Cosmological Special Theory of Relativity and Quantum Physics

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ABSTRACT

In the Cosmological Special Relativity Theory, we study Maxwell equations, electromagnetic wave equation and function. In the Cosmological Special Theory of Relativity, we study energy-momentum relations, Klein-Gordon equation and wave function. We study Yukawa potential dependent about time in cosmological inertial frame. If we solve Klein-Gordon equation, we obtain Yukawa potential dependent about time in cosmological inertial frame. Schrodinger equation is a wave equation. Wave function uses as a probability amplitude in quantum mechanics. We make Schrodinger equation from Klein-Gordon free particle's wave function in cosmological special theory of relativity. Dirac equation is a one order-wave equation. Wave function uses as a probability amplitude in quantum mechanics. We make Dirac Equation from wave function, Type A in cosmological inertial frame. The Dirac equation satisfy Klein-Gordon equation in cosmological inertial frame. We found equations of complex scalar fields and electromagnetic fields on interaction of complex scalar fields and electromagnetic fields in Klein-Gordon-Maxwell theory from Type A of wave function and Type B of expanded distance in cosmological inertial frame. In the Cosmological Special Theory of Relativity, we quantized Klein-Gordon scalar field. We treat Lagrangian density and Hamiltonian in quantized Klein-Gordon scalar field.

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Quantization

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1. Introduction

Our article's aim is that we make cosmological special theory of relativity.

At first, Robertson-Walker metric is

$$d\tau^2 = dt^2 - \frac{1}{c^2}\Omega^2(t)\,[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2] \qquad (1)$$

According to ΛCDM model, our universe's k is zero. In this time, if t_0 is cosmological time[6],

$$k = 0, t = t_0 >> \Delta t\,,\ \ \Delta t \text{ is period of matter's motion} \qquad (2)$$

Hence, the proper time is in cosmological time,

$$d\tau^2 = dt^2 - \frac{1}{c^2}\Omega^2(t_0)\,[dr^2 + r^2 d\Omega^2]$$

$$= dt^2 - \frac{1}{c^2}\Omega^2(t_0)\,[dx^2 + dy^2 + dz^2]$$

$$= dt^2(1 - \frac{1}{c^2}\Omega^2(t_0)V^2)\,, \quad V^2 = \frac{dx^2 + dy^2 + dz^2}{dt^2} \qquad (3)$$

In this time,

$$d\bar{t} = dt, d\bar{x} = \Omega(t_0)dx, d\bar{y} = \Omega(t_0)dy, d\bar{z} = \Omega(t_0)dz \qquad (4)$$

Cosmological special theory of relativity's coordinate transformations are

$$c\bar{t} = ct = \gamma(c\bar{t}\,' + \frac{v_0}{c}\Omega(t_0)\bar{x}\,') = \gamma(ct + \frac{v_0}{c}\Omega(t_0)x'\Omega(t_0))$$

$$\bar{x} = x\Omega(t_0) = \gamma(\bar{x}\,' + v_0\Omega(t_0)\bar{t}\,') = \gamma(\Omega(t_0)x' + v_0\Omega(t_0)t')$$

$$\begin{matrix}\bar{y} = \Omega(t_0)y = \bar{y}\,' = \Omega(t_0)y'\,, \\ \bar{z} = \Omega(t_0)z = \bar{z}\,' = \Omega(t_0)z'\end{matrix}\quad,\ \gamma = 1/\sqrt{1 - \frac{v_0^{\ 2}}{c^2}\Omega^2(t_0)} \qquad (5)$$

Therefore, proper time is

$$d\tau^2 = d\bar{t}^{\,2} - \frac{1}{c^2}[d\bar{x}^2 + d\bar{y}^2 + d\bar{z}^2]$$

$$= dt^2 - \frac{1}{c^2}\Omega^2(t_0)\,[dx^2 + dy^2 + dz^2]$$

$$= dt'^2 - \frac{1}{c^2}\Omega^2(t_0)\,[dx'^2 + dy'^2 + dz'^2]$$

$$= d\bar{t}\,'^2 - \frac{1}{c^2}[d\bar{x}\,'^2 + d\bar{y}\,'^2 + d\bar{z}\,'^2] \qquad (6)$$

Hence, velocities are

$$\frac{dx}{dt}=V_x=\frac{V_x'+v_0}{1+\frac{\Omega^2(t_0)}{c^2}V_x'\cdot v_0},V_x'=\frac{dx'}{dt'}$$

$$\frac{dy}{dt}=V_y=\frac{V_y'}{\gamma(1+\frac{\Omega^2(t_0)}{c^2}V_x'\cdot v_0)},V_y'=\frac{dy'}{dt'}$$

$$\frac{dz}{dt}=V_z=\frac{V_z'}{\gamma(1+\frac{\Omega^2(t_0)}{c^2}V_x'\cdot v_0)},V_z'=\frac{dz'}{dt'} \tag{7}$$

In cosmological special theory of relativity(CSTR)'s differential operators are

$$\frac{1}{c}\frac{\partial}{\partial\bar{t}}=\frac{1}{c}\frac{\partial}{\partial t}=\gamma(\frac{1}{c}\frac{\partial}{\partial\bar{t}'}-\frac{v_0}{c}\Omega(t_0)\frac{\partial}{\partial\bar{x}'})$$

$$=\gamma(\frac{1}{c}\frac{\partial}{\partial t'}-\frac{v_0}{c}\frac{\partial}{\partial x'})$$

$$\frac{\partial}{\partial\bar{x}}=\frac{\partial}{\partial x}\frac{1}{\Omega(t_0)}=\gamma(\frac{\partial}{\partial\bar{x}'}-\frac{v_0}{c}\Omega(t_0)\frac{1}{c}\frac{\partial}{\partial\bar{t}'})$$

$$=\gamma(\frac{\partial}{\partial x'}\frac{1}{\Omega(t_0)}-\frac{v_0}{c}\Omega(t_0)\frac{1}{c}\frac{\partial}{\partial t'})$$

$$\frac{\partial}{\partial\bar{y}}=\frac{\partial}{\partial y}\frac{1}{\Omega(t_0)}=\frac{\partial}{\partial\bar{y}'}=\frac{\partial}{\partial y'}\frac{1}{\Omega(t_0)}$$

$$\frac{\partial}{\partial\bar{z}}=\frac{\partial}{\partial z}\frac{1}{\Omega(t_0)}=\frac{\partial}{\partial\bar{z}'}=\frac{\partial}{\partial z'}\frac{1}{\Omega(t_0)},\gamma=1/\sqrt{1-\frac{{v_0}^2}{c^2}\Omega^2(t_0)} \tag{8}$$

