag{18-i}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \tag{18-ii}$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}\Omega(t_0) \tag{18-iii}$$

$$\vec{\nabla} \times \vec{B} = \Omega(t_0)\left(\frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{j}\right) \tag{18-iv}$$

Therefore, in CSTR, the electric field $\vec{E}$ and the magnetic field $\vec{B}$ are

$$\vec{\bar{E}} = \vec{E}\Omega(t_0) = \Omega(t_0)(-\vec{\nabla}\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t})$$
$$= -\Omega(t_0)\vec{\nabla}\phi - \Omega(t_0)\frac{1}{c}\frac{\partial \vec{A}}{\partial t} = -\vec{\bar{\nabla}}(\bar{\phi}\Omega^2(t_0)) - \frac{1}{c}\frac{\partial \vec{\bar{A}}}{\partial \bar{t}} \tag{19}$$

$$\vec{\bar{B}} = \vec{B}\Omega(t_0) = \Omega(t_0)\vec{\nabla}\times\vec{A} = \Omega(t_0)\vec{\bar{\nabla}}\times\vec{\bar{A}} \tag{20}$$

3. Electromagnetic Wave in CSTR

Electromagnetic wave equation is in CSTR,

$$\frac{1}{c}\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{E}) = -\Omega(t_0)\frac{1}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2}$$
$$= \vec{\nabla}\times(\frac{1}{c}\frac{\partial \vec{E}}{\partial t}) = \vec{\nabla}\times(\frac{1}{\Omega(t_0)}\vec{\nabla}\times\vec{B}), \vec{\nabla}\times\vec{j} = \vec{0}$$
$$= \frac{1}{\Omega(t_0)}\{-\nabla^2\vec{B} + \vec{\nabla}(\vec{\nabla}\cdot\vec{B})\} = -\frac{1}{\Omega(t_0)}\nabla^2\vec{B} \tag{21}$$

Hence, electromagnetic wave equation is

$$\Omega(t_0)\frac{1}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2} - \frac{1}{\Omega(t_0)}\nabla^2\vec{B} = \vec{0} \tag{22}$$

And,

$$\frac{1}{c}\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B}) = \Omega(t_0)\frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}, \frac{1}{c}\frac{\partial \vec{j}}{\partial t} = \vec{0}$$
$$= \vec{\nabla}\times(\frac{1}{c}\frac{\partial \vec{B}}{\partial t}) = \vec{\nabla}\times(-\frac{1}{\Omega(t_0)}\vec{\nabla}\times\vec{E})$$
$$= -\frac{1}{\Omega(t_0)}\{-\nabla^2\vec{E} + \vec{\nabla}(\vec{\nabla}\cdot\vec{E})\} = \frac{1}{\Omega(t_0)}\nabla^2\vec{E}, \vec{\nabla}(4\pi\rho) = \vec{0} \tag{23}$$

Hence, electromagnetic wave equation is

$$\Omega(t_0)\frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} - \frac{1}{\Omega(t_0)}\nabla^2\vec{E} = \vec{0} \tag{24}$$

In CSTR, electromagnetic wave functions are

$$\vec{E} = \vec{E}_0 \sin\Phi, \vec{B} = \vec{B}_0 \sin\Phi$$

$$\Phi = \omega\{\frac{t}{\sqrt{\Omega(t_0)}} - \frac{\sqrt{\Omega(t_0)}}{c}(lx + my + nz)\} \tag{25}$$

Where,

$$l^2 + m^2 + n^2 = 1 \quad (26)$$

According to Maxwell equations are in CSTR,[1]

$$\Omega(t_0)\{\frac{1}{c}\frac{\partial E_x}{\partial t} + \frac{4\pi}{c} j_x\} = (\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}), \qquad \Omega(t_0)\frac{1}{c}\frac{\partial B_x}{\partial t} = (\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y})$$

$$\Omega(t_0)\{\frac{1}{c}\frac{\partial E_y}{\partial t} + \frac{4\pi}{c} j_y\} = (\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}), \qquad \Omega(t_0)\frac{1}{c}\frac{\partial B_y}{\partial t} = (\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z})$$

$$\Omega(t_0)\{\frac{1}{c}\frac{\partial E_z}{\partial t} + \frac{4\pi}{c} j_z) = (\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}), \qquad \Omega(t_0)\frac{1}{c}\frac{\partial B_z}{\partial t} = (\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}) \quad (27)$$

Where,

$$\Omega(t_0)\{\frac{1}{c}\frac{\partial E_x'}{\partial t} + \frac{4\pi}{c} j_x\} = \{\frac{\partial}{\partial y}\gamma(B_z' + \frac{v_0}{c}\Omega(t_0)E_y') - \frac{\partial}{\partial z}\gamma(B_y' - \frac{v_0}{c}\Omega(t_0)E_z')\}$$

$$\Omega(t_0)\{\frac{1}{c}\frac{\partial}{\partial t}\gamma(E_y' + \frac{v_0}{c}\Omega(t_0)B_z') + \frac{4\pi}{c} j_y\} = \{\frac{\partial B_x'}{\partial z} - \frac{\partial}{\partial x}\gamma(B_z' + \frac{v_0}{c}\Omega(t_0)E_y')\}$$

$$\Omega(t_0)\{\frac{1}{c}\frac{\partial}{\partial t}\gamma(E_z' - \frac{v_0}{c}\Omega(t_0)B_y') + \frac{4\pi}{c} j_z\} = \{\frac{\partial}{\partial y}\gamma(B_y' - \frac{v_0}{c}\Omega(t_0)E_z') - \frac{\partial B_x'}{\partial z}\} \quad (28)$$

