

# Problems and Solutions of Black Hole Cosmology <sup>1</sup>

Hyoyoung Choi \* <sup>2</sup>

*\*Independent Researcher, Seoul, Republic of Korea.*

## Abstract

The Black Hole Cosmology that “the universe we observe is the interior of a black hole” was proposed in the 1970’s. However, this Black Hole Cosmology is known to have several fatal flaws. In the black hole, singularity exist in the future, whereas in the real universe, singularity exist in the past. And, while the objects inside the black hole are moving toward the singularity, the observed universe is an expanding universe, and it looks completely opposite to each other. Moreover, since black holes are images that decompose humans into atomic units through strong tidal forces, the claim that humans are living inside black holes has not been seriously considered. In this study, I will solve the singularity problem and prove that humans can live inside a sufficiently large black hole. Inside a universe black hole, there is an almost flat space-time larger than the observable universe. Currently, the observable universe is expanding at an accelerated rate because it exists in a region  $R_{obs} < R_{gs}$  where negative gravitational potential energy is greater than positive mass energy. Therefore, I would like to request new interest and research on Gravitational Self-Energy and Black Hole Cosmology through this study.

## I. Black Hole Cosmology: We are now living in a black hole

The image of a black hole that many people think of is an image that stretches people like a stick because of the presence of strong gravity and tidal force, but the Schwarzschild equation that defines the size of a black hole contains slightly different contents.

There are several types of black holes. However, for simple analysis, we present an analysis using a Schwarzschild black hole. In the Schwarzschild radius, the size (radius)  $R$  of a black hole is proportional to its mass  $M$ . Volume is proportional to  $R^3$  and mass is proportional to  $R^1$ , so the average density  $\rho$  is proportional to  $R/R^3 = 1/R^2 = 1/M^2$ . That is, as the mass  $M$  increases, the average density  $\rho$  decreases rapidly.

$$R_S = \frac{2GM}{c^2} \quad (1)$$

$$\rho = \frac{3c^2}{8\pi G} \frac{1}{R^2} = \frac{3c^6}{32\pi G^3} \frac{1}{M^2} \quad (2)$$

In principle, a black hole is not a collection of special matter, but any ordinary matter can become a black hole as long as it is sufficiently collected within a certain radius.

A thing with a small mass must have a very high mass density to become a black hole, but a thing with a large mass becomes a black hole even if the density is not high in order to become a black hole.

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<sup>1</sup>This paper was written to stimulate interest in Black Hole Cosmology. I have reordered this paper (<https://www.researchgate.net/publication/309786718> or <https://vixra.org/abs/1611.0136>) [1] I’ve written before, and an article I’ve written on my personal blog(<https://blog.naver.com/hbar108>) and scientific community on the topic of Black Hole Cosmology, and added a new research.

<sup>2</sup>E-mail:7icarus7@gmail.com

From observations of the universe in which we live, we have several observations and estimates, the critical mass density of the present universe is on the order of  $5 \sim 6$  hydrogen atoms per  $1/m^3$ , and the size of the observable universe is approximately  $46.5Gly$ . [2] [3]

Currently, the average mass density of the universe is almost identical to the critical mass density  $\rho_c$ . This value depends on the value of Hubble's constant  $H_0$ . Approximately,

$$\rho_{c,0} = \frac{3H_0^2}{8\pi G} \approx 8.64 \times 10^{-27} kgm^{-3} \quad (3)$$

If we find the size at which the mass distribution with the average mass density of the present universe forms a black hole,

$$R_{UB} = \sqrt{\frac{3c^2}{8\pi G\rho}} = 1.36 \times 10^{26} m = 14.3Gly \quad (4)$$

**The above expression means that if the present universe has a critical mass density  $\rho_c$  value and the size is approximately  $R_{UB} = 14.3Gly$  or more, this region becomes a black hole.** As the size of the distribution of matter increases, the mass density  $\rho$  required to become a black hole decreases, so the current mass density of the universe is more than a black hole.

Mass distribution	The size of the universe black hole (created by the mass distribution)
Below 14.3 Gly	Not a black hole
14.3Gly	14.3Gly
20.0Gly	39.1Gly
46.5Gly	491.6Gly

In the above equation, the critical mass density value including dark matter and dark energy is used, but if it is obtained only through normal matter, a value of about  $64Gly$  is obtained.

**Currently, we estimate that the size of the observable universe is larger than  $14.3Gly$ , and the entire universe is estimated to be larger than the observable universe  $46.5Gly$ , so our observable universe inevitably exists inside a huge black hole called the universe.**

As above, "Our universe is a black hole." or "Our universe exists inside a black hole." called the Black Hole Cosmology, and this model appeared in the 1970s. [4] [5]

#### [Black Hole Cosmology]

##### 1) Finite Universe

We do not yet know whether the universe is finite or infinite. However, in my personal opinion, infinity is a mathematical notion, and there seems to be no infinite in physical reality. Even flat space-time does not guarantee an infinite universe (infinite mass-energy distribution). Since the age of the universe is finite and the propagation velocity of the field is also finite, the mass distribution is considered to be finite. Therefore, if we exclude the infinite universe, we face the problems 2)~4).

##### 2) Schwarzschild radius : $R_S = \frac{2GM}{c^2}$

##### 3) Observed average density

##### 4) The observable universe $46.5Gly$ , the entire universe much larger than the observable universe

The Black Hole Cosmology is the inevitable conclusion of the above 4 items. 2) is an equation that has been verified in two different theoretical systems (Newtonian mechanics and general relativity), 3) and 4) is on a very solid foundation, and even if 3) and 4) have some errors, the entire universe is estimated to be much larger than the observable universe. Even if the average density is lower than the current observation, the much

larger entire universe inevitably renders the universe a black hole. This is because when the universe becomes  $R$  times larger, the density required to become a black hole decreases by  $1/R^2$ .

## II. Weaknesses of the Black Hole Cosmology

Problems such as strong tidal force enough to disintegrate people, the movement of all matter in the direction of the singularity, and the expanding universe have been pointed out as fatal weaknesses of the Black Hole Cosmology. If our universe was a black hole, all galaxies should have collapsed into a singularity or exhibit motion in the direction of the singularity, but the real universe does not exhibit such motion characteristics. Therefore, the Black Hole Cosmology was judged to be inconsistent with the current observations, and the Black Hole Cosmology did not become a mainstream cosmological model.

### [Fatal weaknesses of the Black Hole Cosmology] [6] [7]

1) In a black hole, all matter is compressed into a singularity, so there is no space for humans to live. There is no almost flat space-time that could contain the observable universe inside a black hole.

2) In the black hole, singularity exist in the future, and in the universe, singularity exist in the past. Black hole and the universe are opposites.

3) The universe is expanding. Inside a black hole, all matter must contract at a singularity. The two models show opposite phenomena. It is difficult to explain the expansion of the universe inside a black hole.

### [Problems with Fatal Weakness Claims]

Although this weaknesses appears to be clear and well-grounded, in fact, this weaknesses also has its own weaknesses.