Hence,

$$\frac{1}{c^2}\frac{\partial^2}{\partial\bar{t}^2}-\bar{\nabla}^2=\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\frac{1}{\Omega^2(t_0)}\{(\frac{\partial}{\partial x})^2+(\frac{\partial}{\partial y})^2+(\frac{\partial}{\partial z})^2\}$$

$$=\frac{1}{c^2}\frac{\partial^2}{\partial t'^2}-\frac{1}{\Omega^2(t_0)}\{(\frac{\partial}{\partial x'})^2+(\frac{\partial}{\partial y'})^2+(\frac{\partial}{\partial z'})^2\} \tag{9}$$

The electric charge density ρ and the electric current density $\vec{j}$ are

$$j^{\mu} = \rho_0 \frac{dx^{\mu}}{d\tau}, j^0 = c\rho = c\gamma\rho_0, j^i = \vec{j} = \rho\vec{u}, \ i = 1,2,3 \quad (10)$$

In CSTR, transformations of the electric charge density and the electric current density are likely as coordinate transformations are

$$c\bar{\rho} = c\rho = \gamma(c\bar{\rho}' + \frac{v_0}{c}\Omega(t_0)\bar{j}_x') = \gamma(c\rho' + \frac{v_0}{c}\Omega(t_0)j_x'\Omega(t_0))$$

$$\bar{j}_x = j_x\Omega(t_0) = \gamma(\bar{j}_x' + v_0\Omega(t_0)\bar{\rho}') = \gamma(\Omega(t_0)j_x' + v_0\Omega(t_0)\rho')$$

$$\begin{matrix} \bar{j}_y = \Omega(t_0)j_y = \bar{j}_y' = \Omega(t_0)j_y', \\ \bar{j}_z = \Omega(t_0)j_z = \bar{j}_z' = \Omega(t_0)j_z' \end{matrix} \quad , \ \gamma = 1/\sqrt{1 - \frac{v_0^2}{c^2}\Omega^2(t_0)} \quad (11)$$

2. Electrodynamics in CSTR

The electromagnetic potential A^{μ} is 4-vector potential.Hence, transformations of A^{μ} are

$$\bar{\phi} = \phi = \gamma(\bar{\phi}' + \frac{v_0}{c}\Omega(t_0)\bar{A}_x') = \gamma(\phi' + \frac{v_0}{c}\Omega(t_0)A_x'\Omega(t_0))$$

$$\bar{A}_x = A_x\Omega(t_0) = \gamma(\bar{A}_x' + \frac{v_0}{c}\Omega(t_0)\bar{\phi}') = \gamma(\Omega(t_0)A_x' + \frac{v_0}{c}\Omega(t_0)\phi')$$

$$\begin{matrix} \bar{A}_y = \Omega(t_0)A_y = \bar{A}_y' = \Omega(t_0)A_y', \\ \bar{A}_z = \Omega(t_0)A_z = \bar{A}_z' = \Omega(t_0)A_z' \end{matrix} \quad , \ \gamma = 1/\sqrt{1 - \frac{v_0^2}{c^2}\Omega^2(t_0)} \quad (12)$$

In CSTR, electric field $\bar{\vec{E}}$ and magnetic field $\bar{\vec{B}}$ have to satisfy Maxwell equations of special relativity theory. Hence, in CSRT, Maxwell equations are likely as special theory of relativity,

$$\bar{\vec{\nabla}} \cdot \bar{\vec{E}} = 4\pi\bar{\rho} \quad (13\text{-i})$$

$$\bar{\vec{\nabla}} \cdot \bar{\vec{B}} = 0 \quad (13\text{-ii})$$

$$\bar{\vec{\nabla}} \times \bar{\vec{E}} = -\frac{1}{c}\frac{\partial \bar{\vec{B}}}{\partial \bar{t}} \quad (13\text{-iii})$$

$$\bar{\vec{\nabla}} \times \bar{\vec{B}} = \frac{1}{c}\frac{\partial \bar{\vec{E}}}{\partial \bar{t}} + \frac{4\pi}{c}\bar{\vec{j}} \quad (13\text{-iv})$$

In this time, Eq(13-i) is

$$\bar{\vec{\nabla}} \cdot \bar{\vec{E}} = \frac{1}{\Omega(t_0)}\vec{\nabla} \cdot \bar{\vec{E}} = 4\pi\bar{\rho} = 4\pi\rho \quad (14)$$

Hence, $\bar{\vec{E}} = \vec{E}\Omega(t_0)$ .According to special relativity, $\bar{\vec{B}} = \vec{B}\Omega(t_0)$

Eq(13-ii) is

$$\bar{\vec{\nabla}} \cdot \bar{\vec{B}} = \frac{1}{\Omega(t_0)} \vec{\nabla} \cdot \vec{B}\Omega(t_0) = \vec{\nabla} \cdot \vec{B} = 0 \tag{15}$$

Eq(13-iii) is

$$\bar{\vec{\nabla}} \times \bar{\vec{E}} = \frac{1}{\Omega(t_0)} \vec{\nabla} \times \vec{E}\Omega(t_0) = \vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \bar{\vec{B}}}{\partial \bar{t}} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}\Omega(t_0) \tag{16}$$

Eq(13-iv) is

$$\bar{\vec{\nabla}} \times \bar{\vec{B}} = \frac{1}{\Omega(t_0)} \vec{\nabla} \times \vec{B}\Omega(t_0) = \vec{\nabla} \times \vec{B}$$

$$= \frac{1}{c}\frac{\partial \bar{\vec{E}}}{\partial \bar{t}} + \frac{4\pi}{c}\bar{\vec{j}} = \Omega(t_0)(\frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{j}) \tag{17}$$

Hence, in CSTR, Maxwell equations are

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho \tag{18-i}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \tag{18-ii}$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}\Omega(t_0) \tag{18-iii}$$

$$\vec{\nabla} \times \vec{B} = \Omega(t_0)(\frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{j}) \tag{18-iv}$$

Therefore, in CSTR, the electric field $\vec{E}$ and the magnetic field $\vec{B}$ are

$$\bar{\vec{E}} = \vec{E}\Omega(t_0) = \Omega(t_0)(-\vec{\nabla}\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t})$$

$$= -\Omega(t_0)\vec{\nabla}\phi - \Omega(t_0)\frac{1}{c}\frac{\partial \vec{A}}{\partial t} = -\bar{\vec{\nabla}}(\bar{\phi}\Omega^2(t_0)) - \frac{1}{c}\frac{\partial \bar{\vec{A}}}{\partial \bar{t}} \tag{19}$$

$$\bar{\vec{B}} = \vec{B}\Omega(t_0) = \Omega(t_0)\vec{\nabla} \times \vec{A} = \Omega(t_0)\bar{\vec{\nabla}} \times \bar{\vec{A}} \tag{20}$$

