Octonions, Broctonions and Sedenions

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Abstract

There are 128 different eight-dimensional algebras that have seven Quaternion subalgebras with common construction. Sixteen of these are proper Octonion Algebras. The remaining 112 forms are one Quaternion subalgebra orientation off of a proper Octonion Algebra orientation, one may say a broken Octonion Algebra. An appropriate name for them would be "Broctonion Algebras". It is impossible to orient a Sedenion Algebra without Broctonion subalgebras, and they are the source of all Sedenion Algebra primitive zero divisors. It is possible though, to form Sedenion Algebras without a single proper Octonion subalgebra. This paper defines all Broctonion forms and explores their relationship to Sedenion Algebras.

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There are two different chiral ways to orient a Quaternion triplet multiplication rule. This gives us 2^7 = 128 different ways to orient algebras that have seven Quaternion $\mathbb{H}$ subalgebras. Sixteen of these are proper Octonion $\mathbb{O}$ algebras. We are able to derive a norm in an equivalent way for any of the 128 orientation schemes, but one requirement to be a proper Octonion Algebra is that it must be a normed composition algebra where for norm N: N(**A** * **B**) = N(**A**) N(**B**). Also, a proper Octonion orientation is not generally an associative algebra, but is an alternative algebra meaning associative for any two algebraic elements: **A** * (**A** * **B**) = (**A** * **A**) * **B**, **A** * (**B** * **A**) = (**A** * **B**) * **A**, **A** * (**B** * **B**) = (**A** * **B**) * **B**. These are tests we can apply generally to each of our 128 chiral orientation choices. Both requirements limit our choices to the same set of 16 Quaternion subalgebra orientations for proper Octonion Algebras. But what about the other 112 orientations? We cannot call them Octonion Algebras because they do not meet the two tests above, which are both required for Octonions to be a division algebra. To better understand all 128 choices, let's look at all of them together.

The 16 proper Octonion forms (References [1][2][4]) are shown next using the Quaternion triplet orientation product rule (e_a e_b e_c) specifying six basis element products $e_a * e_b = e_c$, $e_b * e_c = e_a$, $e_c * e_a = e_b$, $e_c * e_b = –e_a$, $e_b * e_a = –e_c$, $e_a * e_c = –e_b$. Cyclic shifts on this form produce identical basis element products. The second possible orientation is any permutation exchanging two basis elements e.g. (e_c e_b e_a), all of which are also cyclic shifts of each other, and produce products with signs opposite those of (e_a e_b e_c), thus called an ordered triplet negation.

Right Octonion Algebra

R0	**R1**	**R2**	**R3**	**R4**	**R5**	**R6**	**R7**
(e_1 e_2 e_3)	(e_1 e_2 e_3)	(e_1 e_2 e_3)	(e_1 e_2 e_3)	(e_3 e_2 e_1)	(e_3 e_2 e_1)	(e_3 e_2 e_1)	(e_3 e_2 e_1)
(e_7 e_6 e_1)	(e_7 e_6 e_1)	(e_1 e_6 e_7)	(e_1 e_6 e_7)	(e_1 e_6 e_7)	(e_1 e_6 e_7)	(e_7 e_6 e_1)	(e_7 e_6 e_1)
(e_5 e_7 e_2)	(e_2 e_7 e_5)	(e_5 e_7 e_2)	(e_2 e_7 e_5)	(e_2 e_7 e_5)	(e_5 e_7 e_2)	(e_2 e_7 e_5)	(e_5 e_7 e_2)
(e_6 e_5 e_3)	(e_3 e_5 e_6)	(e_3 e_5 e_6)	(e_6 e_5 e_3)	(e_3 e_5 e_6)	(e_6 e_5 e_3)	(e_6 e_5 e_3)	(e_3 e_5 e_6)
(e_5 e_4 e_1)	(e_5 e_4 e_1)	(e_1 e_4 e_5)	(e_1 e_4 e_5)	(e_5 e_4 e_1)	(e_5 e_4 e_1)	(e_1 e_4 e_5)	(e_1 e_4 e_5)
(e_6 e_4 e_2)	(e_2 e_4 e_6)	(e_6 e_4 e_2)	(e_2 e_4 e_6)	(e_6 e_4 e_2)	(e_2 e_4 e_6)	(e_6 e_4 e_2)	(e_2 e_4 e_6)
(e_7 e_4 e_3)	(e_3 e_4 e_7)	(e_3 e_4 e_7)	(e_7 e_4 e_3)	(e_7 e_4 e_3)	(e_3 e_4 e_7)	(e_3 e_4 e_7)	(e_7 e_4 e_3)

Left Octonion Algebra

L0	**L1**	**L2**	**L3**	**L4**	**L5**	**L6**	**L7**
$(e_3\ e_2\ e_1)$	$(e_3\ e_2\ e_1)$	$(e_3\ e_2\ e_1)$	$(e_3\ e_2\ e_1)$	$(e_1\ e_2\ e_3)$	$(e_1\ e_2\ e_3)$	$(e_1\ e_2\ e_3)$	$(e_1\ e_2\ e_3)$
$(e_1\ e_6\ e_7)$	$(e_1\ e_6\ e_7)$	$(e_7\ e_6\ e_1)$	$(e_7\ e_6\ e_1)$	$(e_7\ e_6\ e_1)$	$(e_7\ e_6\ e_1)$	$(e_1\ e_6\ e_7)$	$(e_1\ e_6\ e_7)$
$(e_2\ e_7\ e_5)$	$(e_5\ e_7\ e_2)$	$(e_2\ e_7\ e_5)$	$(e_5\ e_7\ e_2)$	$(e_5\ e_7\ e_2)$	$(e_2\ e_7\ e_5)$	$(e_5\ e_7\ e_2)$	$(e_2\ e_7\ e_5)$
$(e_3\ e_5\ e_6)$	$(e_6\ e_5\ e_3)$	$(e_6\ e_5\ e_3)$	$(e_3\ e_5\ e_6)$	$(e_6\ e_5\ e_3)$	$(e_3\ e_5\ e_6)$	$(e_3\ e_5\ e_6)$	$(e_6\ e_5\ e_3)$
$(e_1\ e_4\ e_5)$	$(e_1\ e_4\ e_5)$	$(e_5\ e_4\ e_1)$	$(e_5\ e_4\ e_1)$	$(e_1\ e_4\ e_5)$	$(e_1\ e_4\ e_5)$	$(e_5\ e_4\ e_1)$	$(e_5\ e_4\ e_1)$
$(e_2\ e_4\ e_6)$	$(e_6\ e_4\ e_2)$	$(e_2\ e_4\ e_6)$	$(e_6\ e_4\ e_2)$	$(e_2\ e_4\ e_6)$	$(e_6\ e_4\ e_2)$	$(e_2\ e_4\ e_6)$	$(e_6\ e_4\ e_2)$
$(e_3\ e_4\ e_7)$	$(e_7\ e_4\ e_3)$	$(e_7\ e_4\ e_3)$	$(e_3\ e_4\ e_7)$	$(e_3\ e_4\ e_7)$	$(e_7\ e_4\ e_3)$	$(e_7\ e_4\ e_3)$	$(e_3\ e_4\ e_7)$

The following are the optimal enumerations (Reference [4]) for the seven unspecified orientation Quaternion subalgebra triplets:

Triplet[1] = $\{e_2\ e_4\ e_6\}$ Triplet[2] = $\{e_1\ e_4\ e_5\}$ Triplet[3] = $\{e_3\ e_4\ e_7\}$
Triplet[4] = $\{e_1\ e_2\ e_3\}$ Triplet[5] = $\{e_2\ e_5\ e_7\}$ Triplet[6] = $\{e_1\ e_6\ e_7\}$
Triplet[7] = $\{e_3\ e_5\ e_6\}$