1) The proposed weakness does not break the argument 2)  $\sim$  4) of the Black Hole Cosmology. Whatever the weakness, if 1)  $\sim$  4) does not collapse, the Black Hole Cosmology is very likely to hold.

2) Most physicists and astronomers believe that the singularity problem will be solved either using quantum mechanics or in some unknown way, so there will be no singularity. In other words, in the process of solving the singularity problem, there is a possibility that the singularity problem of the Black Hole Cosmology will also be solved.

3) Since the singularity exists in the Schwarzschild solution, the Schwarzschild solution must be changed for the singularity problem to disappear. That is, among the elements constituting the Black Hole Cosmology, the “2)Schwarzschild radius equation” is affected. For the singularity to disappear, there must be a repulsive force inside the black hole. Due to this repulsive force, an uncompressed region inevitably exists inside the black hole.

The remaining problem is, “Could an uncompressed region be larger than the observable universe?”

## III. Solutions to the problems of Black Hole Cosmology

### 1. Incompleteness of Einstein’s Field Equation

Modern cosmology is based on Einstein’s field equation. However, this field equation has a singularity problem inside the black hole. This singularity is an object to which general relativity itself does not apply, and therefore the singularity suggests that the Einstein field equation is incomplete and flawed.

So, what’s wrong with Einstein’s field equation?

The fundamental principle of general relativity states that “all energy is a source of gravity”. However, the field equation created by Einstein did not fully realize this principle.

*In writing the field equation (48) we have assumed that the quantity  $T^{\mu\nu}$  is the energy-momentum tensor of matter. In order to obtain a linear field equation we have left out the effect of the gravitational field upon itself. Because of this omission, our linear field equation has several (related) defects: (1) According to (48) matter acts on the gravitational field (changes the fields), but there is*

no mutual action of gravitational fields on matter; that is, the gravitational field can acquire energy-momentum from matter, but nevertheless the energy-momentum of matter is conserved ( $\partial_\nu T^{\mu\nu} = 0$ ). This is an inconsistency. (2) Gravitational energy does not act as source of gravitation, in contradiction to the principle of equivalence. Thus, although Eq. (48) may be a fair approximation in the case of weak gravitational fields, it cannot be an exact equation.

The obvious way to correct for our sin of omission is to include the energy-momentum tensor of the gravitational field in  $T^{\mu\nu}$ . This means that we take for the quantity  $T^{\mu\nu}$  the total energy-momentum tensor of matter plus gravitation:

$$T^{\mu\nu} = T_{(m)}^{\mu\nu} + t^{\mu\nu} \quad [8]$$

The energy of the gravitational field is also a source of gravity. However, since it was difficult to make a field equation including the energy-momentum tensor of the gravitational field, we now have a field equation containing only the energy-momentum tensor of the matter.

**2. Fatal weakness: 1) In a black hole, all matter is compressed into a singularity, so there is no space for humans to live. There is no almost flat space-time that could contain the observable universe inside a black hole**

A singularity exists inside a black hole, and all matter falls to the singularity and is destroyed. [9] In other words, if we were inside a black hole now, it would have collapsed due to gravity as a singularity, so we would not have been able to form the current structure of the galaxy or the solar system, and we could not exist.

However, most physicists and astronomers believe that the singularity problem will not exist, either using quantum mechanics or solved in some unknown way. [3] In other words, in the process of solving the singularity problem, there is a possibility that the singularity problem of the Black Hole Cosmology will also be solved.

## 2-1. A black hole has no singularity and has a Zero Energy Zone [10] [11]

### 2-1-1. Binding energy in the mass defect problem

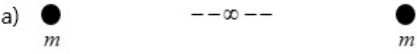
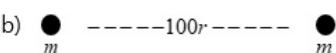
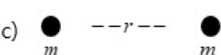
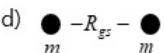

a) 	$E_T = (\text{System}) + (\text{Outside\_of\_the\_system}) = (2mc^2) + (0)$
b) 	$E_T = (2mc^2 - \frac{Gmm}{100r}) + (\frac{Gmm}{100r})$
c) 	$E_T = (2mc^2 - \frac{Gmm}{r}) + (\frac{Gmm}{r})$
d) 	$\text{If } R_g = \frac{Gm}{2c^2}$ $E_T = (2mc^2 - \frac{Gmm}{R_g}) + (\frac{Gmm}{R_g}) = (0) + (2mc^2)$
e) 	$E_T = (?) + (?)$

Figure 1: Description from the mass defect. What if we compress more than  $R_{gs}$  ?

In c), the total energy of the two particle system is

$$E_{T1} = 2mc^2 - \frac{Gmm}{r} \quad (5)$$

In the dimensional analysis of energy,  $E$  has  $kg(m/s)^2$ , so all energy can be expressed in the form of  $(mass)X(velocity)^2$ . So,  $E = mc^2$  holds true for all kinds of energy. Here,  $m$  is the equivalent mass. If we introduce the negative equivalent mass  $-m_{gp}$  for the negative gravitational potential energy,

$$-\frac{Gmm}{r} = -m_{gp}c^2 \quad (6)$$

$$E_{T_1} = 2mc^2 - \frac{Gmm}{r} = 2mc^2 - m_{gp}c^2 = (2m - m_{gp})c^2 = m^*c^2 \quad (7)$$

The gravitational force acting on a relatively distant third mass  $m_3$  is

$$F = -\frac{Gm^*m_3}{R^2} = -\frac{G(2m - m_{gp})m_3}{R^2} = -\frac{G(2m)m_3}{R^2} - \frac{G(-m_{gp})m_3}{R^2} \quad (8)$$

That is, **when considering the gravitational action of a bound system, not only the mass in its free state but also the binding energy term ( $-m_{gp}$ ) should be considered.** Alternatively, the gravitational force acting on the bound system can be decomposed into a free-state mass term and an equivalent mass term of binding energy.

While we usually use the mass  $m^*$  of the bound system, we forget that  $m^*$  is “ $m - m_{binding-energy}$ ”. Gravitational potential energy is also a kind of binding energy. Therefore, the gravitational potential energy, which is the binding energy, must also be considered in the universe.

When a macroscopic object is subjected to gravitational force, the equation includes the gravitational term of the negative gravitational potential energy. In a normal case, since the term of gravitational potential energy is negligible compared to mass energy, it can be neglected. However, since the gravitational self-energy has a fairly large value near the Schwarzschild radius, it must be considered. The same is true of the universe.

When two particles form a binding state, energy corresponding to the binding energy must be released from the system to the outside of the system. In order to keep the two particles close enough so that  $r = R_{gs}$ , the total energy of the system must be zero and the initial (in free state) total mass energy of the system must be released to the outside of the system. Now, in order for these two particles to compress further and achieve a stable state, positive energy must be released from the system to the outside as much as the difference in binding energy. In the case of allowing only positive energy, this compression must be inhibited because there is no more positive energy to withdraw from the system. In other words, gravitational potential energy, a type of binding energy, has the potential to solve the singularity problem.