3. Electromagnetic Wave in CSTR

Electromagnetic wave equation is in CSTR,

$$\frac{1}{c}\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{E}) = -\Omega(t_0)\frac{1}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2}$$

$$= \vec{\nabla} \times (\frac{1}{c}\frac{\partial \vec{E}}{\partial t}) = \vec{\nabla} \times (\frac{1}{\Omega(t_0)}\vec{\nabla} \times \vec{B}), \vec{\nabla} \times \vec{j} = \vec{0}$$

$$= \frac{1}{\Omega(t_0)}\{-\nabla^2\vec{B} + \vec{\nabla}(\vec{\nabla} \cdot \vec{B})\} = -\frac{1}{\Omega(t_0)}\nabla^2\vec{B} \tag{21}$$

Hence, electromagnetic wave equation is

$$\Omega(t_0)\frac{1}{c^2}\frac{\partial^2\vec{B}}{\partial t^2}-\frac{1}{\Omega(t_0)}\nabla^2\vec{B}=\vec{0} \tag{22}$$

And,

$$\frac{1}{c}\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})=\Omega(t_0)\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2},\frac{1}{c}\frac{\partial\vec{j}}{\partial t}=\vec{0}$$

$$=\vec{\nabla}\times(\frac{1}{c}\frac{\partial\vec{B}}{\partial t})=\vec{\nabla}\times(-\frac{1}{\Omega(t_0)}\vec{\nabla}\times\vec{E})$$

$$=-\frac{1}{\Omega(t_0)}\{-\nabla^2\vec{E}+\vec{\nabla}(\vec{\nabla}\cdot\vec{E})\}=\frac{1}{\Omega(t_0)}\nabla^2\vec{E},\vec{\nabla}(4\pi\rho)=\vec{0} \tag{23}$$

Hence, electromagnetic wave equation is

$$\Omega(t_0)\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2}-\frac{1}{\Omega(t_0)}\nabla^2\vec{E}=\vec{0} \tag{24}$$

In CSTR, electromagnetic wave functions are

$$\vec{E}=\vec{E}_0\sin\Phi,\vec{B}=\vec{B}_0\sin\Phi$$

$$\Phi=\omega\{\frac{t}{\sqrt{\Omega(t_0)}}-\frac{\sqrt{\Omega(t_0)}}{c}(lx+my+nz)\} \tag{25}$$

Where,

$$l^2+m^2+n^2=1 \tag{26}$$

According to Maxwell equations are in CSTR,[1]

$$\Omega(t_0)\{\frac{1}{c}\frac{\partial E_x}{\partial t}+\frac{4\pi}{c}j_x\}=(\frac{\partial B_z}{\partial y}-\frac{\partial B_y}{\partial z}),\qquad \Omega(t_0)\frac{1}{c}\frac{\partial B_x}{\partial t}=(\frac{\partial E_y}{\partial z}-\frac{\partial E_z}{\partial y})$$

$$\Omega(t_0)\{\frac{1}{c}\frac{\partial E_y}{\partial t}+\frac{4\pi}{c}j_y\}=(\frac{\partial B_x}{\partial z}-\frac{\partial B_z}{\partial x}),\qquad \Omega(t_0)\frac{1}{c}\frac{\partial B_y}{\partial t}=(\frac{\partial E_z}{\partial x}-\frac{\partial E_x}{\partial z})$$

$$\Omega(t_0)\{\frac{1}{c}\frac{\partial E_z}{\partial t}+\frac{4\pi}{c}j_z)=(\frac{\partial B_y}{\partial x}-\frac{\partial B_x}{\partial y}),\qquad \Omega(t_0)\frac{1}{c}\frac{\partial B_z}{\partial t}=(\frac{\partial E_x}{\partial y}-\frac{\partial E_y}{\partial x}) \tag{27}$$

Where,

$$\Omega(t_0)\{\frac{1}{c}\frac{\partial E_x'}{\partial t}+\frac{4\pi}{c}j_x\}=\{\frac{\partial}{\partial y}\gamma(B_z'+\frac{v_0}{c}\Omega(t_0)E_y')-\frac{\partial}{\partial z}\gamma(B_y'-\frac{v_0}{c}\Omega(t_0)E_z')\}$$

$$\Omega(t_0)\{\frac{1}{c}\frac{\partial}{\partial t}\gamma(E_y'+\frac{v_0}{c}\Omega(t_0)B_z')+\frac{4\pi}{c}j_y\}=\{\frac{\partial B_x'}{\partial z}-\frac{\partial}{\partial x}\gamma(B_z'+\frac{v_0}{c}\Omega(t_0)E_y')\}$$

$$\Omega(t_0)\{\frac{1}{c}\frac{\partial}{\partial t}\gamma(E_z'-\frac{v_0}{c}\Omega(t_0)B_y')+\frac{4\pi}{c}j_z\}=\{\frac{\partial}{\partial y}\gamma(B_y'-\frac{v_0}{c}\Omega(t_0)E_z')-\frac{\partial B_x'}{\partial z}\} \quad (28)$$

Where,[1]

$$\Omega(t_0)\frac{1}{c}\frac{\partial B_x'}{\partial t}=\{\frac{\partial}{\partial z}\gamma(E_y'+\frac{v_0}{c}\Omega(t_0)B_z')-\frac{\partial}{\partial y}\gamma(E_z'-\frac{v_0}{c}\Omega(t_0)B_y')\}$$

$$\Omega(t_0)\frac{1}{c}\frac{\partial}{\partial t}\gamma(B_y'-\frac{v_0}{c}\Omega(t_0)E_z')=\{\frac{\partial}{\partial x}\gamma(E_z'-\frac{v_0}{c}\Omega(t_0)B_y')-\frac{\partial E_x'}{\partial z}\}$$

$$\Omega(t_0)\frac{1}{c}\frac{\partial}{\partial t}\gamma(B_z'+\frac{v_0}{c}\Omega(t_0)E_y')=\{\frac{\partial E_x'}{\partial y}-\frac{\partial}{\partial x}\gamma(E_y'+\frac{v_0}{c}\Omega(t_0)B_z')\} \quad (29)$$

Hence, in CSTR, transformations of electromagnetic field are

$$E_x=E_x',E_y=\gamma(E_y'+\frac{v_0}{c}\Omega(t_0)B_z'),E_z=\gamma(E_z'-\frac{v_0}{c}\Omega(t_0)B_y') \quad (30)$$

$$B_x=B_x',B_y=\gamma(B_y'-\frac{v_0}{c}\Omega(t_0)E_z'),B_z=\gamma(B_z'+\frac{v_0}{c}\Omega(t_0)E_y') \quad (31)$$