We can cover our two orientation choices for each of these with a single binary bit, where 1 represents negating Triplet[m] orientation shown in some select reference proper Octonion Algebra, and 0 represents staying with its select algebra orientation. We can then span all 128 orientations with a single binary integer n over the range 0 to 127, where each of its seven bits m tells us whether or not to negate the orientation of Triplet[m]. Choose for the reference standard Octonion orientation **R0** as defined above, and partition the bit number m in each binary integer n sequentially left to right 1 to 7. Doing so, we can determine how close each of these come to our 16 proper Octonion forms specified above. The results are itemized in the following table.

n	Binary n	Closest match	Base Algebra	Triplet in error	n	Binary n	Closest match	Base Algebra	Triplet in error
0	[0000000]	[0000000]	R0	-	64	[0000001]	[0000000]	R0	7
1	[1000000]	[0000000]	R0	1	65	[1000001]	[1000011]	L6	6
2	[0100000]	[0000000]	R0	2	66	[0100001]	[0100101]	L5	5
3	[1100000]	[1110000]	L4	3	67	[1100001]	[1101001]	R7	4
4	[0010000]	[0000000]	R0	3	68	[0010001]	[0011001]	L3	4
5	[1010000]	[1110000]	L4	2	69	[1010001]	[1010101]	R1	5
6	[0110000]	[1110000]	L4	1	70	[0110001]	[0110011]	R2	6
7	[1110000]	[1110000]	L4	-	71	[1110001]	[1110000]	L4	7
8	[0001000]	[0000000]	R0	4	72	[0001001]	[0011001]	L3	3
9	[1001000]	[1001100]	L2	5	73	[1001001]	[1101001]	R7	2
10	[0101000]	[0101010]	L1	6	74	[0101001]	[1101001]	R7	1
11	[1101000]	[1101001]	R7	7	75	[1101001]	[1101001]	R7	-
12	[0011000]	[0011001]	L3	7	76	[0011001]	[0011001]	L3	-
13	[1011000]	[1011010]	R5	6	77	[1011001]	[0011001]	L3	1
14	[0111000]	[0111100]	R6	5	78	[0111001]	[0011001]	L3	2
15	[1111000]	[1110000]	L4	4	79	[1111001]	[1101001]	R7	3
16	[0000100]	[0000000]	R0	5	80	[0000101]	[0100101]	L5	2
17	[1000100]	[1001100]	L2	4	81	[1000101]	[1010101]	R1	3
18	[0100100]	[0100101]	L5	7	82	[0100101]	[0100101]	L5	-

n	Binary n	Closest match	Base Algebra	Triplet in error	n	Binary n	Closest match	Base Algebra	Triplet in error
19	[1100100]	[1100110]	R3	6	83	[1100101]	[0100101]	L5	1
20	[0010100]	[0010110]	L7	6	84	[0010101]	[1010101]	R1	1
21	[1010100]	[1010101]	R1	7	85	[1010101]	[1010101]	R1	-
22	[0110100]	[0111100]	R6	4	86	[0110101]	[0100101]	L5	3
23	[1110100]	[1110000]	L4	5	87	[1110101]	[1010101]	R1	2
24	[0001100]	[1001100]	L2	1	88	[0001101]	[0001111]	R4	6
25	[1001100]	[1001100]	L2	-	89	[1001101]	[1001100]	L2	7
26	[0101100]	[0111100]	R6	3	90	[0101101]	[0100101]	L5	4
27	[1101100]	[1001100]	L2	2	91	[1101101]	[1101001]	R7	5
28	[0011100]	[0111100]	R6	2	92	[0011101]	[0011001]	L3	5
29	[1011100]	[1001100]	L2	3	93	[1011101]	[1010101]	R1	4
30	[0111100]	[0111100]	R6	-	94	[0111101]	[0111100]	R6	7
31	[1111100]	[0111100]	R6	1	95	[1111101]	[1111111]	L0	6
32	[0000010]	[0000000]	R0	6	96	[0000011]	[1000011]	L6	1
33	[1000010]	[1000011]	L6	7	97	[1000011]	[1000011]	L6	-
34	[0100010]	[0101010]	L1	4	98	[0100011]	[0110011]	R2	3
35	[1100010]	[1100110]	R3	5	99	[1100011]	[1000011]	L6	2
36	[0010010]	[0010110]	L7	5	100	[0010011]	[0110011]	R2	2
37	[1010010]	[1011010]	R5	4	101	[1010011]	[1000011]	L6	3
38	[0110010]	[0110011]	R2	7	102	[0110011]	[0110011]	R2	-
39	[1110010]	[1110000]	L4	6	103	[1110011]	[0110011]	R2	1
40	[0001010]	[0101010]	L1	2	104	[0001011]	[0001111]	R4	5
41	[1001010]	[1011010]	R5	3	105	[1001011]	[1000011]	L6	4
42	[0101010]	[0101010]	L1	-	106	[0101011]	[0101010]	L1	7
43	[1101010]	[0101010]	L1	1	107	[1101011]	[1101001]	R7	6
44	[0011010]	[1011010]	R5	1	108	[0011011]	[0011001]	L3	6
45	[1011010]	[1011010]	R5	-	109	[1011011]	[1011010]	R5	7
46	[0111010]	[0101010]	L1	3	110	[0111011]	[0110011]	R2	4
47	[1111010]	[1011010]	R5	2	111	[1111011]	[1111111]	L0	5
48	[0000110]	[0010110]	L7	3	112	[0000111]	[0001111]	R4	4
49	[1000110]	[1100110]	R3	2	113	[1000111]	[1000011]	L6	5
50	[0100110]	[1100110]	R3	1	114	[0100111]	[0100101]	L5	6
51	[1100110]	[1100110]	R3	-	115	[1100111]	[1100110]	R3	7
52	[0010110]	[0010110]	L7	-	116	[0010111]	[0010110]	L7	7
53	[1010110]	[0010110]	L7	1	117	[1010111]	[1010101]	R1	6
54	[0110110]	[0010110]	L7	2	118	[0110111]	[0110011]	R2	5
55	[1110110]	[1100110]	R3	3	119	[1110111]	[1111111]	L0	4
56	[0001110]	[0001111]	R4	7	120	[0001111]	[0001111]	R4	-
57	[1001110]	[1001100]	L2	6	121	[1001111]	[0001111]	R4	1
58	[0101110]	[0101010]	L1	5	122	[0101111]	[0001111]	R4	2
59	[1101110]	[1100110]	R3	4	123	[1101111]	[1111111]	L0	3
60	[0011110]	[0010110]	L7	4	124	[0011111]	[0001111]	R4	3
61	[1011110]	[1011010]	R5	5	125	[1011111]	[1111111]	L0	2
62	[0111110]	[0111100]	R6	6	126	[0111111]	[1111111]	L0	1
63	[1111110]	[1111111]	L0	7	127	[1111111]	[1111111]	L0	-

We find all 128 possible choices for orienting the seven Quaternion subalgebra triplets produce the 16 proper Octonion Algebras, and 16*7 = 112 not proper Octonion Algebras where each of the 16 proper orientation *base Octonion Algebras* show up seven times, once for each of its seven triplets that in turn are the negation of their proper Octonion orientation. Define then *broken Octonion Algebras (Broctonion Algebras)* as eight dimensional algebras that have seven Quaternion subalgebras, one of which has an orientation negated from that of some proper Octonion Algebra. These algebras are not division algebras, since zero divisors can be produced. We can define a specific Broctonion algebra by referring to its associated proper base Octonion Algebra - its indicated improper triplet, e.g. **R0**-1.

When the Cayley-Dickson doubling process is applied to Octonion Algebra, Sedenion Algebra is produced. Sedenions have 15 separate Octonion subalgebra candidates, but only the original doubled Octonion Algebra and seven more subalgebras that share a common new basis element are proper Octonion Algebras when the doubling process is used. The remaining seven Octonion subalgebra candidates are Broctonion Algebras. The triplet in conflict with proper Octonion orientation for each of these Broctonion subalgebras is its single Quaternion subalgebra triplet intersection with the original doubled proper Octonion Algebra. This is the largest number of proper Octonion subalgebras possible for Sedenion Algebras, the maximal Octonion orientation. It is however possible to build Sedenion orientations that have no proper Octonion subalgebras as we shall see below.