Where,[1]

$$\Omega(t_0)\frac{1}{c}\frac{\partial B_x'}{\partial t} = \{\frac{\partial}{\partial z}\gamma(E_y' + \frac{v_0}{c}\Omega(t_0)B_z') - \frac{\partial}{\partial y}\gamma(E_z' - \frac{v_0}{c}\Omega(t_0)B_y')\}$$

$$\Omega(t_0)\frac{1}{c}\frac{\partial}{\partial t}\gamma(B_y' - \frac{v_0}{c}\Omega(t_0)E_z') = \{\frac{\partial}{\partial x}\gamma(E_z' - \frac{v_0}{c}\Omega(t_0)B_y') - \frac{\partial E_x'}{\partial z}\}$$

$$\Omega(t_0)\frac{1}{c}\frac{\partial}{\partial t}\gamma(B_z' + \frac{v_0}{c}\Omega(t_0)E_y') = \{\frac{\partial E_x'}{\partial y} - \frac{\partial}{\partial x}\gamma(E_y' + \frac{v_0}{c}\Omega(t_0)B_z'\} \quad (29)$$

Hence, in CSTR, transformations of electromagnetic field are

$$E_x = E_x', E_y = \gamma(E_y' + \frac{v_0}{c}\Omega(t_0)B_z'), E_z = \gamma(E_z' - \frac{v_0}{c}\Omega(t_0)B_y') \quad (30)$$

$$B_x = B_x', B_y = \gamma(B_y' - \frac{v_0}{c}\Omega(t_0)E_z'), B_z = \gamma(B_z' + \frac{v_0}{c}\Omega(t_0)E_y') \quad (31)$$

In CSTR, electromagnetic wave functions are

$$E_x' = E_{x0}\sin\Phi', E_y' = \gamma(E_{y0} - \frac{v_0}{c}\Omega(t_0)B_{z0})\sin\Phi', E_z' = \gamma(E_{z0} + \frac{v_0}{c}\Omega(t_0)B_{y0})\sin\Phi' \quad (32)$$

$$B_x' = B_{x0}\sin\Phi', B_y' = \gamma(B_{y0} + \frac{v_0}{c}\Omega(t_0)E_{z0})\sin\Phi', B_z' = \gamma(B_{z0} - \frac{v_0}{c}\Omega(t_0)E_{y0})\sin\Phi' \quad (33)$$

In this time,

$$\Phi' = \omega'\{\frac{t'}{\sqrt{\Omega(t_0)}} - \frac{\sqrt{\Omega(t_0)}}{c}(l'x' + m'y' + n'z')\} \quad (34)$$

$$\Phi = \omega\{\frac{t}{\sqrt{\Omega(t_0)}} - \frac{\sqrt{\Omega(t_0)}}{c}(lx + my + nz)\} \tag{35}$$

If we compare Eq(34) and Eq(35),

$$\omega' = \omega\gamma(1 - l\Omega(t_0)\frac{v_0}{c}), l' = \frac{l - \frac{v_0}{c}\Omega(t_0)}{1 - l\frac{v_0}{c}\Omega(t_0)}$$

$$m' = \frac{m}{\gamma(1 - l\Omega(t_0)\frac{v_0}{c})}, n' = \frac{n}{\gamma(1 - l\Omega(t_0)\frac{v_0}{c})} \tag{36}$$

Where,

$$l'^2 + m'^2 + n'^2 = 1 \tag{37}$$

4. Klein-Gordon Equation and Wave Fuction in CSTR

We make Klein-Gordon equation and wave function in cosmological special theory of relativity.

At first, space-time relations are in cosmological special theory of relativity (CSTR).

$$ct = \gamma(ct' + \frac{v_0}{c}\Omega^2(t_0)x'), \quad x\Omega(t_0) = \gamma(\Omega(t_0)x' + v_0\Omega(t_0)t')$$

$$\begin{matrix}\Omega(t_0)y = \Omega(t_0)y', \\ \Omega(t_0)z = \Omega(t_0)z'\end{matrix}, \quad \gamma = 1/\sqrt{1 - \frac{v_0^2}{c^2}\Omega^2(t_0)}, \quad t_0 \text{ is cosmological time} \tag{38}$$

Therefore, proper time is,

$$d\tau^2 = dt^2 - \frac{1}{c^2}\Omega^2(t_0)[dx^2 + dy^2 + dz^2]$$

$$= dt'^2 - \frac{1}{c^2}\Omega^2(t_0)[dx'^2 + dy'^2 + dz'^2], \quad t_0 \text{ is cosmological time} \tag{39}$$