In CSTR, electromagnetic wave functions are

$$E_x'=E_{x0}\sin\Phi',E_y'=\gamma(E_{y0}-\frac{v_0}{c}\Omega(t_0)B_{z0})\sin\Phi',E_z'=\gamma(E_{z0}+\frac{v_0}{c}\Omega(t_0)B_{y0})\sin\Phi' \quad (32)$$

$$B_x'=B_{x0}\sin\Phi',B_y'=\gamma(B_{y0}+\frac{v_0}{c}\Omega(t_0)E_{z0})\sin\Phi',B_z'=\gamma(B_{z0}-\frac{v_0}{c}\Omega(t_0)E_{y0})\sin\Phi' \quad (33)$$

In this time,

$$\Phi'=\omega'\{\frac{t'}{\sqrt{\Omega(t_0)}}-\frac{\sqrt{\Omega(t_0)}}{c}(l'x'+m'y'+n'z')\} \quad (34)$$

$$\Phi=\omega\{\frac{t}{\sqrt{\Omega(t_0)}}-\frac{\sqrt{\Omega(t_0)}}{c}(lx+my+nz)\} \quad (35)$$

If we compare Eq(34) and Eq(35),

$$\omega'=\omega\gamma(1-l\Omega(t_0)\frac{v_0}{c}),l'=\frac{l-\frac{v_0}{c}\Omega(t_0)}{1-l\frac{v_0}{c}\Omega(t_0)}$$

$$m'=\frac{m}{\gamma(1-l\Omega(t_0)\frac{v_0}{c})},n'=\frac{n}{\gamma(1-l\Omega(t_0)\frac{v_0}{c})} \quad (36)$$

Where,

$$l'^2+m'^2+n'^2=1 \quad (37)$$

4. Klein-Gordon Equation and Wave Fuction in CSTR

We make Klein-Gordon equation and wave function in cosmological special theory of relativity.

At first, space-time relations are in cosmological special theory of relativity (CSTR).

$$ct=\gamma\left(ct'+\frac{v_0}{c}\Omega^2(t_0)x'\right)\quad x\Omega(t_0)=\gamma(\Omega(t_0)x'+v_0\Omega(t_0)t')$$

$$\begin{array}{l}\Omega(t_0)y=\Omega(t_0)y',\\ \Omega(t_0)z=\Omega(t_0)z'\end{array},\quad \gamma=1/\sqrt{1-\frac{v_0^2}{c^2}\Omega^2(t_0)},\quad t_0 \text{ is cosmological time} \qquad (38)$$

Therefore, proper time is,

$$d\tau^2=dt^2-\frac{1}{c^2}\Omega^2(t_0)[dx^2+dy^2+dz^2]$$

$$=dt'^2-\frac{1}{c^2}\Omega^2(t_0)[dx'^2+dy'^2+dz'^2],\quad t_0 \text{ is cosmological time} \qquad (39)$$

Hence, energy-momentum relations are by the fact that energy-momentum are 4-vector in CSTR,

$$E=\gamma(E'+v_0\Omega^2(t_0)p_x')\quad p_x\Omega(t_0)=\gamma\left(\Omega(t_0)p_x'+\frac{v_0}{c^2}\Omega(t_0)E'\right)$$

$$\begin{array}{l}\Omega(t_0)p_y=\Omega(t_0)p_y',\\ \Omega(t_0)p_z=\Omega(t_0)p_z'\end{array},\gamma=1/\sqrt{1-\frac{v_0^2}{c^2}\Omega^2(t_0)}\quad E=m_0c^2\frac{dt}{d\tau},\vec{p}=m_0\frac{d\vec{x}}{d\tau} \qquad (40)$$

Therefore, energy-momentum-mass relation is in CSTR,

$$m_0^2c^4=E^2-\Omega^2(t_0)p^2c^2 \qquad (41)$$

Matter wave function is in CSTR,

$$\phi=\phi_0\exp i\Phi=\phi_0\exp i\left[\frac{\omega t}{\sqrt{\Omega(t_0)}}-\vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)}\right]$$

$$=\phi'=\phi_0\exp i\Phi'=\phi_0\exp i\left[\frac{\omega' t'}{\sqrt{\Omega(t_0)}}-\vec{k}'\cdot\vec{x}'\sqrt{\Omega(t_0)}\right]$$

ϕ_0 is amplitude, ω is angular frequency, $k=|\vec{k}|$ is wave number. (42)

If we use Eq(38) in Eq(42), we obtain angular frequency-wave number relation.

$$\omega'=\gamma(\omega-v_0\Omega(t_0)k_1),\quad k_1'=\gamma\left(k_1-\frac{v_0}{c^2}\Omega(t_0)\omega\right)$$

$$k_2' = k_2, k_3' = k_3, \gamma = 1/\sqrt{1 - \frac{v_0^2}{c^2}\Omega^2(t_0)} \quad (43)$$

In this time, if we define energy-momentum by angular frequency-wave number,

$$E = \hbar\omega, \vec{p} = \frac{\hbar \vec{k}}{\Omega(t_0)} \quad (44)$$

Hence, we obtain the angular frequency-wave number relation about the energy-momentum-mass relation in CSTR,

$$m_0^2 c^4 = E^2 - \Omega^2(t_0) p^2 c^2 = \hbar^2\omega^2 - \hbar^2 k^2 c^2 \quad (45)$$

We obtain next result by the transformation of the angular frequency-wave number relation, Eq(43) in CSTR.

$$m_0^2 c^4 = \hbar^2\omega^2 - \hbar^2 k^2 c^2 = \hbar^2\omega'^2 - \hbar^2 k'^2 c^2 \quad (46)$$

If we define the differential operator about energy-momentum in CSTR,

$$E = i\hbar\frac{\partial}{\partial t}, \vec{p} = -i\hbar\frac{1}{\Omega(t_0)}\vec{\nabla} \quad (47)$$

If we apply Eq(47) to Eq(46),

$$m_0^2 c^4 = E^2 - \Omega^2(t_0) p^2 c^2 = \hbar^2[-(\frac{\partial}{\partial t})^2 + c^2\nabla^2]$$

We finally obtain Klein-Gordon equation in CSTR.

$$\frac{m_0^2 c^2}{\hbar^2}\phi = [-\frac{1}{c^2}(\frac{\partial}{\partial t})^2 + \nabla^2]\phi \quad (48)$$

Wave function, Eq(42) satisfy Klein-Gordon equation, Eq(48).