We can generate Broctonion Algebra orientations with a modification to the two required forms of Directed Fano Planes (References [1][2][4]) by flipping the direction of just the triangle side mid-point connections. This will not modify any of the other six triplet orientations. We can likewise represent Broctonions with a modification to Left and Right Ordered 9-tuple (Reference [2]) representations by keeping all rules for building the triplets from the structure, but replacing the ↓ with ↑ and orienting the cardinal triplet appropriately. Itemizing Ordered 9-tuples we have

Right Octonion Ordered 9-tuple cardinal basis element e_d, cardinal oriented triplet $(e_a\ e_b\ e_c)$

$(e_e\ e_d\ e_a)$
$(e_f\ e_d\ e_b)\downarrow$ Orientations $(e_a\ e_b\ e_c)$, $(e_e\ e_d\ e_a)$, $(e_f\ e_d\ e_b)$, $(e_g\ e_d\ e_c)$, $(e_g\ e_f\ e_a)$, $(e_f\ e_e\ e_c)$, $(e_e\ e_g\ e_b)$
$(e_g\ e_d\ e_c)$

Left Octonion Ordered 9-tuple cardinal basis element e_d, cardinal oriented triplet $(e_c\ e_b\ e_a)$

$(e_c\ e_d\ e_g)$
$\downarrow(e_b\ e_d\ e_f)$ Orientations $(e_c\ e_b\ e_a)$, $(e_c\ e_d\ e_g)$, $(e_b\ e_d\ e_f)$, $(e_a\ e_d\ e_e)$, $(e_e\ e_f\ e_c)$, $(e_f\ e_g\ e_a)$, $(e_g\ e_e\ e_b)$
$(e_a\ e_d\ e_e)$

Right Broctonion Ordered 9-tuple cardinal basis element e_d, cardinal oriented triplet $(e_c\ e_b\ e_a)$

$(e_e\ e_d\ e_a)$
$(e_f\ e_d\ e_b)\uparrow$ Orientations $(e_c\ e_b\ e_a)$, $(e_e\ e_d\ e_a)$, $(e_f\ e_d\ e_b)$, $(e_g\ e_d\ e_c)$, $(e_g\ e_f\ e_a)$, $(e_f\ e_e\ e_c)$, $(e_e\ e_g\ e_b)$
$(e_g\ e_d\ e_c)$

Left Broctonion Ordered 9-tuple cardinal basis element e_d, cardinal oriented triplet $(e_a\ e_b\ e_c)$

$(e_c\ e_d\ e_g)$
$\uparrow(e_b\ e_d\ e_f)$ Orientations $(e_a\ e_b\ e_c)$, $(e_c\ e_d\ e_g)$, $(e_b\ e_d\ e_f)$, $(e_a\ e_d\ e_e)$, $(e_e\ e_f\ e_c)$, $(e_f\ e_g\ e_a)$, $(e_g\ e_e\ e_b)$
$(e_a\ e_d\ e_e)$

Sedenion zero divisors were discussed in detail within Reference [2]. There, we found all primitive zero divisors are constructed within broken Octonion subalgebra candidates, built around their mis-oriented Quaternion subalgebras and their basic quad basis element set. These are best understood and appreciated using the algebraic invariant forms for the 24 resultant primitive zero divisors for a given broken (Broctonion) subalgebra, as follows

$[e_a+e_a*e_d] * [e_b - e_b*e_d] = 0$, $[e_b+e_b*e_d] * [e_c - e_c*e_d] = 0$, $[e_c+e_c*e_d] * [e_a - e_a*e_d] = 0$,
$[e_a - e_a*e_d] * [e_b+e_b*e_d] = 0$, $[e_b - e_b*e_d] * [e_c+e_c*e_d] = 0$, $[e_c - e_c*e_d] * [e_a+e_a*e_d] = 0$,

$[e_a+e_a*e_e] * [e_b - e_b*e_e] = 0$, $[e_b+e_b*e_e] * [e_c - e_c*e_e] = 0$, $[e_c+e_c*e_e] * [e_a - e_a*e_e] = 0$,
$[e_a - e_a*e_e] * [e_b+e_b*e_e] = 0$, $[e_b - e_b*e_e] * [e_c+e_c*e_e] = 0$, $[e_c - e_c*e_e] * [e_a+e_a*e_e] = 0$,

$[e_a+e_a*e_f] * [e_b - e_b*e_f] = 0$, $[e_b+e_b*e_f] * [e_c - e_c*e_f] = 0$, $[e_c+e_c*e_f] * [e_a - e_a*e_f] = 0$,
$[e_a - e_a*e_f] * [e_b+e_b*e_f] = 0$, $[e_b - e_b*e_f] * [e_c+e_c*e_f] = 0$, $[e_c - e_c*e_f] * [e_a+e_a*e_f] = 0$,

$[e_a+e_a*e_g] * [e_b - e_b*e_g] = 0$, $[e_b+e_b*e_g] * [e_c - e_c*e_g] = 0$, $[e_c+e_c*e_g] * [e_a - e_a*e_g] = 0$,
$[e_a - e_a*e_g] * [e_b+e_b* e_g] = 0$, $[e_b - e_b*e_g] * [e_c+e_c*e_g] = 0$, $[e_c - e_c*e_g] * [e_a+e_a*e_g] = 0$

Here, the conflict Quaternion triplet is $\{e_a\ e_b\ e_c\}$ and its basic quad is the set $\{e_d\ e_e\ e_f\ e_g\}$. This is also the case in both Broctonion Ordered 9-tuples defined above.

Sedenion Algebras can be very easily built without referring to the Cayley-Dickson doubling product structure, by using the Ordered 9-tuples shown above as follows. Define the first Octonion form by choosing a 9-tuple cardinal triplet and a cardinal basis element. Either the Left or Right Octonion Ordered 9-tuple can be used to build out the first of 15 Octonion subalgebra candidates as proper Octonion. It is unnecessary but more straightforward to limit the basis set for this first one to e_0 to e_7. Then one of e_8 through e_{15} could be used for the cardinal basis element, with cardinal triplets in turn each of our seven now oriented Quaternion subalgebras from our first Octonion form. This approach will always build seven additional proper Octonion subalgebras, yielding the maximal count of proper Octonion subalgebras: one free choice Octonion and seven more sharing a basis element not in the first. At this point, all 35 Sedenion Algebra Quaternion subalgebra triplet orientations are set, meaning the orientations of the remaining seven Octonion subalgebra candidates are determined. Every one of them will be a Broctonion Algebra, independent of any previous proper Octonion orientation choices.

If we were to build the original first Octonion candidate using a Broctonion Algebra 9-tuple, then use Broctonion 9-tuples for the common basis element set of seven, once again all 35 Quaternion subalgebras will be oriented. Just as with the eight proper Octonion maximal case, the remaining Octonion candidates will still be Broctonion Algebras, leaving all 15 Octonion subalgebra candidates oriented as Broctonion Algebras.

The naming enumerations for the eight Right and eight Left Octonion Algebras are straight-forward for the basis set e_0 through e_7 as defined in References [1][2], but we can also specify particular Right and Left enumerations for the remaining fourteen Sedenion subalgebra candidates. We just need a method to map their seven non-scalar basis elements to the standard e_1 through e_7 set, then use the standard Octonion Algebra naming rules. The options for doing this are specified by the Exclusive-Or Group covered in detail within Reference [3].