### 2-1-2. Gravitational self-energy

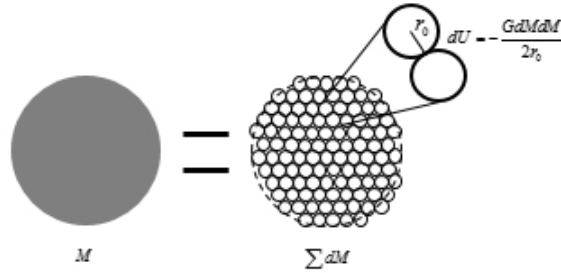


Figure 2: Since all mass  $M$  is a set of infinitesimal mass  $dMs$  and each  $dM$  is gravitational source, too. There exists gravitational potential energy among each of  $dMs$ . Generally, mass of an object measured from its outside corresponds to the value of dividing the total of all energy into  $c^2$ .

The concept of gravitational self-energy( $U_{gs}$ ) is the total of gravitational potential energy possessed by a certain object  $M$  itself. Since a certain object  $M$  itself is a binding state of infinitesimal mass  $dMs$ , it involves the existence of gravitational potential energy among these dMs and is the value of adding up these.  $M = \sum dM$ .

In the case of a spherical uniform distribution, the gravitational self-energy ( $U_{gs}$ ) or the gravitational binding energy ( $-U_{gs}$ ) is

$$U_{gs} = -\frac{3}{5} \frac{GM^2}{R} \quad (9)$$

*Treating the Earth as a continuous, classical mass distribution (with no gravitational self-energy in the elementary, subatomic particles), we find that its gravitational self-energy is about  $4.6 \times 10^{-10}$  times its rest-mass energy. The gravitational self-energy of the Moon is smaller, only about  $0.2 \times 10^{-10}$  times its rest-mass energy. - Gravitation and Spacetime [8]*

All energy is a source of gravity. Therefore, gravitational potential energy is also a gravity source. Since gravitational self-energy is proportional to  $-\frac{M^2}{R}$ , as the mass increases, the ratio of gravitational self-energy increases. In some cases, finding the value,

In the case of Moon,  $U_{gs-Earth} = (-1.89 \times 10^{-11})M_{Moon}c^2$   
 In the case of Earth,  $U_{gs-Earth} = (-4.17 \times 10^{-10})M_{Earth}c^2$   
 In the case of the Sun,  $U_{gs-Sun} = (-1.27 \times 10^{-4})M_{Sun}c^2$   
 In case of a Black hole,  $U_{gs-Black-hole} = (-3.0 \times 10^{-1})M_{Black-hole}c^2$

$$U_{gs-Black-hole}(R = R_S) = -\frac{3}{5} \frac{GM_{fr}^2}{R} \approx -\frac{3}{5} \frac{GM_{fr}^2}{(\frac{2GM_{fr}}{c^2})} = -0.3M_{fr}c^2 \quad (10)$$

Strictly speaking, the mass  $M$  of a black hole is not the mass  $M_{fr}$  in the free state, but the equivalent mass (or effective mass) including the binding energy. Here,  $M_{fr}$  is used for simple estimation.

It can be seen that as the mass increases, the ratio of gravitational self-energy increases. In particular, in the case of a black hole, it can be seen that it has a fairly large value in the event horizon of the black hole.

In the generality of cases, the value of gravitational self-energy is small enough to be negligible, compared to mass energy  $Mc^2$ . So generally, there was no need to consider gravitational self-energy. However the smaller  $R$  becomes, the higher the absolute value of  $U_{gs}$ . For this reason, we can see that  $U_{gs}$  is likely to offset the mass energy in a certain radius.

Thus, **looking for the size in which gravitational self-energy becomes equal to rest mass energy by comparing both,**

$$U_{gs} = |-\frac{3}{5} \frac{GM_{fr}^2}{R_{gs}}| = M_{fr}c^2 \quad (11)$$

$$R_{gs} = \frac{3}{5} \frac{GM_{fr}}{c^2} \quad (12)$$

This equation means that if mass  $M$  is uniformly distributed within the radius  $R_{gs}$ , gravitational self-energy for such an object equals mass energy in size. So, in case of such an object, mass energy and gravitational self-energy can be completely offset while total energy is zero. Since total energy of such an object is 0, gravity exercised on another object outside is also 0.

**Comparing  $R_{gs}$  with  $R_S$ , the radius of Schwarzschild black hole,**

In the rough estimate above, since the gravitational potential energy at the event horizon is  $U_{gs} = -0.3M_{fr}c^2$ , the mass energy of the black hole will be approximately  $E_{BH} = 0.7M_{fr}c^2$ .

$$R_S = \frac{2GM}{c^2} \approx \frac{2G(\frac{7}{10}M_{fr})}{c^2} = \frac{7}{5} \frac{GM_{fr}}{c^2} \quad (13)$$

$$R_{gs} = \frac{3}{5} \frac{GM_{fr}}{c^2} = \frac{3}{7} (\frac{7GM_{fr}}{5c^2}) \approx \frac{3}{7} R_S \approx 0.43R_S \quad (14)$$

This means that there exists the point where gravitational self-energy becomes equal to mass energy within the radius of black hole, and that, supposing a uniform distribution, the value exists approximately at the point  $\frac{3}{7}R_S$ .

Even with kinetic energy and virial theorem applied only the radius diminishes as negative energy counterbalances positive energy, but no effects at all on this point: "There is a zone which cannot be compressed anymore due to the negative gravitational potential energy." Although potential energy changes to kinetic

energy, in order to achieve a stable bonded state, a part of the kinetic energy must be released to the outside of the system.

Considering the virial theorem ( $K=U/2$ ),

$$R_{gs-vir} = \frac{1}{2} R_{gs} \quad (15)$$

Since this value is on a level not negligible against the size of black hole, we should never fail to consider “gravitational self-energy” for case of black hole. In case of the smallest black hole with three times the solar mass,  $R_S = 9km$ .  $R_{gs}$  of this black hole is as far as 3km. In other words, even in a black hole with smallest size that is made by the contraction of a star, the mass distribution can't be reduced to at least radius 3.87km ( $R_{gs-vir} = 1.94km$ ).

### 2-1-3. Black hole does not have a singularity, but it has a Zero Energy Zone

From the equation above, even if some particle comes into the radius of black hole, it is not a fact that it contracts itself infinitely to the point  $r = 0$ . From the point  $R_{gs}$  (or  $R_{gs-vir}$ ), gravity is 0, and when it enters into the area of  $R_{gs}$  (or  $R_{gs-vir}$ ), total energy within  $R_{gs}$  (or  $R_{gs-vir}$ ) region corresponds to negative values enabling anti-gravity to exist. This  $R_{gs}$  (or  $R_{gs-vir}$ ) region comes to exert repulsive gravity effects on the particles outside of it, therefore it interrupting the formation of singularity at the near the area  $r = 0$ .