Hence, energy-momentum relations are by the fact that energy-momentum are 4-vector in CSTR,

$$E = \gamma(E' + v_0\Omega^2(t_0)p_x') \quad p_x\Omega(t_0) = \gamma(\Omega(t_0)p_x' + \frac{v_0}{c^2}\Omega(t_0)E')$$

$$\begin{matrix}\Omega(t_0)p_y = \Omega(t_0)p_y', \\ \Omega(t_0)p_z = \Omega(t_0)p_z'\end{matrix}, \gamma = 1/\sqrt{1 - \frac{v_0^2}{c^2}\Omega^2(t_0)} \quad E = m_0c^2\frac{dt}{d\tau}, \vec{p} = m_0\frac{d\vec{x}}{d\tau} \tag{40}$$

Therefore, energy-momentum-mass relation is in CSTR,

$$m_0^2c^4 = E^2 - \Omega^2(t_0)p^2c^2 \tag{41}$$

Matter wave function is in CSTR,

$$\phi = \phi_0 \exp i\Phi = \phi_0 \exp i[\frac{\omega t}{\sqrt{\Omega(t_0)}} - \vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)}]$$

$$= \phi' = \phi_0 \exp i\Phi' = \phi_0 \exp i[\frac{\omega' t'}{\sqrt{\Omega(t_0)}} - \vec{k}'\cdot\vec{x}'\sqrt{\Omega(t_0)}]$$

ϕ_0 is amplitude, ω is angular frequency, $k = \left|\vec{k}\right|$ is wave number. (42)

If we use Eq(38) in Eq(42), we obtain angular frequency-wave number relation.

$$\omega' = \gamma(\omega - v_0\Omega(t_0)k_1),\ k_1' = \gamma(k_1 - \frac{v_0}{c^2}\Omega(t_0)\omega)$$

$$k_2' = k_2, k_3' = k_3, \gamma = 1/\sqrt{1 - \frac{v_0^{\,2}}{c^2}\Omega^2(t_0)} \tag{43}$$

In this time, if we define energy-momentum by angular frequency-wave number,

$$E = \hbar\omega, \vec{p} = \frac{\hbar\vec{k}}{\Omega(t_0)} \tag{44}$$

Hence, we obtain the angular frequency-wave number relation about the energy-momentum-mass relation in CSTR,

$$m_0^{\,2}c^4 = E^2 - \Omega^2(t_0)p^2c^2 = \hbar^2\omega^2 - \hbar^2k^2c^2 \tag{45}$$

We obtain next result by the transformation of the angular frequency-wave number relation, Eq(43) in CSTR.

$$m_0^{\,2}c^4 = \hbar^2\omega^2 - \hbar^2k^2c^2 = \hbar^2\omega'^2 - \hbar^2k'^2c^2 \tag{46}$$

If we define the differential operator about energy-momentum in CSTR,

$$E = i\hbar\frac{\partial}{\partial t}, \vec{p} = -i\hbar\frac{1}{\Omega(t_0)}\vec{\nabla} \tag{47}$$

If we apply Eq(47) to Eq(46),

$$m_0^{\,2}c^4 = E^2 - \Omega^2(t_0)p^2c^2 = \hbar^2[-(\frac{\partial}{\partial t})^2 + c^2\nabla^2]$$

We finally obtain Klein-Gordon equation in CSTR.

$$\frac{m_0^{\,2}c^2}{\hbar^2}\phi = [-\frac{1}{c^2}(\frac{\partial}{\partial t})^2 + \nabla^2]\phi \tag{48}$$

Wave function, Eq(42) satisfy Klein-Gordon equation, Eq(48).

5. Yukawa potential in Klein-Gordon equation in cosmological inertial frame

If we focus Klein-Gordon equation about Yukawa potential ϕ dependent about time,

$$\frac{m_\pi{}^2 c^2}{\hbar^2}\phi + \partial_\mu \partial^\mu \phi = \frac{m_\pi{}^2 c^2}{\hbar^2}\phi + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\phi - \nabla^2 \phi = 0 \qquad (49)$$

In this time, Yukawa potential ϕ dependent about time is.

$$\phi = -\frac{g^2}{r}\exp(-\frac{m_\pi r c}{\hbar}) + A_0 \sin \omega t$$

$$\text{Frequency } \omega = \frac{m_\pi c^2}{\hbar}, \quad m_\pi \text{ is meson's mass} \qquad (50)$$

Eq(49)-Klein-Gordon equation is satisfied by Eq(50)-Yukawa potential dependent about time

In cosmological inertial frame, Klein-Gordon equation is

$$-\Omega(t_0)\frac{1}{c^2}\frac{\partial^2 \phi'}{\partial t^2} + \frac{1}{\Omega(t_0)}\nabla^2 \phi' = \frac{m_\pi{}^2 c^2}{\hbar^2}\phi' \qquad (51)$$

In this point, in cosmological inertial frame, space-time transformations in the type A of wave function and the other type B of the expanded distance are

$$\text{Type A: } r \to r\sqrt{\Omega(t_0)}, t \to \frac{t}{\sqrt{\Omega(t_0)}}, \text{ Type B: } r \to r\Omega(t_0), t \to t \qquad (52)$$

Space-time transformation of Yukawa potential ϕ' is depend on Type A

Hence, Yukawa potential ϕ' dependent about time is

$$\phi' = -\frac{g^2}{r\sqrt{\Omega(t_0)}}\exp[-\frac{m_\pi r\sqrt{\Omega(t_0)}c}{\hbar}] + A_0 \sin(\frac{\omega t}{\sqrt{\Omega(t_0)}})$$

$$\text{Frequency } \omega = \frac{m_\pi c^2}{\hbar}, \quad m_\pi \text{ is meson's mass} \qquad (53)$$

Eq(51)-Klein-Gordon equation is satisfied by Eq(53)-the solution.