Notice first that all fourteen candidates we wish to enumerate share a Quaternion subalgebra with our initial standard Octonion subalgebra. We can use these as a starting point. In Reference [3], it was shown that every Quaternion subalgebra correspondence subgroup is a normal subgroup of the order 16 exclusive-or group corresponding to Sedenion Algebra. Each of these may be used as the kernel for a quotient group. The three additional cosets going with the kernel in these quotient groups are the indexes for the basic quads for our three appearances for the kernel triplet in our 15 Octonion subalgebra candidates. One of these is for our standard Octonion first assignment, and the other two are for the two candidates we now wish to name. Let's work out one example, the two sharing the Quaternion subalgebra triplet index set {2, 6, 7}. The quotient group members representing this subalgebra are the cosets using the basis element indexes in

[e_0 e_2 e_4 e_6], [e_1 e_3 e_5 e_7], [e_8 e_{10} e_{12} e_{14}], [e_9 e_{11} e_{13} e_{15}]

Our three Octonion subalgebra candidates sharing [e_0 e_2 e_4 e_6] are built then from the basis sets

[e_0 e_2 e_4 e_6], [e_1 e_3 e_5 e_7]
[e_0 e_2 e_4 e_6], [e_8 e_{10} e_{12} e_{14}]
[e_0 e_2 e_4 e_6], [e_9 e_{11} e_{13} e_{15}]

If we have a one-to-one map from [e_8 e_{10} e_{12} e_{14}] to [e_1 e_3 e_5 e_7], and from [e_9 e_{11} e_{13} e_{15}] to [e_1 e_3 e_5 e_7], we can map the two Octonion candidates we wish to name to our standard Octonion form we have a naming method for, without changing any of the intrinsic structure of the "from" algebra. Familiarity with cosets and this quotient group structure tells us for our exclusive-or group we can exclusive-or each of the indexes in [e_8 e_{10} e_{12} e_{14}] with any one of the indexes in [e_9 e_{11} e_{13} e_{15}] and produce the standard Octonion coset [e_1 e_3 e_5 e_7]. Likewise taking the exclusive-or of any one of the indexes in [e_8 e_{10} e_{12} e_{14}] on the coset [e_9 e_{11} e_{13} e_{15}] forms the coset [e_1 e_3 e_5 e_7]. So, this is our mapping scheme prior to naming. The four index choices to exclusive-or with form four different results, giving four different names, our naming scheme can't give unique names. This is not because the four mapping schemes changed the structure, it is because there are four separate ways to map the seven consistent Quaternion subalgebra orientations, injected into the standard Octonion form subalgebras, and the naming convention is entirely based on their relative orientations.

There is nothing better than demonstrated examples, first the maximal proper Octonion subalgebras. Enumerate the 15 Octonion subalgebra candidates {basic quad : Quaternion triplet} forms as

C0 = [e_7 e_1 e_3 e_5 : e_6 e_4 e_2}

C1 = [e_8 e_{14} e_{12} e_{10} : e_6 e_4 e_2} C2 = [e_8 e_{13} e_{12} e_9 : e_5 e_4 e_1} C3 = [e_8 e_{15} e_{12} e_{11} : e_7 e_4 e_3}
C4 = [e_8 e_9 e_{10} e_{11} : e_1 e_2 e_3} C5 = [e_8 e_{13} e_{15} e_{10} : e_5 e_7 e_2} C6 = [e_8 e_{15} e_{14} e_9 : e_7 e_6 e_1}
C7 = [e_8 e_{14} e_{13} e_{11} : e_6 e_5 e_3}

C8 = [e_9 e_{15} e_{13} e_{11} : e_6 e_4 e_2} C9 = [e_{10} e_{15} e_{14} e_{11} : e_5 e_4 e_1} C10 = [e_9 e_{14} e_{13} e_{10} : e_7 e_4 e_3}
C11 = [e_{12} e_{13} e_{14} e_{15} : e_1 e_2 e_3} C12 = [e_9 e_{12} e_{14} e_{11} : e_5 e_7 e_2} C13 = [e_{10} e_{13} e_{12} e_{11} : e_7 e_6 e_1}
C14 = [e_9 e_{15} e_{12} e_{10} : e_6 e_5 e_3}

For the first proper Octonion subalgebra for Sedenion Algebra make the free choice of **R0** using C0 with triplet orientation (e_6 e_4 e_2) for the cardinal triplet and e_7 for the cardinal basis element. The Right Octonion Ordered 9-tuple is then

(e_1 e_7 e_6)
(e_3 e_7 e_4)↓ Orientation (e_6 e_4 e_2), (e_1 e_7 e_6), (e_3 e_7 e_4), (e_5 e_7 e_2), (e_5 e_3 e_6), (e_3 e_1 e_2), (e_1 e_5 e_4)
(e_5 e_7 e_2)

These orientations are equivalent cyclic shifts on the **R0** definition given above. Next do Right

Octonion 9-tuples using each of the seven now oriented triplets for cardinal triplets, using C1 through C7 with e_8 for the cardinal basis element. We get the following:

$(e_{14}\ e_8\ e_6)$
$(e_{12}\ e_8\ e_4)\downarrow$ Orientation $(e_6\ e_4\ e_2)$, $(e_{14}\ e_8\ e_6)$, $(e_{12}\ e_8\ e_4)$, $(e_{10}\ e_8\ e_2)$, $(e_{10}\ e_{12}\ e_6)$, $(e_{12}\ e_{14}\ e_2)$, $(e_{14}\ e_{10}\ e_4)$
$(e_{10}\ e_8\ e_2)$

$(e_9\ e_8\ e_1)$
$(e_{13}\ e_8\ e_5)\downarrow$ Orientation $(e_1\ e_5\ e_4)$, $(e_9\ e_8\ e_1)$, $(e_{13}\ e_8\ e_5)$, $(e_{12}\ e_8\ e_4)$, $(e_{12}\ e_{13}\ e_1)$, $(e_{13}\ e_9\ e_4)$, $(e_9\ e_{12}\ e_5)$
$(e_{12}\ e_8\ e_4)$

$(e_{15}\ e_8\ e_7)$
$(e_{12}\ e_8\ e_4)\downarrow$ Orientation $(e_7\ e_4\ e_3)$, $(e_{15}\ e_8\ e_7)$, $(e_{12}\ e_8\ e_4)$, $(e_{11}\ e_8\ e_3)$, $(e_{11}\ e_{12}\ e_7)$, $(e_{12}\ e_{15}\ e_3)$, $(e_{15}\ e_{11}\ e_4)$
$(e_{11}\ e_8\ e_3)$

$(e_{11}\ e_8\ e_3)$
$(e_9\ e_8\ e_1)\downarrow$ Orientation $(e_3\ e_1\ e_2)$, $(e_{11}\ e_8\ e_3)$, $(e_9\ e_8\ e_1)$, $(e_{10}\ e_8\ e_2)$, $(e_{10}\ e_9\ e_3)$, $(e_9\ e_{11}\ e_2)$, $(e_{11}\ e_{10}\ e_1)$
$(e_{10}\ e_8\ e_2)$

$(e_{13}\ e_8\ e_5)$
$(e_{15}\ e_8\ e_7)\downarrow$ Orientation $(e_5\ e_7\ e_2)$, $(e_{13}\ e_8\ e_5)$, $(e_{15}\ e_8\ e_7)$, $(e_{10}\ e_8\ e_2)$, $(e_{10}\ e_{15}\ e_5)$, $(e_{15}\ e_{13}\ e_2)$, $(e_{13}\ e_{10}\ e_7)$
$(e_{10}\ e_8\ e_2)$

$(e_9\ e_8\ e_1)$
$(e_{15}\ e_8\ e_7)\downarrow$ Orientation $(e_1\ e_7\ e_6)$, $(e_9\ e_8\ e_1)$, $(e_{15}\ e_8\ e_7)$, $(e_{14}\ e_8\ e_6)$, $(e_{14}\ e_{15}\ e_1)$, $(e_{15}\ e_9\ e_6)$, $(e_9\ e_{14}\ e_7)$
$(e_{14}\ e_8\ e_6)$