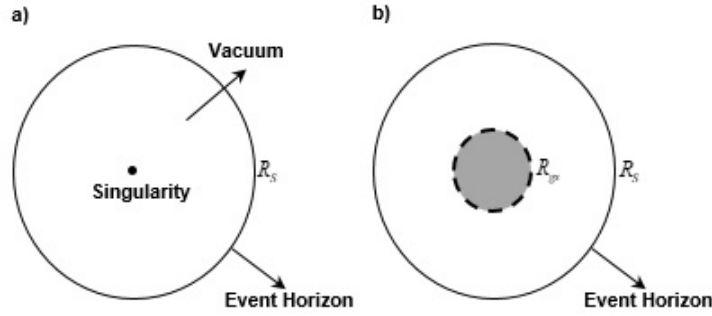


Figure 3: a) Existing Model. b) New Model. The area of within  $R_{gs}$ (or  $R_{gs-vir}$ ) has gravitational self-energy(potential energy) of negative value, which is larger than mass energy of positive value. If  $r$  is less than  $R_{gs}$ , this area becomes negative energy (mass) state. There is a repulsive gravitational effect between the negative masses, which causes it to expand again. This area (within  $R_{gs}$ ) exercises anti-gravity on all particles entering this area, and accordingly prevents all masses from gathering to  $r = 0$ . Therefore the distribution of mass (energy) can't be reduced to at least radius  $R_{gs}$ (or  $R_{gs-vir}$ ).

### 2-1-4. Expansion of the general relativity

In all existing solutions, the mass term  $M$  must be replaced by  $(M_{fr} - M_{gs})$

We can solve the problem of singularity by separating the term  $(-M_{gs})$  of gravitational self-energy from mass and including it in the solutions of field equation.

$M \rightarrow (M_{fr}) + (-M_{gs})$ ,  $M_{fr}$  is total mass when all components of the object are in a free state,  $-M_{gs}$  is the equivalent mass of gravitational self-energy. In all existing solutions(Schwarzschild, Kerr, Reissner-Nordström, ... ), the mass term  $M$  must be replaced by  $(M_{fr} - M_{gs})$ .

For example, Schwarzschild solution is,

$$ds^2 = -(1 - \frac{2GM}{c^2 r})c^2 dt^2 + \frac{1}{(1 - \frac{2GM}{c^2 r})}dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (16)$$

**Schwarzschild-Choi solution** is

$$ds^2 = -\left(1 - \frac{2G(M_{fr} - M_{gs})}{c^2 r}\right)c^2 dt^2 + \frac{1}{\left(1 - \frac{2G(M_{fr} - M_{gs})}{c^2 r}\right)}dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (17)$$

For the sphere with uniform density,

$$-M_{gs} = -\frac{3}{5} \frac{GM_{fr}^2}{Rc^2} \quad (18)$$

- 1) If  $M_{fr} \gg |-M_{gs}|$ , in other words if  $r \gg R_S$ , we get the Schwarzschild solution.
- 2) If  $M_{fr} = |-M_{gs}|$

$$ds^2 = -c^2 dt^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (19)$$

It has a flat space-time.

- 3) If  $M_{fr} \ll |-M_{gs}|$ , in other words if  $0 \leq r \ll R_{gs}$ ,

$$ds^2 \simeq -\left(1 + \frac{2GM_{gs}}{c^2 r}\right)c^2 dt^2 + \frac{1}{\left(1 + \frac{2GM_{gs}}{c^2 r}\right)}dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (20)$$

In the domain of  $0 \leq r \ll R_{gs}$ ,

The area of within  $R_{gs}$  has gravitational self-energy of negative value, which is larger than mass energy of positive value. Negative mass has gravitational effect which is repulsive to each other. So, we can assume that  $-M_{gs}$  is almost evenly distributed. Therefore  $\rho_{gs}$  is constant. And we must consider the Shell Theorem.

$$-M_{gs} = -\frac{4\pi r^3}{3} \rho_{gs} \quad (21)$$

$$\left(1 + \frac{2GM_{gs}}{c^2 r}\right) = 1 + \frac{2G\left(\frac{4\pi}{3} r^3 \rho_{gs}\right)}{c^2 r} = 1 + \frac{8\pi G \rho_{gs} r^2}{3c^2} \quad (22)$$

$$ds^2 \simeq -\left(1 + \frac{8\pi G \rho_{gs} r^2}{3c^2}\right)c^2 dt^2 + \frac{1}{\left(1 + \frac{8\pi G \rho_{gs} r^2}{3c^2}\right)}dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (23)$$

If  $r \rightarrow 0$ ,

$$ds^2 \simeq -c^2 dt^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (24)$$

There is no singularity.

In practice, mass contraction must be stopped at the point where  $M_{shell} = M_{shell-gs}$ .

## 2-1-5. Internal structure of black hole considering gravitational self-energy

When the mass distribution inside the black hole is reduced from  $R_S$  to  $R_{gs}$  (or  $R_{gs-vir}$ ), the energy must be released from the inside of the system to the outside of the system in order to reach this binding state. Here, the system refers to the mass distribution within the radius  $0 \leq r \leq R_{gs}$  (or  $R_{gs-vir}$ ). Although potential energy changes to kinetic energy, in order to achieve a stable bonded state, a part of the kinetic energy must be released to the outside of the system. We need to consider the virial theorem.

At this time, the emitted energy does not go out of the black hole. This energy is distributed in the  $R_{gs}$  (or  $R_{gs-vir}$ )  $< r \leq R_S$  region.

**If you have only the concept of positive energy, please refer to the following explanation.**

From the point of view of mass defect,  $r = R_{gs}$  (or  $R_{gs-vir}$ ) is the point where the total energy of the system is zero. For the system to compress more than this point, there must be an positive energy release from the system. However, since the total energy of the system is zero, there is no positive energy that the system can release. Therefore, the system cannot be more compressed than  $r = R_{gs}$  (or  $R_{gs-vir}$ ). So black hole doesn't have singularity.



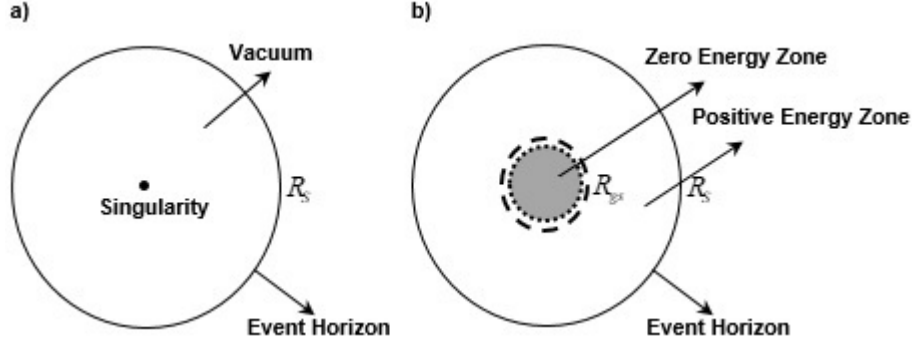


Figure 4: Internal structure of the black hole. a) Existing model b) New model. If, over time, the black hole stabilizes, the black hole does not have a singularity in the center, but it has a Zero (total) Energy Zone.