6. Schrodinger Equation from Klein-Gordon Free Particle Field in Cosmological Inertial Frame

At first, Klein-Gordon equation is for free particle field ϕ in cosmological inertial frame.

$$\frac{m^2 c^2}{\hbar^2}\phi + \Omega(t_0)\frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} - \frac{1}{\Omega(t_0)}\nabla^2 \phi = 0$$

m is free particle's mass , $\Omega(t_0)$ is the ratio of universe's expansion in cosmological time t_0 (54)

If we write wave function as solution of Klein-Gordon equation for free particle,

$$\phi = A_0 \exp[i(\frac{\omega t}{\sqrt{\Omega(t_0)}} - \vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)})]$$

A_0 is amplitude, ω is angular frequency, $k = |\vec{k}|$ is wave number (55)

Energy and momentum is in cosmological inertial frame,

$$E = \hbar\omega, \vec{p} = \hbar\vec{k} / \Omega(t_0) \tag{56}$$

Hence, energy-momentum relation is in cosmological inertial frame

$$E^2 = \hbar^2\omega^2 = \Omega^2(t_0)p^2c^2 + m^2c^4 = \hbar^2k^2c^2 + m^2c^4 \tag{57}$$

Or angular frequency- wave number relation is

$$\frac{\omega^2}{c^2} = k^2 + \frac{m^2c^2}{\hbar^2} \tag{58}$$

Hence, wave function is in cosmological inertial frame,

$$\phi = A_0 \exp[(-\frac{i}{\hbar})(\hbar\frac{\omega t}{\sqrt{\Omega(t_0)}} - \hbar\vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)})]$$

$$= A_0 \exp[(-\frac{i}{\hbar})(\frac{Et}{\sqrt{\Omega(t_0)}} - \vec{p}\cdot\vec{x}\Omega(t_0)\sqrt{\Omega(t_0)}] \tag{59}$$

Because, Schrodinger equation is made from Klein-Gordon free particle's wave function in cosmological special theory of relativity,

$$\phi = A_0 \exp[(-\frac{i}{\hbar})(\frac{Et}{\sqrt{\Omega(t_0)}} - \vec{p}\cdot\vec{x}\Omega(t_0)\sqrt{\Omega(t_0)}] \tag{60}$$

If we calculate the derivation of Schrodinger equation,

$$\sum_i (\frac{\partial}{\partial x^i})^2\phi = -\sum_i \frac{(p^i)^2}{\hbar^2}\Omega^3(t_0)\phi = -\frac{p^2}{\hbar^2}\Omega^3(t_0)\phi \tag{61}$$

$$\frac{\partial\phi}{\partial t} = -\frac{i}{\hbar}E\frac{\phi}{\sqrt{\Omega(t_0)}} \tag{62}$$

Energy E is

$$E = \frac{p^2}{2m}\Omega^2(t_0) + V, \quad V \text{ is the potential energy} \tag{63}$$

Hence,

$$E\phi = \frac{p^2}{2m}\Omega^2(t_0)\phi + V\phi \quad , \quad V \text{ is the potential energy} \tag{64}$$

Therefore, by Eq(61),Eq(62)

$$E\phi = i\hbar\frac{\partial\phi}{\partial t}\sqrt{\Omega(t_0)}\ , \ \ \Omega^2(t_0)\,p^2\phi = -\hbar^2\nabla^2\phi\frac{1}{\Omega(t_0)} \tag{65}$$

Therefore, Schrodinger equation in cosmological inertial frame,

$$E\phi = i\hbar\frac{\partial\phi}{\partial t}\sqrt{\Omega(t_0)} = -\frac{\hbar^2}{2m}\frac{1}{\Omega(t_0)}\nabla^2\phi + V\phi \tag{66}$$

If the energy E is not concerned by time t,

$$\frac{\partial E}{\partial t} = 0 \tag{67}$$

$$\begin{aligned}\phi &= A_0\exp[-(\frac{i}{\hbar})(\frac{Et}{\sqrt{\Omega(t_0)}} - \vec{p}\cdot\vec{x}\,\Omega(t_0)\sqrt{\Omega(t_0)})]\\ &= \varphi\exp[-(\frac{i}{\hbar})(\frac{Et}{\sqrt{\Omega(t_0)}})]\end{aligned} \tag{68}$$

Hence, stationary state of Schrodinger equation is in cosmological inertial frame,

$$\begin{aligned}&E\varphi\exp[-(\frac{iE}{\hbar})\frac{t}{\sqrt{\Omega(t_0)}}]\\ &= [-\frac{\hbar^2}{2m}\frac{1}{\Omega(t_0)}\nabla^2\varphi + V\varphi]\exp[-(\frac{iE}{\hbar})\frac{t}{\sqrt{\Omega(t_0)}}]\end{aligned} \tag{69}$$

Hence, stationary state of Schrodinger equation is

$$E\varphi = -\frac{\hbar^2}{2m}\frac{1}{\Omega(t_0)}\nabla^2\varphi + V\varphi \tag{70}$$