$(e_{13}\ e_8\ e_5)$
$(e_{11}\ e_8\ e_3)\downarrow$ Orientation $(e_5\ e_3\ e_6)$, $(e_{13}\ e_8\ e_5)$, $(e_{11}\ e_8\ e_3)$, $(e_{14}\ e_8\ e_6)$, $(e_{14}\ e_{11}\ e_5)$, $(e_{11}\ e_{13}\ e_6)$, $(e_{13}\ e_{14}\ e_3)$
$(e_{14}\ e_8\ e_6)$

All 35 Quaternion subalgebra triplet orientations are now defined, so the orientations for C8 through C14 are determined. This was accomplished with none of the duplications having orientation conflicts. Collecting all we have:

C0:	$(e_6\ e_4\ e_2)$, $(e_1\ e_5\ e_4)$, $(e_3\ e_7\ e_4)$, $(e_3\ e_1\ e_2)$, $(e_5\ e_7\ e_2)$, $(e_1\ e_7\ e_6)$, $(e_5\ e_3\ e_6)$	**R0**-0 (-)
C1:	$(e_6\ e_4\ e_2)$, $(e_{14}\ e_{10}\ e_4)$, $(e_{12}\ e_8\ e_4)$, $(e_{12}\ e_{14}\ e_2)$, $(e_{10}\ e_8\ e_2)$, $(e_{14}\ e_8\ e_6)$, $(e_{10}\ e_{12}\ e_6)$	**R0**-0 (^15)
C2:	$(e_{12}\ e_8\ e_4)$, $(e_1\ e_5\ e_4)$, $(e_{13}\ e_9\ e_4)$, $(e_{12}\ e_{13}\ e_1)$, $(e_9\ e_{12}\ e_5)$, $(e_9\ e_8\ e_1)$, $(e_{13}\ e_8\ e_5)$	**R0**-0 (^14)
C3:	$(e_{15}\ e_{11}\ e_4)$, $(e_{12}\ e_8\ e_4)$, $(e_3\ e_7\ e_4)$, $(e_{12}\ e_{15}\ e_3)$, $(e_{15}\ e_8\ e_7)$, $(e_{11}\ e_{12}\ e_7)$, $(e_{11}\ e_8\ e_3)$	**R0**-0 (^13)
C4:	$(e_{10}\ e_8\ e_2)$, $(e_9\ e_8\ e_1)$, $(e_{11}\ e_8\ e_3)$, $(e_3\ e_1\ e_2)$, $(e_9\ e_{11}\ e_2)$, $(e_{11}\ e_{10}\ e_1)$, $(e_{10}\ e_9\ e_3)$	**R0**-0 (^12)
C5:	$(e_{15}\ e_{13}\ e_2)$, $(e_{13}\ e_8\ e_5)$, $(e_{13}\ e_{10}\ e_7)$, $(e_{10}\ e_8\ e_2)$, $(e_5\ e_7\ e_2)$, $(e_{15}\ e_8\ e_7)$, $(e_{10}\ e_{15}\ e_5)$,	**R0**-0 (^9)
C6:	$(e_{15}\ e_9\ e_6)$, $(e_{14}\ e_{15}\ e_1)$, $(e_{15}\ e_8\ e_7)$, $(e_9\ e_8\ e_1)$, $(e_9\ e_{14}\ e_7)$, $(e_1\ e_7\ e_6)$, $(e_{14}\ e_8\ e_6)$	**R0**-0 (^11)
C7:	$(e_{14}\ e_8\ e_6)$, $(e_{14}\ e_{11}\ e_5)$, $(e_{13}\ e_{14}\ e_3)$, $(e_{11}\ e_8\ e_3)$, $(e_{13}\ e_8\ e_5)$, $(e_{11}\ e_{13}\ e_6)$, $(e_5\ e_3\ e_6)$	**R0**-0 (^10)
C8:	$(e_6\ e_4\ e_2)$, $(e_{13}\ e_9\ e_4)$, $(e_{15}\ e_{11}\ e_4)$, $(e_9\ e_{11}\ e_2)$, $(e_{15}\ e_{13}\ e_2)$, $(e_{15}\ e_9\ e_6)$, $(e_{11}\ e_{13}\ e_6)$	**L0**-1 (^8)
C9:	$(e_{14}\ e_{10}\ e_4)$, $(e_1\ e_5\ e_4)$, $(e_{15}\ e_{11}\ e_4)$, $(e_{11}\ e_{10}\ e_1)$, $(e_{10}\ e_{15}\ e_5)$, $(e_{14}\ e_{15}\ e_1)$, $(e_{14}\ e_{11}\ e_5)$	**L0**-2 (^8)
C10:	$(e_{14}\ e_{10}\ e_4)$, $(e_{13}\ e_9\ e_4)$, $(e_3\ e_7\ e_4)$, $(e_{10}\ e_9\ e_3)$, $(e_{13}\ e_{10}\ e_7)$, $(e_9\ e_{14}\ e_7)$, $(e_{13}\ e_{14}\ e_3)$	**L0**-3 (^8)
C11:	$(e_{12}\ e_{14}\ e_2)$, $(e_{12}\ e_{13}\ e_1)$, $(e_{12}\ e_{15}\ e_3)$, $(e_3\ e_1\ e_2)$, $(e_{15}\ e_{13}\ e_2)$, $(e_{14}\ e_{15}\ e_1)$, $(e_{13}\ e_{14}\ e_3)$	**L0**-4 (^8)
C12:	$(e_{12}\ e_{14}\ e_2)$, $(e_9\ e_{12}\ e_5)$, $(e_{11}\ e_{12}\ e_7)$, $(e_9\ e_{11}\ e_2)$, $(e_5\ e_7\ e_2)$, $(e_9\ e_{14}\ e_7)$, $(e_{14}\ e_{11}\ e_5)$	**L0**-5 (^8)

C13: $(e_{10}\ e_{12}\ e_6)$, $(e_{12}\ e_{13}\ e_1)$, $(e_{11}\ e_{12}\ e_7)$, $(e_{11}\ e_{10}\ e_1)$, $(e_{13}\ e_{10}\ e_7)$, $(e_1\ e_7\ e_6)$, $(e_{11}\ e_{13}\ e_6)$ **L0**-6 (^8)
C14: $(e_{10}\ e_{12}\ e_6)$, $(e_9\ e_{12}\ e_5)$, $(e_{12}\ e_{15}\ e_3)$, $(e_{10}\ e_9\ e_3)$, $(e_{10}\ e_{15}\ e_5)$, $(e_{15}\ e_9\ e_6)$, $(e_5\ e_3\ e_6)$ **L0**-7 (^8)

One of four choices of Octonion orientation names for C1 through C7 are given, and since all are proper Octonion, no triplet negations are required, hence **R0**-0. The coset exclusive-or (xor operator ^) value used for the naming map is shown inside the (). C8 through C14 are Broctonion Algebras named with their base Octonion and its negated triplet number as defined above. For an example, do C14. The triplet in C14 also present in our first assignment C0 is $(e_5\ e_3\ e_6)$, which has for its basic quad $\{e_9\ e_{10}\ e_{12}\ e_{15}\}$. Taking the exclusive-or of each index with 8 forms the one-to-one positional replacement map $\{e_9\ e_{10}\ e_{12}\ e_{15}\} \rightarrow \{e_1\ e_2\ e_4\ e_7\}$. Doing the replacements, we have the mapped C14 as follows

C14: $(e_2\ e_4\ e_6)$, $(e_2\ e_7\ e_5)$, $(e_2\ e_1\ e_3)$, $(e_4\ e_7\ e_3)$, $(e_1\ e_4\ e_5)$, $(e_7\ e_1\ e_6)$, $(e_5\ e_3\ e_6)$

If we changed the seventh triplet $(e_5\ e_3\ e_6)$ to $(e_6\ e_3\ e_5)$, the mapped C14 would indeed be **L0**. Hence the name **L0**-7.