By locking horns between gravitational self-energy and mass energy, particles inside black hole or distribution of energy can be stabilized. As a final state, the black hole does not have a singularity in the center, but it has a Zero (total) Energy Zone (ZEE).  $R_{gs}$  is the maximum of ZEE.

## 2-2. Inside the huge black hole, there is enough space for intelligent life to exist

A black hole has no singularity, has a Zero Energy Zone with a total energy of zero, and this region is very large, reaching 15% ~ 30% of the radius of the black hole. It suggests an internal structure of a black hole that is completely different from the existing model. Inside the huge black hole, there is an area where intelligent life can live.

Therefore, by considering gravitational self-energy, it is possible to solve the problem of singularity of black hole, which is the most important problem in general relativity. And, this discovery provides a logic for why we are surviving in universe black hole (formed when only mass energy is considered) without collapsing into singularity.

Size of mass distribution	The size of the universe black hole created by the mass distribution	Size of Zero Energy Zone (15% ~ 30%)
14.3Gly	14.3Gly	2.2Gly ~ 4.3Gly
20.0Gly	39.1Gly	5.9Gly ~ 11.7Gly
46.5Gly	491.6Gly	73.7Gly ~ 147.5Gly
100.0Gly	4883.9Gly	732.6Gly ~ 1465.2gly

Figure 5: Under the observed average density, the size of Universe Black Hole and Zero Energy Zone. If the size of the mass distribution increases  $R$  times, the Universe Black Hole and ZEE created by the new mass distribution become  $R^3$  times larger.

For example, if the masses are distributed approximately 46.5Gly with the average density of the current universe, the size of the black hole created by this mass distribution will be 491.6Gly, and the size of the Zero Energy Zone will be approximately 73.7Gly ~ 147.5Gly. In other words, there is no strong tidal force and a region with almost flat space-time that can form a stable galaxy structure is much larger than the observable range of 46.5Gly. The entire universe is estimated to be much larger than the observable universe, so it may not be at all unusual for us to observe only the Zero Energy Zone (nearly flat space-time).

Even if humans live inside a black hole called the universe, a sufficiently stable survival area for intelligent life is guaranteed.

**3. Fatal weakness: 2) In the black hole, singularity exist in the future, and in the universe, singularity exist in the past. Black hole and the universe are opposites**

This problem will be further explained in “Fatal weakness: 3) The problem of cosmic expansion inside a black hole”. However, many people use this assertion as the basis for “the universe is not a black hole”, but I think this claim is wrong.

**3-1. Even within a black hole, singularity can exist in the past**

In order to solve the singularity problem of black hole, there must be a situation in which repulsion outweighs attraction (gravity) below a certain size. In a situation in which repulsion outweighs attraction, the area must expand. If this expansion is converged into the past, a singularity appears, and thus the singularity direction becomes a form that exists in the past. Like the universe ~

**3-2. In a black hole, an object enters the black hole from the event horizon, and in the case of the universe, it is only a case of expanding from a singularity toward the event horizon. It is still a phenomenon that occurs inside a universe black hole**

When an object is thrown upwards in Earth’s gravitational field, it looks different when it rises up and when it comes down from its apex, but both events are just two aspects of a single event in the same gravitational field.

**4. Fatal weakness: 3) The problem of cosmic expansion inside a black hole. The universe is expanding. It is difficult to explain the distance between galaxies inside a black hole**

Within  $R_{gs}$ , negative gravitational self-energy is larger than positive mass energy, and the region within  $R_{gs}$  corresponds to a negative mass state, and repulsive force or antigravity exists.

**Now, we have repulsion or anti-gravity on a cosmic scale. Therefore, this force will be applicable to various phenomena that require repulsive force on a cosmic scale.** For example, Inflation, Dark energy, and the Force that displace the expansion of space and move galaxies.

There are several ways to explain the expansion of the universe inside a black hole (formed when only mass energy is considered without considering gravitational self-energy).

**4-1. Expansion within  $R_{gs}$  from the initial mass-energy distribution at the birth of the universe**

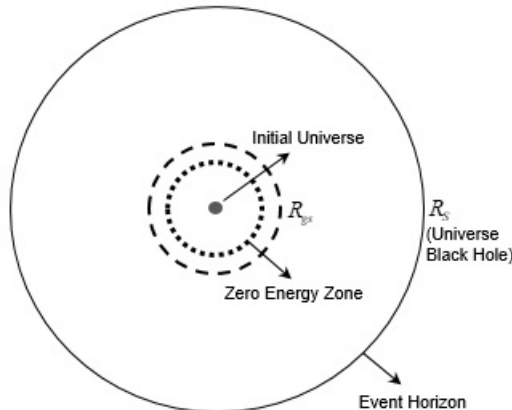


Figure 6: Comparison of the size of the initial universe, the size of the ZEZ, and the size of the Universe Black Hole

Consider the initial state of the universe. The entire universe is larger than the present observable universe,  $46.5Gly$ . Since we do not know the size of the entire universe, after thinking about the state in which all the

mass-energy in the present observable universe is concentrated in a very small area, let's apply this logic to the entire universe.

As calculated above, the size of the ZEZ produced by all mass-energy in the observable universe is approximately  $73.7Gly \sim 147.5Gly$ , and the size of the universe black hole is  $491.6Gly$ . Since these materials are concentrated in a very small area, the negative gravitational potential energy of this area exceeds the positive mass energy and corresponds to a negative mass state as a whole. Because there is a repulsive gravitational effect between negative masses, it expands.

This expansion is accelerated up to at least ZEZ ( $73.7Gly \sim 147.5Gly$ ), and since it is in an accelerated state, expansion continues beyond ZEZ. As time passes, when the distribution of mass is outside the ZEZ, the mass state within the ZEZ is a state in which the positive mass energy is greater than the negative gravitational potential energy, so the total mass (within the ZEZ) is a positive mass, and the attraction is applied to the masses outside the ZEZ. This will have the effect of slowing the expansion.

**The universe expansion at the time of the big bang is because all matter inside the cosmic black hole started in a region smaller than the ZEZ**, and there is a possibility that it corresponds to the accelerated expansion process up to the ZEZ. The size of the ZEZ created by the mass distribution of the observable universe is  $73.7Gly \sim 147.5Gly$ , but the present observable universe is passing  $46.5Gly$ .

#### **4-1-1. In the early days of the universe, why didn't the universe become a singularity or black hole?**

If the total mass of the universe is collected in a very small area, the negative gravitational potential energy is greater than the positive mass energy, and the whole is placed in a negative mass state. There is a repulsive gravitational effect between the negative masses, so expansion occurs away from each other. At least up to the  $R_{gs}$  (Maximum of ZEZ) region, there is an expansion. The singularity problem can be explained as "The singularity itself cannot be formed because of negative gravitational potential energy."

The universe has already become a black hole. However, because the existing general relativity was not perfect, it had a singularity problem. If the general theory of relativity includes the action of gravitational potential energy, the internal structure of a black hole is completely different from the present one. The current state of the universe we are seeing is the image of ZEZ inside a black hole.