5. Yukawa potential in Klein-Gordon equation in cosmological inertial frame

If we focus Klein-Gordon equation about Yukawa potential ϕ dependent about time,

$$\frac{m_\pi^2 c^2}{\hbar^2}\phi + \partial_\mu\partial^\mu\phi = \frac{m_\pi^2 c^2}{\hbar^2}\phi + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\phi - \nabla^2\phi = 0 \quad (49)$$

In this time, Yukawa potential ϕ dependent about time is.

$$\phi = -\frac{g^2}{r}\exp(-\frac{m_\pi r c}{\hbar} + i) A_0 \sin \omega t$$

Frequency $\omega = \frac{m_\pi c^2}{\hbar}$, m_π is meson's mass (50)

Eq(49)-Klein-Gordon equation is satisfied by Eq(50)-Yukawa potential dependent about time

In cosmological inertial frame, Klein-Gordon equation is

$$-\Omega(t_0)\frac{1}{c^2}\frac{\partial^2\phi'}{\partial t^2}+\frac{1}{\Omega(t_0)}\nabla^2\phi'=\frac{m_\pi{}^2c^2}{\hbar^2}\phi' \tag{51}$$

In this point, in cosmological inertial frame, space-time transformations in the type A of wave function and the other type B of the expanded distance are

Type A: $r\to r\sqrt{\Omega(t_0)},t\to\frac{t}{\sqrt{\Omega(t_0)}}$, Type B: $r\to r\Omega(t_0),t\to t$ (52)

Space-time transformation of Yukawa potential ϕ' is depend on Type A

Hence, Yukawa potential ϕ' dependent about time is

$$\phi'=-\frac{g^2}{r\sqrt{\Omega(t_0)}}\exp[-\frac{m_\pi r\sqrt{\Omega(t_0)}c}{\hbar}]+A_0\sin(\frac{\omega t}{\sqrt{\Omega(t_0)}})$$

Frequency $\omega=\frac{m_\pi c^2}{\hbar}$, m_π is meson's mass (53)

Eq(51)-Klein-Gordon equation is satisfied by Eq(53)-the solution.

6. Schrodinger Equation from Klein-Gordon Free Particle Field in Cosmological Inertial Frame

At first, Klein-Gordon equation is for free particle field ϕ in cosmological inertial frame.

$$\frac{m^2c^2}{\hbar^2}\phi+\Omega(t_0)\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}-\frac{1}{\Omega(t_0)}\nabla^2\phi=0$$

m is free particle's mass , $\Omega(t_0)$ is the ratio of universe's expansion in cosmological time t_0 (54)

If we write wave function as solution of Klein-Gordon equation for free particle,

$$\phi=A_0\exp[i(\frac{\omega t}{\sqrt{\Omega(t_0)}}-\vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)})]$$

A_0 is amplitude, ω is angular frequency, $k=|\vec{k}|$ is wave number (55)

Energy and momentum is in cosmological inertial frame,

$$E=\hbar\omega,\vec{p}=\hbar\vec{k}/\Omega(t_0) \tag{56}$$

Hence, energy-momentum relation is in cosmological inertial frame

$$E^2=\hbar^2\omega^2=\Omega^2(t_0)p^2c^2+m^2c^4=\hbar^2k^2c^2+m^2c^4 \tag{57}$$

Or angular frequency- wave number relation is

$$\frac{\omega^2}{c^2} = k^2 + \frac{m^2c^2}{\hbar^2} \tag{58}$$

Hence, wave function is in cosmological inertial frame,

$$\phi = A_0 \exp[(-\frac{i}{\hbar})(\hbar\frac{\omega t}{\sqrt{\Omega(t_0)}} - \hbar\vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)})]$$

$$= A_0 \exp[(-\frac{i}{\hbar})(\frac{Et}{\sqrt{\Omega(t_0)}} - \vec{p}\cdot\vec{x}\Omega(t_0)\sqrt{\Omega(t_0)}] \tag{59}$$

Because, Schrodinger equation is made from Klein-Gordon free particle's wave function in cosmological special theory of relativity,

$$\phi = A_0 \exp[(-\frac{i}{\hbar})(\frac{Et}{\sqrt{\Omega(t_0)}} - \vec{p}\cdot\vec{x}\Omega(t_0)\sqrt{\Omega(t_0)}] \tag{60}$$

If we calculate the derivation of Schrodinger equation,

$$\sum_i(\frac{\partial}{\partial x^i})^2\phi = -\sum_i\frac{(p^i)^2}{\hbar^2}\Omega^3(t_0)\phi = -\frac{p^2}{\hbar^2}\Omega^3(t_0)\phi \tag{61}$$

$$\frac{\partial\phi}{\partial t} = -\frac{i}{\hbar}E\frac{\phi}{\sqrt{\Omega(t_0)}} \tag{62}$$

Energy E is

$$E = \frac{p^2}{2m}\Omega^2(t_0) + V\,, \quad V \text{ is the potential energy} \tag{63}$$

Hence,

$$E\phi = \frac{p^2}{2m}\Omega^2(t_0)\phi + V\phi\,, \quad V \text{ is the potential energy} \tag{64}$$

Therefore, by Eq(61),Eq(62)

$$E\phi = i\hbar\frac{\partial\phi}{\partial t}\sqrt{\Omega(t_0)}\,,\ \ \Omega^2(t_0)p^2\phi = -\hbar^2\nabla^2\phi\frac{1}{\Omega(t_0)} \tag{65}$$

Therefore, Schrodinger equation in cosmological inertial frame,

$$E\phi = i\hbar\frac{\partial\phi}{\partial t}\sqrt{\Omega(t_0)} = -\frac{\hbar^2}{2m}\frac{1}{\Omega(t_0)}\nabla^2\phi + V\phi \tag{66}$$

If the energy E is not concerned by time t,

$$\frac{\partial E}{\partial t} = 0 \tag{67}$$

$$\phi = A_0 \exp[-(\frac{i}{\hbar})(\frac{Et}{\sqrt{\Omega(t_0)}} - \vec{p}\cdot\vec{x}\Omega(t_0)\sqrt{\Omega(t_0)})]$$

$$=\varphi\exp[-(\frac{i}{\hbar})(\frac{Et}{\sqrt{\Omega(t_0)}})] \qquad (68)$$

Hence, stationary state of Schrodinger equation is in cosmological inertial frame,

$$E\varphi\exp[-(\frac{iE}{\hbar})\frac{t}{\sqrt{\Omega(t_0)}}]$$

$$=[-\frac{\hbar^2}{2m}\frac{1}{\Omega(t_0)}\nabla^2\varphi+V\varphi]\exp[-(\frac{iE}{\hbar})\frac{t}{\sqrt{\Omega(t_0)}}] \qquad (69)$$