Now let's do a maximal Broctonion subalgebra orientation for Sedenion Algebra. Start again with C0 but now orient it with the Right Broctonion Ordered 9-tuple. Reversing the orientation of the cardinal triplet we used above to $(e_2\ e_4\ e_6)$ and using once more e_7 for the cardinal basis element, we build a C0 representation that differs from the proper **R0** form only by the orientation $(e_2\ e_4\ e_6)$. We have now

C0: $(e_2\ e_4\ e_6)$, $(e_1\ e_5\ e_4)$, $(e_3\ e_7\ e_4)$, $(e_3\ e_1\ e_2)$, $(e_5\ e_7\ e_2)$, $(e_1\ e_7\ e_6)$, $(e_5\ e_3\ e_6)$ **R0**-1 (-)

Build out now the seven subalgebra candidates sharing e_8 as above, but this time using the Right Broctonion Ordered 9-tuple. The resultant orientations are the following

C1: $(e_2\ e_4\ e_6)$, $(e_{14}\ e_{10}\ e_4)$, $(e_{12}\ e_8\ e_4)$, $(e_{12}\ e_{14}\ e_2)$, $(e_{10}\ e_8\ e_2)$, $(e_{14}\ e_8\ e_6)$, $(e_{10}\ e_{12}\ e_6)$ **R0**-1 (^15)
C2: $(e_{12}\ e_8\ e_4)$, $(e_1\ e_5\ e_4)$, $(e_9\ e_{13}\ e_4)$, $(e_{13}\ e_{12}\ e_1)$, $(e_{12}\ e_9\ e_5)$, $(e_9\ e_8\ e_1)$, $(e_{13}\ e_8\ e_5)$ **R6**-2 (^14)
C3: $(e_{11}\ e_{15}\ e_4)$, $(e_{12}\ e_8\ e_4)$, $(e_3\ e_7\ e_4)$, $(e_{15}\ e_{12}\ e_3)$, $(e_{15}\ e_8\ e_7)$, $(e_{12}\ e_{11}\ e_7)$, $(e_{11}\ e_8\ e_3)$ **R5**-3 (^13)
C4: $(e_{10}\ e_8\ e_2)$, $(e_9\ e_8\ e_1)$, $(e_{11}\ e_8\ e_3)$, $(e_3\ e_1\ e_2)$, $(e_{11}\ e_9\ e_2)$, $(e_{10}\ e_{11}\ e_1)$, $(e_9\ e_{10}\ e_3)$ **R4**-4 (^12)
C5: $(e_{13}\ e_{15}\ e_2)$, $(e_{13}\ e_8\ e_5)$, $(e_{10}\ e_{13}\ e_7)$, $(e_{10}\ e_8\ e_2)$, $(e_5\ e_7\ e_2)$, $(e_{15}\ e_8\ e_7)$, $(e_{15}\ e_{10}\ e_5)$ **R1**-5 (^9)
C6: $(e_9\ e_{15}\ e_6)$, $(e_{15}\ e_{14}\ e_1)$, $(e_{15}\ e_8\ e_7)$, $(e_9\ e_8\ e_1)$, $(e_{14}\ e_9\ e_7)$, $(e_1\ e_7\ e_6)$, $(e_{14}\ e_8\ e_6)$ **R3**-6 (^11)
C7: $(e_{14}\ e_8\ e_6)$, $(e_{11}\ e_{14}\ e_5)$, $(e_{14}\ e_{13}\ e_3)$, $(e_{11}\ e_8\ e_3)$, $(e_{13}\ e_8\ e_5)$, $(e_{13}\ e_{11}\ e_6)$, $(e_5\ e_3\ e_6)$ **R2**-7 (^10)

Once again, all 35 Quaternion subalgebra triplet orientations have been built with none of the duplications showing orientation conflicts. The remaining C8 through C14 orientations are set, and the results are

C8: $(e_2\ e_4\ e_6)$, $(e_9\ e_{13}\ e_4)$, $(e_{11}\ e_{15}\ e_4)$, $(e_{11}\ e_9\ e_2)$, $(e_{13}\ e_{15}\ e_2)$, $(e_9\ e_{15}\ e_6)$, $(e_{13}\ e_{11}\ e_6)$ **R0**-1 (^8)
C9: $(e_{14}\ e_{10}\ e_4)$, $(e_1\ e_5\ e_4)$, $(e_{11}\ e_{15}\ e_4)$, $(e_{10}\ e_{11}\ e_1)$, $(e_{15}\ e_{10}\ e_5)$, $(e_{15}\ e_{14}\ e_1)$, $(e_{11}\ e_{14}\ e_5)$ **R0**-1 (^8)
C10: $(e_{14}\ e_{10}\ e_4)$, $(e_9\ e_{13}\ e_4)$, $(e_3\ e_7\ e_4)$, $(e_9\ e_{10}\ e_3)$, $(e_{10}\ e_{13}\ e_7)$, $(e_{14}\ e_9\ e_7)$, $(e_{14}\ e_{13}\ e_3)$ **R0**-1 (^8)
C11: $(e_{12}\ e_{14}\ e_2)$, $(e_{13}\ e_{12}\ e_1)$, $(e_{15}\ e_{12}\ e_3)$, $(e_3\ e_1\ e_2)$, $(e_{13}\ e_{15}\ e_2)$, $(e_{15}\ e_{14}\ e_1)$, $(e_{14}\ e_{13}\ e_3)$ **R0**-1 (^8)
C12: $(e_{12}\ e_{14}\ e_2)$, $(e_{12}\ e_9\ e_5)$, $(e_{12}\ e_{11}\ e_7)$, $(e_{11}\ e_9\ e_2)$, $(e_5\ e_7\ e_2)$, $(e_{14}\ e_9\ e_7)$, $(e_{11}\ e_{14}\ e_5)$ **R0**-1 (^8)
C13: $(e_{10}\ e_{12}\ e_6)$, $(e_{13}\ e_{12}\ e_1)$, $(e_{12}\ e_{11}\ e_7)$, $(e_{10}\ e_{11}\ e_1)$, $(e_{10}\ e_{13}\ e_7)$, $(e_1\ e_7\ e_6)$, $(e_{13}\ e_{11}\ e_6)$ **R0**-1 (^8)
C14: $(e_{10}\ e_{12}\ e_6)$, $(e_{12}\ e_9\ e_5)$, $(e_{15}\ e_{12}\ e_3)$, $(e_9\ e_{10}\ e_3)$, $(e_{15}\ e_{10}\ e_5)$, $(e_9\ e_{15}\ e_6)$, $(e_5\ e_3\ e_6)$ **R0**-1 (^8)

Call this type of Sedenion Algebra, having zero proper Octonion subalgebras, *Broctonion Sedenion Algebra*. Each Broctonion subalgebra provides 24 primitive zero divisors. The general form for these zero divisors shown above uses two basis elements from the "broken" Quaternion subalgebra and one of its basic quad basis elements. The three basic quads for the three appearances of any triplet within Sedenion Algebra are disjoint, there are no duplications of any basis elements within. Thus, the 15*24 = 360 primitive zero divisors are unique. This is the largest number of primitive zero divisors possible for a Sedenion Algebra orientation.

Using any Broctonion Algebra and algebraic elements **A** and **B**, we do not have a normed composition algebra, for generally N(**A*B**) is not equal to N(**A**)N(**B**). For algebraic element multiplication, all Broctonion Algebra orientations are not commutative, not associative, not alternative. For the latter, we do have **A***(**B*A**) = (**A*B**)***A** for all Octonion and Broctonion orientations, but we only have **A***(**A*B**) = (**A*A**)***B** and **A***(**B*B**) = (**A*B**)***B** for Octonion orientations since they are alternative.