Or,

$$\frac{1}{\Omega(t_0)}\nabla^2\varphi + \frac{2m}{\hbar^2}(E - V)\varphi = 0 \tag{71}$$

7. Dirac Equation from Wave Function-Type A in Cosmological Inertial Frame

Dirac equation is in special relativity theory,

$$(i\hbar\gamma^\mu\partial_\mu - mcI)\psi = 0 \quad ,$$

$$I \text{ is } 4\times4 \text{ unit matrix ,}$$

$$\gamma^0 = \begin{pmatrix} I' & 0 \\ 0 & -I' \end{pmatrix},\ \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix},\ \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$I' \text{ is } 2\times2 \text{ unit matrix, } \sigma^i \text{ is Pauli's matrix.} \tag{72}$$

Dirac equation is the wave equation. Therefore, Dirac equation is in cosmological inertial frame,

Wave function Type A:

$$r \to r\sqrt{\Omega(t_0)} \quad , \quad t \to \frac{t}{\sqrt{\Omega(t_0)}} \, ,$$

t_0 is the cosmological time. $\Omega(t_0)$ is the expanding ratio of universe in the cosmological time t_0 .

$$(i\hbar\sqrt{\Omega(t_0)}\gamma^0\partial_0 + i\hbar\frac{1}{\sqrt{\Omega(t_0)}}\gamma^i\partial_i - mcI)\phi = 0 \qquad (73)$$

If $\overline{\partial}_\mu$ is

$$\overline{\partial}_\mu = (\sqrt{\Omega(t_0)}\partial_0, \frac{1}{\sqrt{\Omega(t_0)}}\partial_i) \qquad (74)$$

Dirac equation is in cosmological inertial frame,

$$(i\hbar\gamma^\mu\overline{\partial}_\mu - mcI)\phi = 0 \qquad (75)$$

Eq(75) multiply $i\hbar\gamma^\upsilon\overline{\partial}_\upsilon$, hence

$$(-\hbar^2(\gamma^\mu\overline{\partial}_\mu)(\gamma^\upsilon\overline{\partial}_\upsilon) - i\hbar(\gamma^\upsilon\overline{\partial}_\upsilon)mcI)\phi = 0 \qquad (76)$$

In this time,

$$i\hbar\gamma^\upsilon\overline{\partial}_\upsilon\phi = mcI\phi \qquad (77)$$

Hence, Eq(76) is

$$(-\hbar^2\gamma^\mu\gamma^\upsilon\overline{\partial}_\mu\overline{\partial}_\upsilon - m^2c^2I)\phi = 0 \qquad (78)$$

In this time, matrix γ^μ is

$$\frac{1}{2}(\gamma^\mu\gamma^\upsilon + \gamma^\upsilon\gamma^\mu) = \frac{1}{2}\{\gamma^\mu, \gamma^\upsilon\} = \eta^{\mu\upsilon}I \qquad (79)$$

Therefore,

$$\frac{1}{2}(\gamma^\mu\gamma^\upsilon + \gamma^\upsilon\gamma^\mu)\overline{\partial}_\mu\overline{\partial}_\upsilon\phi + \frac{m^2c^2}{\hbar^2}I\phi$$
$$= (\eta^{\mu\upsilon}\overline{\partial}_\mu\overline{\partial}_\upsilon + \frac{m^2c^2}{\hbar^2})I\phi = 0 \qquad (80)$$

Eq(80) is the matrix equation of Klein-Gordon.

Dirac spinor ϕ is $\phi = (\phi_1, \phi_2, \phi_3, \phi_4)$. ϕ 's hermitian conjugate $\phi^+ = (\phi^*{}_1, \phi^*{}_2, \phi^*{}_3, \phi^*{}_4)$.

Hence, ϕ 's adjoint spinor $\overline{\phi}$ is

$$\bar{\phi} = \phi^{+}\gamma^{0}, \ \bar{\phi}(i\gamma^{\mu}\overleftarrow{\partial}_{\mu} + mcI) = 0 \tag{81}$$

Hence, positive probability density j^0 is

$$j^0 = \bar{\phi}\gamma^0\phi = \phi^{+}\phi = |\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2 + |\phi_4|^2 \tag{82}$$

8. Equations of Interaction of Complex Scalar Fields and Electromagnetic Fields in Cosmological Inertial Frame

The Lagrangian L of complex scalar fields ϕ, ϕ^* and Electromagnetic fields $F^{\mu\upsilon}, F_{\mu\upsilon}$ is Klein-Gordon-Maxwell theory in special relativity theory,

$$L = (\partial_{\mu}\phi + ieA_{\mu}\phi)(\partial^{\mu}\phi^* - ieA^{\mu}\phi^*) - \frac{m^2c^2}{\hbar^2}\phi\phi^* - \frac{1}{4}F^{\mu\upsilon}F_{\mu\upsilon}$$

ϕ^* is ϕ's adjoint scalar, m is the mass of scalar fields ϕ, ϕ^*

$$F^{\mu\upsilon} = \partial^{\mu}A^{\upsilon} - \partial^{\upsilon}A^{\mu}, F_{\mu\upsilon} = \partial_{\mu}A_{\upsilon} - \partial_{\upsilon}A_{\mu} \tag{83}$$