#### **4-1-2. At the beginning of the universe, the problem of escape from a black hole created by the total mass of the universe?**

When considering the expansion in the early high-density state of the universe, there is a problem that people mistakenly think that this event is the escape of matter from the inside of the black hole created by the total mass of the universe to the outside to form galaxies or stars.

The black hole event horizon created by the total mass of the universe is very large compared to the area where the total mass of the universe is gathered. In this case, the black hole refers to a black hole formed by considering only mass-energy without considering gravitational potential energy. In other words, in the Black Hole Cosmology model, **matter does not escape the universe black hole, but has not yet reached the event horizon of the universe black hole (formed when only mass energy is considered without considering gravitational potential energy).**

#### **4-1-3. About the inflation mechanism**

In the above analysis, we hypothesized that all matter in the present observable universe was gathered in a very small area.

By relaxing the conditions, we can assume the sequential birth of mass, and exploit the fact that fields such as Higgs and gravitational fields also have limited propagation speeds. Also, because of the finite time after birth, the range of interaction is limited. In other words, not everything interacts at the same moment, but it has certain characteristics sequentially according to the birth of the field, the propagation speed of the field, and time (age of the universe). Such circumstances makes it possible to adjust the size of  $R_{gs}$  or ZEZ in the early universe to be smaller than the current  $73.7Gly \sim 147.5Gly$ .

In the standard cosmology, we postulate a rapid accelerated expansion process called inflation before the big bang model. **The accelerated expansion caused by negative gravitational potential energy may be used to explain the inflation mechanism.**

At the birth of the universe, with all matter gathered in a very small area, rapid accelerated expansion occurred due to negative gravitational potential energy, and this expansion was annihilated after expansion up to  $R_{gs}$  formed by the initial materials (end of the inflation mechanism). Afterwards, it can be explained that the decelerated expansion proceeds in a region larger than the  $R_{gs}$  region formed by the initial materials, and then enters the period of accelerated expansion due to the growth of dark energy.

Since the force from the mass distribution within  $R_{gs}$  is anti-gravity, it will ensure the expansion and uniform density of the universe. Anyway, repulsion is now possible on a cosmic scale, so please try using it for various purposes.

The universe was initially born as a white hole by negative gravitational potential energy, and as the mass distribution passes through the  $R_{gs}$  formed by the initial materials, the universe may have entered the period of a black hole.

#### **4-2. Decelerating expansion From $R_{gs}$ to $R_S$ (expansion within the universe black hole)**

If we do not assume the birth or influx of new mass-energy, a decelerated expansion occurs between  $R_{gs}$  and  $R_S$ . This is because, in the mass distribution within  $R_{gs}$ , the positive mass energy is greater than the negative gravitational potential energy. However, if we assume the birth or influx of new mass-energy, such as vacuum energy or cosmological constant, the situation becomes more complicated and will depend on the model.

Even in the case of an observable universe  $46.5Gly$ , the event horizon ( $R_S$ ) of a cosmic black hole, is ten times that of the observable universe. The larger the mass distribution region, the larger this difference becomes, so it is difficult for us to observe the phenomenon near the event horizon at present.

#### **4-3. Expansion of $R_S$ (expansion of the universe black hole)**

The observable universe did not reach even  $R_{gs}$ . In the region up to  $R_{gs}$ , there is an accelerated expansion effect, and thereafter, there is a decelerated expansion effect. If the total mass of the universe was fixed, there would be no expansion of the cosmic black hole. However, if factors such as vacuum energy or cosmological constants are introduced, the expansion of cosmic black holes can also be created.

The size  $R$  of a black hole is proportional to its mass  $M$ . Therefore, as the mass  $M$  increases, the size of the universe black hole also increases. There are candidates for this mass-energy increase in the universe: vacuum energy, cosmological constant, creation tensor, and dark energy.

If new mass-energy is created or increased in the universe, the event horizon of the universe black hole will also increase according to the Schwarzschild equation. The universe black hole could grow larger, and the entire universe could continue to expand. Due to inflation or other reasons, we can imagine a nearly infinite universe with uniform density. In this state, as time passes, the range of gravitational interaction increases, which will have the effect of introducing new mass into the range of this gravitational interaction. If the mass  $M$  increases in this way, the event horizon of the cosmic black hole can also increase. At this time, if the newly created mass energy acts as a repulsive force (or negative pressure), there is a possibility of creating the effect of dark energy.

It is possible to generate new mass-energy while satisfying energy conservation. It is possible if mass-energy is generated as much as the amount offset by negative gravitational potential energy.

#### **4-4. Accelerating expansion and decelerating expansion at the $R_{gs}$ boundary + mass increase (extension of $R_{gs}$ )**

The general argument of standard cosmology is that accelerated expansion in the early universe > decelerated expansion > accelerated expansion from 5-7 billion years ago, so the following model can be established.

##### **4-4-1. Vibration situation near $r = R_{gs}$**

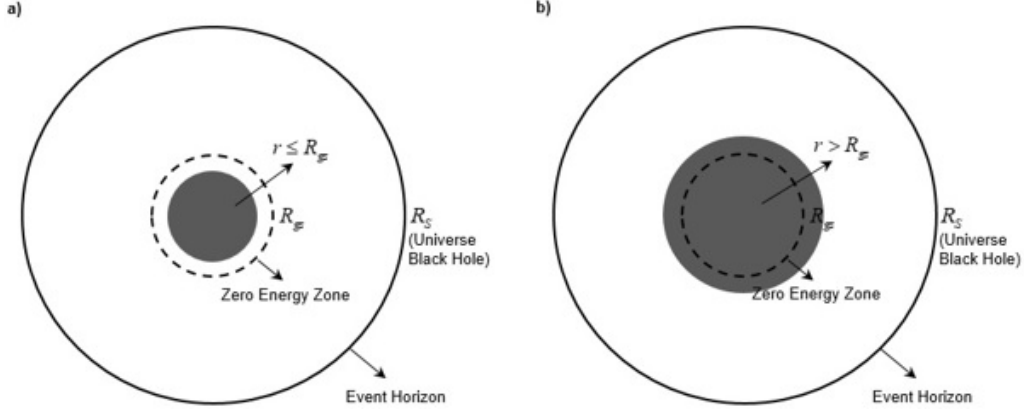


Figure 7: Accelerating expansion and decelerating expansion at the  $R_{gs}$  boundary. The shaded area is the mass-energy distribution area.

Accelerated expansion at  $r < R_{gs}$  --> Decelerated expansion at  $r > R_{gs}$  --> Accelerated expansion at  $r < R_{gs}$  --> Decelerated expansion at  $r > R_{gs}$   
 ( $r$  is the radius of the mass or energy distribution with gravitational interaction)

#### 4-4-2. Expansion of $R_{gs}$ due to the increase in mass-energy of the universe

If the mass of the universe increases with the introduction of vacuum energy or dark energy,  $R_{gs}$  will also increase. If the vacuum has some basic energy, such as the vacuum expectation value, the mass of the universe will increase as the universe expands.