Hence, stationary state of Schrodinger equation is

$$E\varphi=-\frac{\hbar^2}{2m}\frac{1}{\Omega(t_0)}\nabla^2\varphi+V\varphi \qquad (70)$$

Or,

$$\frac{1}{\Omega(t_0)}\nabla^2\varphi+\frac{2m}{\hbar^2}(E-V)\varphi=0 \qquad (71)$$

7. Dirac Equation from Wave Function-Type A in Cosmological Inertial Frame

Dirac equation is in special relativity theory,

$$(i\hbar\gamma^\mu\partial_\mu-mcI)\psi=0 \quad ,$$

I is 4×4 unit matrix ,

$$\gamma^0=\begin{pmatrix} I' & 0 \\ 0 & -I' \end{pmatrix},\ \gamma^i=\begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix},\ \sigma^1=\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\sigma^2=\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\sigma^3=\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

I' is 2×2 unit matrix, σ^i is Pauli's matrix. (72)

Dirac equation is the wave equation. Therefore, Dirac equation is in cosmological inertial frame,

Wave function Type A:

$$r\to r\sqrt{\Omega(t_0)} \quad , \quad t\to\frac{t}{\sqrt{\Omega(t_0)}} \ ,$$

t_0 is the cosmological time. $\Omega(t_0)$ is the expanding ratio of universe in the cosmological time t_0 .

$$(i\hbar\sqrt{\Omega(t_0)}\gamma^0\partial_0+i\hbar\frac{1}{\sqrt{\Omega(t_0)}}\gamma^i\partial_i-mcI)\phi=0 \qquad (73)$$

If $\overline{\partial}_\mu$ is

$$\overline{\partial}_\mu=(\sqrt{\Omega(t_0)}\partial_0,\frac{1}{\sqrt{\Omega(t_0)}}\partial_i) \qquad (74)$$

Dirac equation is in cosmological inertial frame,

$$(i\hbar\gamma^{\mu}\overline{\partial}_{\mu}-mcI)\phi=0 \quad (75)$$

Eq(75) multiply $i\hbar\gamma^{\upsilon}\overline{\partial}_{\upsilon}$, hence

$$(-\hbar^{2}(\gamma^{\mu}\overline{\partial}_{\mu})(\gamma^{\upsilon}\overline{\partial}_{\upsilon})-i\hbar(\gamma^{\upsilon}\overline{\partial}_{\upsilon})mcI)\phi=0 \quad (76)$$

In this time,

$$i\hbar\gamma^{\upsilon}\overline{\partial}_{\upsilon}\phi=mcI\phi \quad (77)$$

Hence, Eq(76) is

$$(-\hbar^{2}\gamma^{\mu}\gamma^{\upsilon}\overline{\partial}_{\mu}\overline{\partial}_{\upsilon}-m^{2}c^{2}I)\phi=0 \quad (78)$$

In this time, matrix γ^{μ} is

$$\frac{1}{2}(\gamma^{\mu}\gamma^{\upsilon}+\gamma^{\upsilon}\gamma^{\mu})=\frac{1}{2}\{\gamma^{\mu},\gamma^{\upsilon}\}=\eta^{\mu\upsilon}I \quad (79)$$

Therefore,

$$\frac{1}{2}(\gamma^{\mu}\gamma^{\upsilon}+\gamma^{\upsilon}\gamma^{\mu})\overline{\partial}_{\mu}\overline{\partial}_{\upsilon}\phi+\frac{m^{2}c^{2}}{\hbar^{2}}I\phi$$

$$=(\eta^{\mu\upsilon}\overline{\partial}_{\mu}\overline{\partial}_{\upsilon}+\frac{m^{2}c^{2}}{\hbar^{2}})I\phi=0 \quad (80)$$

Eq(80) is the matrix equation of Klein-Gordon.

Dirac spinor ϕ is $\phi=(\phi_{1},\phi_{2},\phi_{3},\phi_{4})$. ϕ 's hermitian conjugate $\phi^{+}=(\phi^{*}{}_{1},\phi^{*}{}_{2},\phi^{*}{}_{3},\phi^{*}{}_{4})$.

Hence, ϕ 's adjoint spinor $\overline{\phi}$ is

$$\overline{\phi}=\phi^{+}\gamma^{0},\ \ \overline{\phi}(i\gamma^{\mu}\overleftarrow{\overline{\partial}}_{\mu}+mcI)=0 \quad (81)$$

Hence, positive probability density j^{0} is

$$j^{0}=\overline{\phi}\gamma^{0}\phi=\phi^{+}\phi=|\phi_{1}|^{2}+|\phi_{2}|^{2}+|\phi_{3}|^{2}+|\phi_{4}|^{2} \quad (82)$$

8. Equations of Interaction of Complex Scalar Fields and Electromagnetic Fields in Cosmological Inertial Frame

The Lagrangian L of complex scalar fields ϕ,ϕ^{*} and Electromagnetic fields $F^{\mu\upsilon},F_{\mu\upsilon}$ is Klein-Gordon-Maxwell theory in special relativity theory,

$$L=(\partial_{\mu}\phi+ieA_{\mu}\phi)(\partial^{\mu}\phi^{*}-ieA^{\mu}\phi^{*})-\frac{m^{2}c^{2}}{\hbar^{2}}\phi\phi^{*}-\frac{1}{4}F^{\mu\upsilon}F_{\mu\upsilon}$$

ϕ^* is ϕ 's adjoint scalar, m is the mass of scalar fields ϕ, ϕ^*

$$F^{\mu\upsilon} = \partial^{\mu} A^{\upsilon} - \partial^{\upsilon} A^{\mu}, F_{\mu\upsilon} = \partial_{\mu} A_{\upsilon} - \partial_{\upsilon} A_{\mu} \tag{83}$$

The Lagrangian L of interaction of complex scalar fields and Electromagnetic fields is Klein-Gordon-Maxwell theory in cosmological inertial frame,

$$L = (\bar{\partial}_{\mu}\phi + ie\bar{A}_{\mu}'\phi)(\bar{\partial}^{\mu}\phi^* - ie\bar{A}^{\mu}'\phi^*) - \frac{m^2c^2}{\hbar^2}\phi\phi^* - \frac{1}{4}F^{\mu\upsilon}{}'F_{\mu\upsilon}{}' \tag{84}$$

We consider Type A of wave function and Type B of expanded distance,

Type A of wave function: $r \to r\sqrt{\Omega(t_0)}$, $t \to \frac{t}{\sqrt{\Omega(t_0)}}$,

Type B of expanded distance: $r \to r\Omega(t_0), t \to t$

$$\bar{\partial}_{\mu} = (\sqrt{\Omega(t_0)}\frac{\partial}{c\partial t}, \frac{1}{\sqrt{\Omega(t_0)}}\vec{\nabla}), \bar{\partial}^{\mu} = (\sqrt{\Omega(t_0)}\frac{\partial}{c\partial t}, -\frac{1}{\sqrt{\Omega(t_0)}}\vec{\nabla})$$

$$\bar{A}_{\mu}' = (\phi, \vec{A}\Omega(t_0)), \bar{A}^{\mu}' = (\phi, -\vec{A}\Omega(t_0)), \bar{F}_{\mu\upsilon}' = F_{\mu\upsilon}\Omega(t_0), \bar{F}^{\mu\upsilon}' = F^{\mu\upsilon}\Omega(t_0)$$

t_0 is the cosmological time. $\Omega(t_0)$ is the expanding ratio of universe in the cosmological time t_0 .