The Lagrangian L of interaction of complex scalar fields and Electromagnetic fields is Klein-Gordon-Maxwell theory in cosmological inertial frame,

$$L = (\bar{\partial}_{\mu}\phi + ie\bar{A}_{\mu}'\phi)(\bar{\partial}^{\mu}\phi^* - ie\bar{A}^{\mu}'\phi^*) - \frac{m^2c^2}{\hbar^2}\phi\phi^* - \frac{1}{4}F^{\mu\upsilon}{}'F_{\mu\upsilon}{}' \tag{84}$$

We consider Type A of wave function and Type B of expanded distance,

Type A of wave function: $r \to r\sqrt{\Omega(t_0)}$, $t \to \frac{t}{\sqrt{\Omega(t_0)}}$,

Type B of expanded distance: $r \to r\Omega(t_0), t \to t$

$$\bar{\partial}_{\mu} = (\sqrt{\Omega(t_0)}\frac{\partial}{c\partial t}, \frac{1}{\sqrt{\Omega(t_0)}}\vec{\nabla}), \bar{\partial}^{\mu} = (\sqrt{\Omega(t_0)}\frac{\partial}{c\partial t}, -\frac{1}{\sqrt{\Omega(t_0)}}\vec{\nabla})$$

$$\bar{A}_{\mu}' = (\phi, \vec{A}\Omega(t_0)), \bar{A}^{\mu}' = (\phi, -\vec{A}\Omega(t_0)), \bar{F}_{\mu\upsilon}' = F_{\mu\upsilon}\Omega(t_0), \bar{F}^{\mu\upsilon}' = F^{\mu\upsilon}\Omega(t_0)$$

t_0 is the cosmological time. $\Omega(t_0)$ is the expanding ratio of universe in the cosmological time t_0 .

(85)

Complex scalar field equations are in Klein-Gordon-Maxwell theory in cosmological inertial frame,

$$\bar{\partial}_{\mu}(\frac{\partial L}{\partial(\bar{\partial}_{\mu}\phi)}) - \frac{\partial L}{\partial\phi} = (\bar{\partial}_{\mu} - ie\bar{A}_{\mu}')(\bar{\partial}^{\mu}\phi^* - ie\bar{A}^{\mu}'\phi^*) + \frac{m^2c^2}{\hbar^2}\phi^* = 0 \tag{86}$$

The other equation is in Klein-Gordon-Maxwell theory in cosmological inertial frame,

$$\bar{\partial}_{\mu}(\frac{\partial L}{\partial(\bar{\partial}_{\mu}\phi^*)}) - \frac{\partial L}{\partial\phi^*} = (\bar{\partial}^{\mu} + ie\bar{A}^{\mu}')(\bar{\partial}_{\mu}\phi + ie\bar{A}_{\mu}'\phi) + \frac{m^2c^2}{\hbar^2}\phi = 0 \tag{87}$$

If operator $\bar{\partial}_\mu{}', \bar{\partial}^\mu{}'$ are in cosmological inertial frame,

$$\bar{\partial}_\mu{}' = (\frac{\partial}{c\partial t}, \frac{1}{\Omega(t_0)}\vec{\nabla}), \bar{\partial}^\mu{}' = (\frac{\partial}{c\partial t}, -\frac{1}{\Omega(t_0)}\vec{\nabla})$$

$$\bar{F}^{\mu\upsilon}{}' = \bar{\partial}^\mu{}'\bar{A}^\upsilon{}' - \bar{\partial}^\upsilon{}'\bar{A}^\mu{}', \bar{F}_{\mu\upsilon}{}' = \bar{\partial}_\mu{}'\bar{A}_\upsilon{}' - \bar{\partial}_\upsilon{}'\bar{A}_\mu{}' \quad (88)$$

Electromagnetic field equations are in Klein-Gordon-Maxwell theory in cosmological inertial frame,

$$\bar{\partial}_\upsilon{}'(\frac{\partial L}{\partial(\bar{\partial}_\upsilon{}'\bar{A}_\mu{}')}) - \frac{\partial L}{\partial \bar{A}_\mu{}'}$$

$$= \frac{1}{4}\bar{\partial}_\upsilon{}'(\bar{\partial}^\mu{}'\bar{A}^\upsilon{}' - \bar{\partial}^\upsilon{}'\bar{A}^\mu{}') - ie\phi(\bar{\partial}^\mu\phi^* - ie\bar{A}^\mu{}'\phi^*) + ie\phi^*(\bar{\partial}^\mu\phi + ie\bar{A}^\mu{}'\phi)$$

$$= \frac{1}{4}\bar{\partial}_\upsilon{}'\bar{F}^{\mu\upsilon}{}' - ie\phi(\bar{\partial}^\mu\phi^* - ie\bar{A}^\mu{}'\phi^*) + ie\phi^*(\bar{\partial}^\mu\phi + ie\bar{A}^\mu{}'\phi) = 0 \quad (89)$$

