Einstein’s gravitational theory of relativity with a fourdimensional sphere as metric

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Abstract
We examine Einstein’s theory, where the metric is a fourdimensional sphere. The stress-energy tensor $T_{\mu\nu}$ describes the interaction between a body at the center, i.e. f.e. the sun and the orbiting earth.

1 Introduction
We depart from the equivalence principle, i.e. Newton’s gravitational law:

$$F = \gamma \frac{mM}{r^2}$$ (1)

Since the force is depending on the distance of the origin of gravity, it can be interpreted as non-instantaneously. Einstein’s equation of motion which describes gravitation is:

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$ (2)

The Einstein tensor $G_{\mu\nu}$ describes the so-called metric of the gravitational field. The metric is the form of the geometry which is the result of the source $T_{\mu\nu}$.

2 A source of the fourdimensonal spherical metric
Our metric is following from a fourdimensional sphere, i.e.:

$$x = r \cos(\phi) \cos(\psi) \cos(\varphi)$$ (3)
$$y = r \cos(\phi) \cos(\psi) \sin(\varphi)$$ (4)
$$z = r \cos(\phi) \sin(\psi)$$ (5)
$$ct = r \sin(\psi)$$ (6)
The Einstein tensor is the determinant of second derivations w.r.t. spacetime of the metric. It follows:

\[ G_{\mu\nu} = r \]  

(7)

We write the equivalence principle as:

\[ F r^2 = mg r^2 = \gamma m M \]  

(8)

We look for a solution of a quadropol. The l.h.s. is simply (after dividing by \( mg \)):

\[ G_{\mu\nu} = r^2 \]  

(9)

The right hand side looks like:

\[ 8\pi T_{\mu\nu} = 8\pi \frac{\gamma}{mg} \alpha (x_\mu - x_\nu)(x_\nu - x_\mu) \]  

(10)

We interprete the factor \( 8\pi \) as describing the Earth orbiting around the sun. Herein describes \( 4\pi \) the angular momentum of the earth and \( 2\pi \) the resulting orbit around the sun, i.e. \( 4\pi + 2\pi = 8\pi \).

The unit \([kg]\) is assume to be partly divided into distance multiplied some not yet defined unit \( \alpha \). Thus \([kg] = [\alpha m^2]\).

We consider a concrete example that is:

\[ r^2 = 8\pi \gamma' \alpha (x - y)(y - x) \]  

(11)

\[ = 8\pi 2\gamma' (xy - R^2) \]  

(12)

with \( \gamma' = (1/mg)\gamma \sim \frac{10^{-43}}{10^{12}} \frac{s^4}{kg^3 m^2} \)

### 3 Conclusions

We have examined a system of two objects, at the center the Sun and orbiting the earth.

The orbit of Earth is an ellipse. The ellipse is assumed to change their extrema. So the minimum result is a time dependent quadropol.

The ripples or gravitational waves are characterized by \( r^2 \equiv \lambda^2 \) where \( \lambda \) is the fourdimensional wavelength of the gravitational waves. They are of negative value what we interprete as leaving Sun.