For completeness, the following tables of naming possibilities for the above vs. coset exclusive-or choices. Note the parallelism between the algebra indexes and the Quaternion triplet used as cardinal to generate their orientations.

Maximal Octonion Subalgebra Sedenions

C1 coset xor	9	11	13	15
$\{e_2\ e_4\ e_6\}$ C1 name	**R2**-0	**R6**-0	**R4**-0	**R0**-0
C2 coset xor	10	11	14	15
$\{e_1\ e_4\ e_5\}$ C2 name	**R5**-0	**R1**-0	**R0**-0	**R4**-0
C3 coset xor	9	10	13	14
$\{e_3\ e_4\ e_7\}$ C3 name	**R7**-0	**R3**-0	**R0**-0	**R4**-0
C4 coset xor	12	13	14	15
$\{e_1\ e_2\ e_3\}$ C4 name	**R0**-0	**R3**-0	**R1**-0	**R2**-0
C5 coset xor	9	11	12	14
$\{e_2\ e_5\ e_7\}$ C5 name	**R0**-0	**R7**-0	**R2**-0	**R5**-0
C6 coset xor	10	11	12	13
$\{e_1\ e_7\ e_6\}$ C6 name	**R6**-0	**R0**-0	**R1**-0	**R7**-0
C7 coset xor	9	10	12	15
$\{e_3\ e_5\ e_6\}$ C7 name	**R5**-0	**R0**-0	**R3**-0	**R6**-0
C8 coset xor	8	10	12	14
$\{e_2\ e_4\ e_6\}$ C8 name	**L0**-1	**L4**-1	**L6**-1	**L2**-1
C9 coset xor	8	9	12	13
$\{e_1\ e_4\ e_5\}$ C9 name	**L0**-2	**L4**-2	**L5**-2	**L1**-2
C10 coset xor	8	11	12	15
$\{e_3\ e_4\ e_7\}$ C10 name	**L0**-3	**L4**-3	**L7**-3	**L3**-3
C11 coset xor	8	9	10	11
$\{e_1\ e_2\ e_3\}$ C11 name	**L0**-4	**L3**-4	**L1**-4	**L2**-4
C12 coset xor	8	10	13	15
$\{e_2\ e_5\ e_7\}$ C12 name	**L0**-5	**L7**-5	**L2**-5	**L5**-5
C13 coset xor	8	9	14	15
$\{e_1\ e_7\ e_6\}$ C13 name	**L0**-6	**L6**-6	**L7**-6	**L1**-6
C14 coset xor	8	11	13	14
$\{e_3\ e_5\ e_6\}$ C14 name	**L0**-7	**L5**-7	**L6**-7	**L3**-7

Broctonion Subalgebra Only Sedenion Algebra

C0 coset xor	0	0	0	0
$\{e_2\ e_4\ e_6\}$ C0 name	**R0**-1	**R0**-1	**R0**-1	**R0**-1
C1 coset xor	9	11	13	15
$\{e_2\ e_4\ e_6\}$ C1 name	**R2**-1	**R6**-1	**R4**-1	**R0**-1
C2 coset xor	10	11	14	15
$\{e_1\ e_4\ e_5\}$ C1 name	**R7**-2	**R2**-2	**R6**-2	**R3**-2
C3 coset xor	9	10	13	14
$\{e_3\ e_4\ e_7\}$ C3 name	**R6**-3	**R1**-3	**R5**-3	**R2**-3
C4 coset xor	12	13	14	15
$\{e_1\ e_2\ e_3\}$ C4 name	**R4**-4	**R6**-4	**R7**-4	**R5**-4
C5 coset xor	9	11	12	14
$\{e_2\ e_5\ e_7\}$ C5 name	**R1**-5	**R4**-5	**R6**-5	**R3**-5
C6 coset xor	10	11	12	13
$\{e_1\ e_7\ e_6\}$ C6 name	**R4**-6	**R3**-6	**R5**-6	**R2**-6
C7 coset xor	9	10	12	15
$\{e_3\ e_5\ e_6\}$ C1 name	**R4**-7	**R2**-7	**R7**-7	**R1**-7
C8 coset xor	8	10	12	14
$\{e_2\ e_4\ e_6\}$ C8 name	**R0**-1	**R4**-1	**R6**-1	**R2**-1
C9 coset xor	8	9	12	13
$\{e_1\ e_4\ e_5\}$ C9 name	**R0**-1	**R4**-3	**R5**-1	**R1**-3
C10 coset xor	8	11	12	15
$\{e_3\ e_4\ e_7\}$ C10 name	**R0**-1	**R4**-2	**R7**-1	**R3**-2
C11 coset xor	8	9	10	11
$\{e_1\ e_2\ e_3\}$ C11 name	**R0**-1	**R3**-5	**R1**-1	**R2**-5
C12 coset xor	8	10	13	15
$\{e_2\ e_5\ e_7\}$ C12 name	**R0**-1	**R7**-1	**R2**-4	**R5**-4
C13 coset xor	8	9	14	15
$\{e_1\ e_6\ e_7\}$ C13 name	**R0**-1	**R6**-7	**R7**-1	**R1**-7
C14 coset xor	8	11	13	14
$\{e_3\ e_5\ e_6\}$ C1 name	**R0**-1	**R5**-6	**R6**-6	**R3**-1

What if we built our initial C0 as a proper Octonion, then built common e_8 C1 through C7 as just done with Broctonion orientations? The answer is, as usual there are no duplicated triplet orientation conflicts, the 35 Quaternion subalgebras for Sedenions are fully specified, but now with all triplet orientations in hand, C8 through C14 are now proper Octonion Algebras. We once again have a maximal Octonion subalgebra Sedenion Algebra, but the Octonion/Broctonion roles of C1-C7 and C8-C14 are switched up. Now the common basis element set is Broctonion, and the remaining seven are proper Octonion Algebras. Itemizing the results, we have

C0: $(e_6\ e_4\ e_2)$, $(e_1\ e_5\ e_4)$, $(e_3\ e_7\ e_4)$, $(e_3\ e_1\ e_2)$, $(e_5\ e_7\ e_2)$, $(e_1\ e_7\ e_6)$, $(e_5\ e_3\ e_6)$ **R0**-0 (-)

C1: $(e_6\ e_4\ e_2)$, $(e_{10}\ e_{14}\ e_4)$, $(e_{12}\ e_8\ e_4)$, $(e_{14}\ e_{12}\ e_2)$, $(e_{10}\ e_8\ e_2)$, $(e_{14}\ e_8\ e_6)$, $(e_{12}\ e_{10}\ e_6)$ **R7**-1 (^15)
C2: $(e_{12}\ e_8\ e_4)$, $(e_1\ e_5\ e_4)$, $(e_9\ e_{13}\ e_4)$, $(e_{13}\ e_{12}\ e_1)$, $(e_{12}\ e_9\ e_5)$, $(e_9\ e_8\ e_1)$, $(e_{13}\ e_8\ e_5)$ **R6**-2 (^14)
C3: $(e_{11}\ e_{15}\ e_4)$, $(e_{12}\ e_8\ e_4)$, $(e_3\ e_7\ e_4)$, $(e_{15}\ e_{12}\ e_3)$, $(e_{15}\ e_8\ e_7)$, $(e_{12}\ e_{11}\ e_7)$, $(e_{11}\ e_8\ e_3)$ **R5**-3 (^13)
C4: $(e_{10}\ e_8\ e_2)$, $(e_9\ e_8\ e_1)$, $(e_{11}\ e_8\ e_3)$, $(e_3\ e_1\ e_2)$, $(e_{11}\ e_9\ e_2)$, $(e_{10}\ e_{11}\ e_1)$, $(e_9\ e_{10}\ e_3)$ **R4**-4 (^12)
C5: $(e_{13}\ e_{15}\ e_2)$, $(e_{13}\ e_8\ e_5)$, $(e_{10}\ e_{13}\ e_7)$, $(e_{10}\ e_8\ e_2)$, $(e_5\ e_7\ e_2)$, $(e_{15}\ e_8\ e_7)$, $(e_{15}\ e_{10}\ e_5)$ **R1**-5 (^9)
C6: $(e_9\ e_{15}\ e_6)$, $(e_{15}\ e_{14}\ e_1)$, $(e_{15}\ e_8\ e_7)$, $(e_9\ e_8\ e_1)$, $(e_{14}\ e_9\ e_7)$, $(e_1\ e_7\ e_6)$, $(e_{14}\ e_8\ e_6)$ **R3**-6 (^11)