(85)

Complex scalar field equations are in Klein-Gordon-Maxwell theory in cosmological inertial frame,

$$\bar{\partial}_{\mu}(\frac{\partial L}{\partial(\bar{\partial}_{\mu}\phi)}) - \frac{\partial L}{\partial\phi} = (\bar{\partial}_{\mu} - ie\bar{A}_{\mu}')(\bar{\partial}^{\mu}\phi^* - ie\bar{A}^{\mu}'\phi^*) + \frac{m^2c^2}{\hbar^2}\phi^* = 0 \tag{86}$$

The other equation is in Klein-Gordon-Maxwell theory in cosmological inertial frame,

$$\bar{\partial}_{\mu}(\frac{\partial L}{\partial(\bar{\partial}_{\mu}\phi^*)}) - \frac{\partial L}{\partial\phi^*} = (\bar{\partial}^{\mu} + ie\bar{A}^{\mu}')(\bar{\partial}_{\mu}\phi + ie\bar{A}_{\mu}'\phi) + \frac{m^2c^2}{\hbar^2}\phi = 0 \tag{87}$$

If operator $\bar{\partial}_{\mu}', \bar{\partial}^{\mu}'$ are in cosmological inertial frame,

$$\bar{\partial}_{\mu}' = (\frac{\partial}{c\partial t}, \frac{1}{\Omega(t_0)}\vec{\nabla}), \bar{\partial}^{\mu}' = (\frac{\partial}{c\partial t}, -\frac{1}{\Omega(t_0)}\vec{\nabla})$$

$$\bar{F}^{\mu\upsilon}{}' = \bar{\partial}^{\mu}{}'\bar{A}^{\upsilon}{}' - \bar{\partial}^{\upsilon}{}'\bar{A}^{\mu}{}', \bar{F}_{\mu\upsilon}{}' = \bar{\partial}_{\mu}{}'\bar{A}_{\upsilon}{}' - \bar{\partial}_{\upsilon}{}'\bar{A}_{\mu}{}' \tag{88}$$

Electromagnetic field equations are in Klein-Gordon-Maxwell theory in cosmological inertial frame,

$$\bar{\partial}_{\upsilon}'(\frac{\partial L}{\partial(\bar{\partial}_{\upsilon}'\bar{A}_{\mu}')}) - \frac{\partial L}{\partial\bar{A}_{\mu}'}$$

$$= \frac{1}{4}\bar{\partial}_{\upsilon}'(\bar{\partial}^{\mu}{}'\bar{A}^{\upsilon}{}' - \bar{\partial}^{\upsilon}{}'\bar{A}^{\mu}{}') - ie\phi(\bar{\partial}^{\mu}\phi^* - ie\bar{A}^{\mu}'\phi^*) + ie\phi^*(\bar{\partial}^{\mu}\phi + ie\bar{A}^{\mu}'\phi)$$

$$=\frac{1}{4}\bar{\partial}_{\upsilon}'\bar{F}^{\mu\upsilon}'-ie\phi(\bar{\partial}^{\mu}\phi^{*}-ie\bar{A}^{\mu}'\phi^{*})+ie\phi^{*}(\bar{\partial}^{\mu}\phi+ie\bar{A}^{\mu}'\phi)=0 \quad (89)$$

Hence,

$$\bar{\partial}_{\upsilon}'\bar{F}^{\mu\upsilon}'=-4\pi e\bar{J}^{\mu}'=4ie[\phi(\bar{\partial}^{\mu}\phi^{*}-ie\bar{A}^{\mu}'\phi^{*})-\phi^{*}(\bar{\partial}^{\mu}\phi+ie\bar{A}^{\mu}'\phi)]$$

$$\bar{J}^{\mu}'=-\frac{1}{\pi}i[\phi(\bar{\partial}^{\mu}\phi^{*}-ie\bar{A}^{\mu}'\phi^{*})-\phi^{*}(\bar{\partial}^{\mu}\phi+ie\bar{A}^{\mu}'\phi)]$$

$$=\frac{1}{\pi}i[\phi^{*}(\bar{\partial}^{\mu}\phi+ie\bar{A}^{\mu}'\phi)-\phi(\bar{\partial}^{\mu}\phi^{*}-ie\bar{A}^{\mu}'\phi^{*})] \quad (90)$$

The other equation is in Klein-Gordon-Maxwell theory in cosmological inertial frame,

$$\bar{\partial}^{\upsilon}'(\frac{\partial L}{\partial(\bar{\partial}^{\upsilon}'\bar{A}^{\mu}')})-\frac{\partial L}{\partial\bar{A}^{\mu}'}$$

$$=\frac{1}{4}\bar{\partial}^{\upsilon}'(\bar{\partial}_{\mu}'\bar{A}_{\upsilon}'-\bar{\partial}_{\upsilon}'\bar{A}_{\mu}')+ie\phi^{*}(\bar{\partial}_{\mu}\phi+ie\bar{A}_{\mu}'\phi)-ie\phi(\bar{\partial}_{\mu}\phi^{*}-ie\bar{A}_{\mu}'\phi^{*})$$

$$=\frac{1}{4}\bar{\partial}^{\upsilon}'\bar{F}_{\mu\upsilon}'+ie\phi^{*}(\bar{\partial}_{\mu}\phi+ie\bar{A}_{\mu}'\phi)-ie\phi(\bar{\partial}_{\mu}\phi^{*}-ie\bar{A}_{\mu}'\phi^{*})=0 \quad (91)$$

Hence,

$$\bar{\partial}^{\upsilon}'\bar{F}_{\mu\upsilon}'=-4\pi e\bar{J}_{\mu}'=-4ie[\phi^{*}(\bar{\partial}_{\mu}\phi+ie\bar{A}_{\mu}'\phi)-\phi(\bar{\partial}_{\mu}\phi^{*}-ie\bar{A}_{\mu}'\phi^{*})]$$

$$\bar{J}_{\mu}'=i\frac{1}{\pi}[\phi^{*}(\bar{\partial}_{\mu}\phi+ie\bar{A}_{\mu}'\phi)-\phi(\bar{\partial}_{\mu}\phi^{*}-ie\bar{A}_{\mu}'\phi^{*})] \quad (92)$$