Hence,

$$\bar{\partial}_\upsilon{}'\bar{F}^{\mu\upsilon}{}' = -4\pi e\bar{J}^\mu{}' = 4ie[\phi(\bar{\partial}^\mu\phi^* - ie\bar{A}^\mu{}'\phi^*) - \phi^*(\bar{\partial}^\mu\phi + ie\bar{A}^\mu{}'\phi)]$$

$$\bar{J}^\mu{}' = -\frac{1}{\pi}i[\phi(\bar{\partial}^\mu\phi^* - ie\bar{A}^\mu{}'\phi^*) - \phi^*(\bar{\partial}^\mu\phi + ie\bar{A}^\mu{}'\phi)]$$

$$= \frac{1}{\pi}i[\phi^*(\bar{\partial}^\mu\phi + ie\bar{A}^\mu{}'\phi) - \phi(\bar{\partial}^\mu\phi^* - ie\bar{A}^\mu{}'\phi^* \quad (90)$$

The other equation is in Klein-Gordon-Maxwell theory in cosmological inertial frame,

$$\bar{\partial}^\upsilon{}'(\frac{\partial L}{\partial(\bar{\partial}^\upsilon{}'\bar{A}^\mu{}')}) - \frac{\partial L}{\partial \bar{A}^\mu{}'}$$

$$= \frac{1}{4}\bar{\partial}^\upsilon{}'(\bar{\partial}_\mu{}'\bar{A}_\upsilon{}' - \bar{\partial}_\upsilon{}'\bar{A}_\mu{}') + ie\phi^*(\bar{\partial}_\mu\phi + ie\bar{A}_\mu{}'\phi) - ie\phi(\bar{\partial}_\mu\phi^* - ie\bar{A}_\mu{}'\phi^*)$$

$$= \frac{1}{4}\bar{\partial}^\upsilon{}'\bar{F}_{\mu\upsilon}{}' + ie\phi^*(\bar{\partial}_\mu\phi + ie\bar{A}_\mu{}'\phi) - ie\phi(\bar{\partial}_\mu\phi^* - ie\bar{A}_\mu{}'\phi^*) = 0 \quad (91)$$

Hence,

$$\bar{\partial}^\upsilon{}'\bar{F}_{\mu\upsilon}{}' = -4\pi e\bar{J}_\mu{}' = -4ie[\phi^*(\bar{\partial}_\mu\phi + ie\bar{A}_\mu{}'\phi) - \phi(\bar{\partial}_\mu\phi^* - ie\bar{A}_\mu{}'\phi^*)]$$

$$\bar{J}_\mu{}' = i\frac{1}{\pi}[\phi^*(\bar{\partial}_\mu\phi + ie\bar{A}_\mu{}'\phi) - \phi(\bar{\partial}_\mu\phi^* - ie\bar{A}_\mu{}'\phi^*)] \quad (92)$$

9. Quantization of Klein-Gordon Scalar Field in CSTR

We make quantization of Klein-Gordon scalar field in Cosmological Special Theory of Relativity (CSTR).

At first, space-time relations are in cosmological special theory of relativity (CSTR).

$$ct = \gamma(ct' + \frac{v_0}{c}\Omega^2(t_0)x'),\ x\Omega(t_0) = \gamma(\Omega(t_0)x' + v_0\Omega(t_0)t')$$

$$\begin{matrix} \Omega(t_0)y = \Omega(t_0)y', \\ \Omega(t_0)z = \Omega(t_0)z' \end{matrix},\ \gamma = 1/\sqrt{1 - \frac{{v_0}^2}{c^2}\Omega^2(t_0)},\ t_0 \text{ is cosmological time} \quad (93)$$

Proper time is

$$d\tau^2 = dt^2 - \frac{1}{c^2}\Omega^2(t_0)[dx^2 + dy^2 + dz^2]$$
$$= dt'^2 - \frac{1}{c^2}\Omega^2(t_0)[dx'^2 + dy'^2 + dz'^2],\ t_0 \text{ is cosmological time} \quad (94)$$

Angular frequency-wave number relation is in CSTR.

$$\omega' = \gamma(\omega - v_0\Omega(t_0)k_1),\ k_1' = \gamma(k_1 - \frac{v_0}{c^2}\Omega(t_0)\omega)$$

$$k_2' = k_2, k_3' = k_3, \gamma = 1/\sqrt{1 - \frac{{v_0}^2}{c^2}\Omega^2(t_0)} \quad (95)$$

Lagrangian density of Klein-Gordon scalar field in CSTR,

$$L = -\frac{1}{2}[-(\frac{1}{c}\frac{\partial\phi}{\partial t})^2\Omega(t_0) + \frac{1}{\Omega(t_0)}\vec{\nabla}\phi\cdot\vec{\nabla}\phi - \frac{{m_0}^2c^2}{\hbar^2}\phi^2] \quad (96)$$

Hence, Euler-Lagrange equation is in CSTR,

$$\partial_\mu[\frac{\partial L}{\partial(\partial_\mu\phi)}] - \frac{\partial L}{\partial\phi} = [\Omega(t_0)\frac{1}{c^2}(\frac{\partial}{\partial t})^2 - \frac{1}{\Omega(t_0)}\nabla^2 + \frac{{m_0}^2c^2}{\hbar^2}]\phi = 0) \quad (97)$$

Hamiltonian of Klein-Gordon scalar field is in CSTR,

$$H = \frac{1}{2}[(\frac{1}{c}\frac{\partial\phi}{\partial t})^2\Omega(t_0) + \frac{1}{\Omega(t_0)}\vec{\nabla}\phi\cdot\vec{\nabla}\phi + \frac{{m_0}^2c^2}{\hbar^2}\phi^2] \quad (98)$$