C7:	$(e_{14}\, e_8\, e_6), (e_{11}\, e_{14}\, e_5), (e_{14}\, e_{13}\, e_3), (e_{11}\, e_8\, e_3), (e_{13}\, e_8\, e_5), (e_{13}\, e_{11}\, e_6), (e_5\, e_3\, e_6)$	**R2**-7 (^10)
C8:	$(e_6\, e_4\, e_2), (e_9\, e_{13}\, e_4), (e_{11}\, e_{15}\, e_4), (e_{11}\, e_9\, e_2), (e_{13}\, e_{15}\, e_2), (e_9\, e_{15}\, e_6), (e_{13}\, e_{11}\, e_6)$	**R0**-0 (^8)
C9:	$(e_{10}\, e_{14}\, e_4), (e_1\, e_5\, e_4), (e_{11}\, e_{15}\, e_4), (e_{10}\, e_{11}\, e_1), (e_{15}\, e_{10}\, e_5), (e_{15}\, e_{14}\, e_1), (e_{11}\, e_{14}\, e_5)$	**R0**-0 (^8)
C10:	$(e_{10}\, e_{14}\, e_4), (e_9\, e_{13}\, e_4), (e_3\, e_7\, e_4), (e_9\, e_{10}\, e_3), (e_{10}\, e_{13}\, e_7), (e_{14}\, e_9\, e_7), (e_{14}\, e_{13}\, e_3)$	**R0**-0 (^8)
C11:	$(e_{14}\, e_{12}\, e_2), (e_{13}\, e_{12}\, e_1), (e_{15}\, e_{12}\, e_3), (e_3\, e_1\, e_2), (e_{13}\, e_{15}\, e_2), (e_{15}\, e_{14}\, e_1), (e_{14}\, e_{13}\, e_3)$	**R0**-0 (^8)
C12:	$(e_{14}\, e_{12}\, e_2), (e_{12}\, e_9\, e_5), (e_{12}\, e_{11}\, e_7), (e_{11}\, e_9\, e_2), (e_5\, e_7\, e_2), (e_{14}\, e_9\, e_7), (e_{11}\, e_{14}\, e_5)$	**R0**-0 (^8)
C13:	$(e_{12}\, e_{10}\, e_6), (e_{13}\, e_{12}\, e_1), (e_{12}\, e_{11}\, e_7), (e_{10}\, e_{11}\, e_1), (e_{10}\, e_{13}\, e_7), (e_1\, e_7\, e_6), (e_{13}\, e_{11}\, e_6)$	**R0**-0 (^8)
C14:	$(e_{12}\, e_{10}\, e_6), (e_{12}\, e_9\, e_5), (e_{15}\, e_{12}\, e_3), (e_9\, e_{10}\, e_3), (e_{15}\, e_{10}\, e_5), (e_9\, e_{15}\, e_6), (e_5\, e_3\, e_6)$	**R0**-0 (^8)

Finally, the fourth build option is to build C0 as Broctonion and C1 through C7 as proper Octonion. The result, which I will not itemize is C8, whose intersection with C0 is its improper Octonion orientation triplet, now becomes proper Octonion. We have once again a maximal Octonion eight proper Octonions in C1 through C8, and seven Broctonions in C0, C9 through C14.

Let's have some more fun. Look at our Sedenion correspondence quotient group for the exclusive-or group correspondence for kernel Quaternion $\{e_0\, e_2\, e_4\, e_6\}$. We had above three {basic quads : $e_2\, e_4\, e_6$}

$\{e_1\, e_3\, e_5\, e_7 : e_2\, e_4\, e_6\}$
$\{e_8\, e_{10}\, e_{12}\, e_{14} : e_2\, e_4\, e_6\}$
$\{e_9\, e_{11}\, e_{13}\, e_{15} : e_2\, e_4\, e_6\}$

Since the three basic quad indexes are cosets of the Sedenion correspondence exclusive-or quotient group isomorphic to the Klein 4-group using [0, 2, 4, 6] as kernel, we can find multiple sets of four Quaternion subalgebra triplets that use a single choice of basic quad member on each of the above three rows. Shown next is one possible choice that does not duplicate any basis elements, appended to the original $\{e_2\, e_4\, e_6\}$ with the three Octonion subalgebra candidate triplets C0 through C14 the triplets show up in.

$\{e_2\, e_4\, e_6\}$	C0, C1, C8
$\{e_1\, e_8\, e_9\}$	C2, C4, C6
$\{e_3\, e_{12}\, e_{15}\}$	C3, C11. C14
$\{e_5\, e_{14}\, e_{11}\}$	C7, C9, C12
$\{e_7\, e_{10}\, e_{13}\}$	C5, C10, C13

Doing this, we have managed to span all 15 Octonion subalgebra candidates without duplication. This will be the case for any initial triplet appended to any four of its cross-coset Quaternion subalgebra triplets without basis element intersection. Notice the 15 basis elements in the triplets are the full complement of non-scalar basis elements without duplication.

Our Sedenion exclusive-or group correspondence quotient groups for these five triplets are the following basis index cosets

[0, 2, 4, 6], [1, 3, 5, 7], [8, 10, 12, 14], [9, 11, 13, 15]
[0, 1, 8, 9], [2, 3, 10, 11], [4, 5, 12, 13], [6, 7, 14, 15]
[0, 3, 12, 15], [1, 2, 13, 14], [4, 7, 8, 11], [5, 6, 9, 10]
[0, 5, 11, 14], [1, 4, 10, 15], [2, 7, 9, 12], [3, 6, 8, 13]
[0, 7, 10, 13], [1, 6, 11, 12], [2, 5, 8, 15], [3, 4, 9, 14]

Looking closely at these, the three non-scalar basis indexes in any kernel are found one up in each of the three non-kernel cosets of the other four.

In Reference [1] a general rule to select five Octonion subalgebra candidates from effectively C0 through C14 that can't all be oriented as proper Octonion Algebras was specified for a given arbitrary selection candidate O: a proper Octonion/Broctonion basis {basic quad : triplet}, and h not a member of it:

Restricted K5 Set General Rule:
O = {d e f g : a b c}
Op = {h a^h b^h c^h : a b c}
Oq = {e^h   b^h c^h d^h : g f a}
Or = {g^h   b^h   h   e^h : e g b}
Os = {f^h   h   c^h   e^h : f e c}

Take O = C0: {4, 5, 6, 7 : 1, 2, 3} giving a=1 b=2 c=3 d=4, e=5, f=6 g=7

For h=9

Op = {9, 8, 11, 10 : 1, 2, 3}	[0, 1, 8, 9], [2, 3, 10, 11]	this is C4
Oq = {12, 11, 10, 13 ; 7, 6, 1}	[0, 7, 10, 13], [1, 6, 11, 12]	this is C13
Or = {14, 11, 9, 12 : 5, 7, 2}	[0, 5, 11, 14], [2, 7, 9, 12]	this is C12
Os = {15, 9, 10, 12 : 6, 5, 3}	[0, 3, 12, 15], [5, 6, 9, 10]	this is C14

These match a possible Cx selection for each of our five select triplets above, showing the above cross-coset process generates K5 sets, gleaming a bit more information about their structure.

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