9. Quantization of Klein-Gordon Scalar Field in CSTR

We make quantization of Klein-Gordon scalar field in Cosmological Special Theory of Relativity (CSTR). At first, space-time relations are in cosmological special theory of relativity (CSTR).

$$ct=\gamma(ct'+\frac{v_0}{c}\Omega(t_0)x'),\ x\Omega(t_0)=\gamma(\Omega(t_0)x'+v_0\Omega(t_0)t')$$

$$\begin{matrix}\Omega(t_0)y=\Omega(t_0)y', \\ \Omega(t_0)z=\Omega(t_0)z'\end{matrix},\ \gamma=1/\sqrt{1-\frac{v_0^2}{c^2}\Omega^2(t_0)},\ t_0 \text{ is cosmological time} \quad (93)$$

Proper time is

$$d\tau^2=dt^2-\frac{1}{c^2}\Omega^2(t_0)[dx^2+dy^2+dz^2]$$

$$=dt'^2-\frac{1}{c^2}\Omega^2(t_0)[dx'^2+dy'^2+dz'^2],\ t_0 \text{ is cosmological time} \quad (94)$$

Angular frequency-wave number relation is in CSTR.

$$\omega' = \gamma(\omega - v_0\Omega(t_0)k_1),\ \ k_1' = \gamma(k_1 - \frac{v_0}{c^2}\Omega(t_0)\omega)$$

$$k_2' = k_2, k_3' = k_3, \gamma = 1/\sqrt{1 - \frac{{v_0}^2}{c^2}\Omega^2(t_0)} \quad (95)$$

Lagrangian density of Klein-Gordon scalar field in CSTR,

$$L = -\frac{1}{2}[-(\frac{1}{c}\frac{\partial\phi}{\partial t})^2\Omega(t_0) + \frac{1}{\Omega(t_0)}\vec{\nabla}\phi\cdot\vec{\nabla}\phi - \frac{{m_0}^2c^2}{\hbar^2}\phi^2] \quad (96)$$

Hence, Euler-Lagrange equation is in CSTR,

$$\partial_\mu[\frac{\partial L}{\partial(\partial_\mu\phi)}] - \frac{\partial L}{\partial\phi} = [\Omega(t_0)\frac{1}{c^2}(\frac{\partial}{\partial t})^2 - \frac{1}{\Omega(t_0)}\nabla^2 + \frac{{m_0}^2c^2}{\hbar^2}]\phi = 0) \quad (97)$$

Hamiltonian of Klein-Gordon scalar field is in CSTR,

$$H = \frac{1}{2}[(\frac{1}{c}\frac{\partial\phi}{\partial t})^2\Omega(t_0) + \frac{1}{\Omega(t_0)}\vec{\nabla}\phi\cdot\vec{\nabla}\phi + \frac{{m_0}^2c^2}{\hbar^2}\phi^2] \quad (98)$$

The Klein-Gordon scalar field is divided by positive frequency mode and negative frequency mode.

$$\phi(x) = \phi^{(+)}(x) + \phi^{(-)}(x) \quad (99)$$

The positive frequency mode is

$$\phi^{(+)}(x) = \int\frac{d^3k}{[(2\pi)^3 2\omega_k]^{\frac{1}{2}}}a(k)f_k(x) \quad (100)$$

The negative frequency mode is

$$\phi^{(-)}(x) = \int\frac{d^3k}{[(2\pi)^3 2\omega_k]^{\frac{1}{2}}}a^{(+)}(k)f_k(x) \quad (101)$$

In this time, $f_k(x)$ is

$$f_k(x) = \frac{1}{[(2\pi)^3 2\omega_k]^{\frac{1}{2}}}\exp[i(\frac{\omega_k t}{\sqrt{\Omega(t_0)}} - \vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)})] \quad (102)$$

In this time,

$$\frac{\omega_k}{c} = (k^2 + \frac{{m_0}^2c^2}{\hbar^2})^{\frac{1}{2}} \quad (103)$$

Quantization of complex scalar field is in CSTR,

$$\phi(x) = \int \frac{d^3k}{(2\pi)^3 2\omega_k}[a(k)\exp\{i(\frac{\omega_k t}{\sqrt{\Omega(t_0)}} - \vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)})\}]$$

$$+\int \frac{d^3k}{(2\pi)^2 2\omega_k}[b^+(k)\exp\{-i(\frac{\omega_k t}{\sqrt{\Omega(t_0)}} - \vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)})\}] \quad (104)$$

$$\phi^+(x) = \int \frac{d^3k}{(2\pi)^3 2\omega_k}[b(k)\exp\{i(\frac{\omega_k t}{\sqrt{\Omega(t_0)}} - \vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)})\}]$$

$$+\int \frac{d^3k}{(2\pi)^2 2\omega_k}[a^+(k)\exp\{-i(\frac{\omega_k t}{\sqrt{\Omega(t_0)}} - \vec{k}\cdot\vec{x}\sqrt{\Omega(t_0)})\}] \quad (105)$$

Hence, Hamiltonian H is in CSTR,

$$H = \int \frac{d^3k}{(2\pi)^3 2\omega_k}[a^+(k)a(x) + b^+(k)b(k)] \quad (106)$$

In this time,

$$[a(k), a^+(k')] = (2\pi)^3 2\omega_k \delta^3(\vec{k} - \vec{k}')$$

$$[b(k), b^+(k')] = (2\pi)^3 2\omega_k \delta^3(\vec{k} - \vec{k}') \quad (107)$$

10. Conclusion

We know Maxwell equations, electromagnetic wave equations and functions in Cosmological Special Theory of Relativity. We are able to describe free particle by Klein-Gordon equation and wave function in CSTR. We solve Klein-Gordon equation in cosmological inertial frame. Hence, we found Yukawa potential dependent time in cosmological inertial frame. We found Schrodinger equation from Klein-Gordon's free particle equation in cosmological special theory of relativity. The wave function uses as a probability amplitude. We found Dirac equation from Wave Function-Type A in cosmological special theory of relativity. The wave function uses as a probability amplitude. We found equations of complex scalar fields and electromagnetic fields on interaction of complex scalar fields and electromagnetic fields in Klein-Gordon-Maxwell theory from Type A of wave function and Type B of expanded distance in cosmological inertial frame. We quantized Klein-Gordon scalar field in CSTR. We treat Lagranian density and Hamiltonian.

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