We can consider the model “vibration near  $r = R_{gs}$  + expansion of  $R_{gs}$  due to mass-energy increase”.

#### 4-5. Expansion under zero energy condition [1]

Consider a model in which positive mass energy and negative energy cancel each other out during the expansion of the universe. Since gravitational potential energy is a representative negative energy, consider a model in which negative gravitational potential energy cancels positive mass energy.

$$E_T = (+E) + (-E) = \sum mc^2 + \sum U = Mc^2 - \frac{3}{5} \frac{GM^2}{R} = 0 \quad (25)$$

Let us consider the constant velocity expansion in the zero energy state.

If the universe is in a state of constant expansion, as  $R$  increases, the magnitude of negative gravitational potential energy decreases. On the other hand, the magnitude of positive mass energy remains the same. Therefore, the Zero Energy state is broken. Therefore, let's think about how to achieve the zero energy condition while coping with the constant velocity or accelerated expansion of the universe.

Looking at the equation, it can be seen that if the numerator  $M$  also increases when the denominator  $R$  increases, the equation is likely to hold.

$$E_T = Mc^2 - \frac{3}{5} \frac{GM^2}{R} = 0 \quad (26)$$

$$E'_T = M'c^2 - \frac{3}{5} \frac{GM'^2}{R'} = 0 \quad (27)$$

Let  $R' = kR$ ,  $M' = M + \Delta M$

$$k = \frac{M + \Delta M}{M} = \frac{R + \Delta R}{R} = 1 + \frac{\dot{a}}{a} \Delta t = 1 + H \Delta t \quad (28)$$

$$\frac{\Delta M}{M} = k - 1 = H \Delta t \quad (29)$$

$$R = \frac{3}{5} \frac{GM}{c^2} \quad (30)$$

$$\rho = \frac{5c^2}{4\pi GR^2} \quad (31)$$

In standard cosmology, if  $R$  is doubled, the dark energy is 8 times. On the other hand, in this model, if  $R$  is doubled, dark energy is doubled. In the cosmological constant model, the energy density is kept constant, whereas in this model, the energy density of dark energy is proportional to  $R^{-2}$ . Therefore, there is a change in dark energy density with time in this model.

In this model, the density of matter is proportional to  $a^{-3}$ , whereas the density of dark energy is proportional to  $a^{-2}$ . Therefore, there is a point where the density of dark energy naturally overtakes the density of matter. In other words, it has a point where dark energy appears important.

The principle that “to establish zero energy or conservation of energy, new energy must be born” is very beautiful. Also, since the density of newly born energy changes differently from the density of matter, it is good that over time, a situation more dominant than the density of matter appears. Therefore, I think that more competent researchers need to study this model.

## IV. How can we validate this model?

This model argues that an improved general theory of relativity including negative gravitational potential energy should be built. Strictly, when calculating the energy of the gravitational field, the gravitational field has the gravitational potential energy of the object itself, and there is the gravitational potential energy between the gravitational source and another body.

However, when considering the gravitational field in the vicinity of a high-density object such as a black hole, the gravitational potential energy between the gravitational source and other objects is relatively small and can be neglected. Therefore, in this case, it is sufficient to consider only the gravitational self-energy of a high-density object such as a black hole.

Due to my lack of knowledge and skills, I have not been able to present a solution including gravitational potential energy through a traditional method, but I am presenting a solution through an intuitive method. To the readers of this paper, I encourage you to find solution through the traditional method of solving field equation.

Please refer to the chapter II-2-1-4.

By inserting the equivalent mass of the gravitational energy of the object itself into the existing Schwarzschild equation, I proposed a new solution.

**In all existing solutions, the mass term  $M$  must be replaced by  $(M_{fr} - M_{gs})$ .**

$M \rightarrow (M_{fr}) + (-M_{gs})$ ,  $M_{fr}$  is total mass when all components of the object are in a free state,  $-M_{gs}$  is the equivalent mass of gravitational self-energy. In all existing solutions(Schwarzschild, Kerr, Reissner-Nordström, ... ), the mass term  $M$  must be replaced by  $(M_{fr} - M_{gs})$ .

**Schwarzschild-Choi solution is**

$$ds^2 = -(1 - \frac{2G(M_{fr} - M_{gs})}{c^2 r})c^2 dt^2 + \frac{1}{(1 - \frac{2G(M_{fr} - M_{gs})}{c^2 r})}dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \quad (32)$$

Since we cannot see the inside of a stellar black hole, verification of the inside structure of a black hole is difficult. **However, even near the black hole's event horizon, the above solution will be different from the conventional Schwarzschild solution. Even near the event horizon,  $-M_{gs}$  will be large enough to show a verifiable difference.**

Therefore, you can think of this part as a means of validation for this model. Note that the mass of an already formed black hole is  $m^* = M - M_{gs}$ . We need to measure the mass  $m^*$  of the celestial body when its free mass  $M$  forms a celestial body. That way, we can prove the magnitude and significance of the binding energy. This has already been proven in the atomic world.

Another method of proof is that if a model of the expansion of the universe inside a black hole is well constructed, it is thought that the possibility of verification will arise.

Through additional research [12], I developed a model that can explain dark energy using gravitational potential energy. If we express the existing dark energy or cosmological constant term in this model as gravitational potential energy, we get the following equation:

**Dark energy term [12]**

$$\frac{\Lambda(t)c^2}{3} = \frac{\beta(t)}{15} \left( \frac{4\pi G a(t) R \rho(t)}{c} \right)^2 = 3 \left( \frac{2\pi G a R \rho}{5c} \right)^2 \quad (33)$$

$a(t)R$  is the particle horizon at the point in time to be analyzed. Since  $a(t)$  and  $\rho(t)$  are functions of time, the dark energy is a function of time. **Since the dark energy term is clearly presented as a function of time, by verifying this equation, Black Hole Cosmology can also be proven.**

## V. Discussion

Black Hole Cosmology is based on one validated equation and two validated observations. These three arguments seem to exist on a very strong foundation. Thus, unless these three arguments collapse, the Black Hole Cosmology must be an inevitable fact.

However, the singularity problem of over 100 years still exists in general relativity, which requires a change in general relativity itself. If the field equation is changed to a form containing the gravitational potential energy, the Schwarzschild radius equation must also be changed. If the Schwarzschild equation is changed to an equation that includes gravitational potential energy, the result may be different because the core of the three logics constituting the Black Hole Cosmology is changed.

For example, we take it for granted that the universe will form a black hole when the total mass of the universe is gathered into a very small area. However, this is also an explanation that holds true under the assumption that the existing field equation is correct and the existing Schwarzschild equation is correct.

As argued in the text, if we consider gravitational potential energy or gravitational self-energy, it can be said that the initial state of the universe is not a black hole state in a strict sense. At the beginning of the universe, the region where the total mass is gathered is a region in which the negative gravitational potential energy is greater than the positive mass energy, and the region can be viewed as a negative mass state rather than a black hole state.