The Klein-Gordon scalar field is divided by positive frequency mode and negative frequency mode.

$$\phi(x) = \phi^{(+)}(x) + \phi^{(-)}(x) \quad (99)$$

The positive frequency mode is

$$\phi^{(+)}(x) = \int\frac{d^3k}{[(2\pi)^3 2\omega_k]^{\frac{1}{2}}}a(k)f_k(x) \quad (100)$$

The negative frequency mode is

$$\phi^{(-)}(x) = \int \frac{d^3k}{[(2\pi)^3 2\omega_k]^{\frac{1}{2}}} a^{(+)}(k) f_k(x) \tag{101}$$

In this time, $f_k(x)$ is

$$f_k(x) = \frac{1}{[(2\pi)^3 2\omega_k]^{\frac{1}{2}}} \exp[i(\frac{\omega_k t}{\sqrt{\Omega(t_0)}} - \vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)})] \tag{102}$$

In this time,

$$\frac{\omega_k}{c} = (k^2 + \frac{m_0{}^2 c^2}{\hbar^2})^{\frac{1}{2}} \tag{103}$$

Quantization of complex scalar field is in CSTR,

$$\phi(x) = \int \frac{d^3k}{(2\pi)^3 2\omega_k} [a(k)\exp\{i(\frac{\omega_k t}{\sqrt{\Omega(t_0)}} - \vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)})\}$$

$$+\int \frac{d^3k}{(2\pi)^2 2\omega_k} [b^+(k)\exp\{-i(\frac{\omega_k t}{\sqrt{\Omega(t_0)}} - \vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)})\}] \tag{104}$$

$$\phi^+(x) = \int \frac{d^3k}{(2\pi)^3 2\omega_k} [b(k)\exp\{i(\frac{\omega_k t}{\sqrt{\Omega(t_0)}} - \vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)})\}$$

$$+\int \frac{d^3k}{(2\pi)^2 2\omega_k} [a^+(k)\exp\{-i(\frac{\omega_k t}{\sqrt{\Omega(t_0)}} - \vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)})\}] \tag{105}$$

Hence, Hamiltonian H is in CSTR,

$$H = \int \frac{d^3k}{(2\pi)^3 2\omega_k} [a^+(k)a(x) + b^+(k)b(k)] \tag{106}$$

In this time,

$$[a(k), a^+(k')] = (2\pi)^3 2\omega_k \delta^3(\vec{k} - \vec{k}')$$

$$[b(k), b^+(k')] = (2\pi)^3 2\omega_k \delta^3(\vec{k} - \vec{k}') \tag{107}$$

10. Conclusion

We know Maxwell equations, electromagnetic wave equations and functions in Cosmological Special Theory of Relativity (CSTR). We are able to describe free particle by Klein-Gordon equation and wave function in CSTR. We solve Klein-Gordon equation in cosmological inertial frame. Hence, we found Yukawa potential dependent time in cosmological inertial frame. We found Schrodinger equation from Klein-Gordon's free particle equation in cosmological special theory of relativity. The wave function uses as a probability amplitude. We found Dirac equation from Wave Function-Type A in cosmological special theory of relativity. The wave function uses as a probability amplitude. We found equations of complex scalar fields and electromagnetic fields on interaction of complex scalar fields and electromagnetic fields in Klein-Gordon-Maxwell theory from Type A of wave function and Type B of expanded distance in cosmological inertial frame. We quantized Klein-Gordon scalar field in CSTR. We treat Lagranian density and Hamiltonian.

References

[1]A. Einstein, " Zur Elektrodynamik bewegter K¨orper", Annalen der Physik. 17:891(1905)

[2]Friedman-Lemaitre-Robertson-Walker metric-Wikipedia

[3]A.Miller, Albert Einstein's Special Theory of Relativity(Addison-Wesley Publishing Co., Inc., 1981)

[4]W.Rindler, Special Relativity(2nd ed., Oliver and Boyd, Edinburg,1966)

[5]D.J. Griffith," Introduction To Electrodynamics",(2nd ed.,Prentice Hall,Inc.1981)

[6]Lambda-CDM model –Wikipedia

[7]J.D. Bjorken & S. D. Drell, Relativistic Quantum Field(McGraw- Hill Co., 1965)

[8]P.Bergman,Introduction to the Theory of Relativity(Dover Pub. Co.,Inc., New York,1976),Chapter V

[9]R.L.Liboff, Quantum Mechanics(Addison-Wesley Publishing Co., Inc.,1990)

[10]A.Beiser, Concept of Modern Physics(McGraw-Hill,Inc.,1991)

[11]H. Yukawa, "On the interaction of elementary particle. I", Proceedings of the Physico-Mathematical Society of Japan. 3rd Series 17(1935):48-573

[12]Dirac equation-Wikipedia

[13]C.Itzykson & J.B.Zuber, Quantum Field Theory, McGraw-Hill, (1980)

[14]S.S.Schweber, An Introduction to Relativistic Quantum Field Theory, Haper and Raw, (1962)

[15]G.Kallen, Quatum Electrodynamics, Springer-Verlag, (1972)

[16]T.D.Lee, Particle Physics and Introduction to Field Theory, Ch. 2 Harwood Academic Publishers, (1981)

[17]L.H.Ryder, Quantum Field Theory, Cambridge University Press, (1985)