However, as explained by mainstream physics, it can be explained that the early universe region exists inside a black hole created by the total mass of the universe. In addition, even in a new solution that considers gravitational potential energy, a state in which negative gravitational potential energy is greater than positive mass energy may exist temporarily, so it can be treated as a phenomenon that can occur inside a universe black hole in the new model. That is, the radius of the black hole created by the total mass is  $R_S$ , but the current state can be thought of as a region within  $R_{gs}$  that is smaller than the radius  $R_S$  of the black hole.

Negative masses expand because of the repulsive gravitational effect. It may be possible to explain this situation in the form of introducing a negative equivalent mass corresponding to negative gravitational potential energy and acting as a repulsive force on galaxies with positive mass.

Between the negative and positive masses, the gravitational potential energy has a positive value ( $U_{-+} = -\frac{G(-m_-)(+m_+)}{r} = +\frac{Gm_-m_+}{r}$ ). Now, in a state where positive gravitational potential energy exists and the kinetic energy of the particles is 0, the positive gravitational potential energy decreases, and a process occurs in which the particles gain kinetic energy by the reduced value. In other words, the universe expands.

However, since the total gravitational potential energy is inversely proportional to  $R$ , as the universe expands and  $R$  increases, the total gravitational potential energy decreases. That is, as the universe expands, the negative energy component becomes smaller and smaller. As the expansion of the universe passes through  $R_{gs}$ , the region within  $R_{gs}$  becomes a positive mass state, so there is a possibility that the entire universe may form a black hole formed by a mass distribution with positive mass.

Also, even when a black hole is formed by gravitational contraction, since ZEZ exists inside the black hole, if our universe is a universe created by the gravitational contraction of the preceding universe, it becomes a black hole universe.

We also have to consider the case of the universe, which is much larger than the range of interactions with the age of the universe.

In cases where the entire universe is much larger than the observable universe, at any observable point, a situation can exist in which the distribution of matter within the observable range is perfectly symmetrical. In this case, the gravitational force acting on it will be zero because the mass distribution is symmetrical at all points in the observable region. Accordingly, the effect of contraction toward the center may disappear. That is, the range of interactions is limited by the age of the universe, and this may have the effect of preventing the observable universe from becoming a black hole.

In any case, in the explanation of the phenomenon in this paper, the explanation of the mainstream physics and the explanation from a new perspective exist together. Therefore, it is necessary to pay attention to this.

### **Are we living in a black hole now? Is the observable universe inside a black hole?**

The observable universe exists in a region where negative gravitational potential energy is greater than positive mass energy. Therefore, the observable universe is located within  $R < R_{gs}$  inside the black hole.

This also appears in the logic behind the success of standard cosmology.

### ***The logical structure of standard cosmology*** [12]

*From the second Friedmann equation or acceleration equation,*

$$\frac{1}{R} \left( \frac{d^2 R}{dt^2} \right) = - \left( \frac{4\pi G}{3} \right) (\rho + 3P) \quad (34)$$

*Let's look at the expression expressing  $(\rho + 3P)$  as the critical density of the universe.*

*Matter + Dark Matter :  $\rho_m \approx \frac{1}{3}\rho_c$*

*Dark energy density :  $\rho_\Lambda \approx \frac{2}{3}\rho_c$*

*(Matter + Dark Matter)'s pressure :  $P_m \approx 0$*

*Dark energy's pressure :  $P_\Lambda = (-\rho_\Lambda) = (-\frac{2}{3}\rho_c)$*

$$\rho + 3P \simeq \rho_m + \rho_\Lambda + 3(P_m + P_\Lambda) \simeq \left(\frac{1}{3}\right)\rho_c + \left(\frac{2}{3}\right)\rho_c + 3\left(-\frac{2}{3}\right)\rho_c = (+1)\rho_c + (-2)\rho_c = (-1)\rho_c \quad (35)$$

$$\rho + 3P \simeq (+1)\rho_c + (-2)\rho_c = (-1)\rho_c \quad (36)$$

***Standard cosmology is a form of positive mass density of  $+1\rho_c$  and negative mass density of  $-2\rho_c$ . So, finally, the universe has a negative mass density of “ $-\rho_c$ ”, so accelerated expansion is taking place. The current universe is similar to a state where the negative mass density is twice the positive mass density. And the total mass of the observable universe is the negative mass state.***



## VI. Conclusion

The Black Hole Cosmology is clearly established by 3 proven facts (2) Schwarzschild radius equation 3) average mass density of the universe 4) observable size of the universe).

This paper solves the problem of singularity of black holes, which has been raised as a key problem, and proves that there is a very large, almost flat space-time that we humans can live in inside a sufficiently large black hole. And, since gravitational self-energy (or gravitational potential energy) provides repulsion on a cosmic scale, it can be used in various situations requiring repulsion on a cosmic scale. It also provides an explanation for the problem of escaping from a singularity or black hole that gathered the total mass of the universe at the beginning of the universe. In addition, it is necessary to reexamine the Black Hole Cosmology because it is also possible to explain the expansion problem inside the universe black hole.

Therefore, there is a need for continuous interest and research on Black Hole Cosmology and Gravitational Self-Energy that can solve the singularity problem.

## References

- [1] Hyoyoung Choi. Size and Expansion of the Universe in Zero Energy Universe (Logical defenses for the Model“*We are living in a black hole*” ), (2016)  
[<https://www.researchgate.net/publication/309786718>][<https://vixra.org/abs/1611.0136>].
- [2] Planck Collaboration et al., *Astron. Astrophys.* **571**, A16 (2014).
- [3] Bradley W. Carroll, Dale A. Ostlie. *Introduction to Modern Astrophysics*. 2nd Edition. Pearson Education, Inc.. (2007).
- [4] Pathria, R. K. “The Universe as a Black Hole”. *Nature*. 240 (5379): 298-299. (1972).
- [5] Good, I. J. “Chinese universes”. *Physics Today*. 25 (7): 15. (1972)
- [6] Philip Gibbs. *Is the Big Bang a black hole?*. (1997)  
[<http://math.ucr.edu/home/baez/physics/Relativity/BlackHoles/universe.html>].
- [7] Multiple Authors. *Is the Universe a Black Hole?* (2015).  
[<https://www.physicsforums.com/insights/universe-black-hole/>].
- [8] Hans C. Ohanian, *Gravitation and Spacetime*, (W. W. Norton & Company, New Yorl), (1976).
- [9] Penrose, R., *Phys. Rev. Lett.* **14** (3), 57 (1965).
- [10] Hyoyoung Choi. On Problems and Solutions of General Relativity (Commemoration of the 100th Anniversary of General Relativity), (2015).  
[<https://www.researchgate.net/publication/286935998>][<http://vixra.org/abs/1511.0240>].
- [11] Hyoyoung Choi. Solution of the Singularity Problem of Black Hole, (2016).  
[<https://www.researchgate.net/publication/313314666>][<http://vixra.org/abs/1701.0667>].
- [12] Hyoyoung Choi. Dark Energy is Gravitational Potential Energy or Energy of the Gravitational Field, (2022).  
[<https://www.researchgate.net/publication/360096238>][<https://vixra.org/abs/2205.0035>].