Mass Spectrum of the Kaons

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Abstract

All subatomic particle masses can be expressed as either integer multiples of, or, small denominator fraction multiples of n-sphere surface volumess times ‘h’ – Planck’s constant’s coefficient. For eaxample, the Eta meson’s mass can be expressed as (8/3)S6h, where S6 represents the formula for the surface volume of a 6-sphere: (8/3)S6h = 547.866 MeV/c^2. One experimental mass of the Eta meson reported by Particle Data Group is: 547.865 +/- 0.031 MeV/c^2, which matches (8/3)S6h closely. The purpose of this paper is to show that the masses of all kaons can also be expressed as simply defined multiples of S6h, by matching experimental kaon masses with their theoretical values in the mass spectra generated by S6h that are presented in this paper.

Contents

1. Introduction

Kaons are mesons, which are subatomic particles composed of two quarks - the 'down' and 'strange' quarks. According to Quark Theory, the two quarks inside mesons orbit one another in 3d space under the influence of a central force called the Strong Force. The mathematics for calculating meson masses using this model is very complicated and uncertain since two key elements necessary for making accurate mass calculations are missing, namely the exact masses of the quarks, and an exact mathematical expression for the Strong Force. In this paper, much simpler mathematics is used for specifying meson masses, based on the assumption that matter at the subatomic particle level occupies n-sphere surface volumes, and that therefore, subatomic particle masses can be expressed as multiples of n-sphere surface volumes times a constant, with the help of 'h', which seems to act as a conversion factor. It converts higher dimensional matter to 3d mass. This is what the kaon mass spectra presented here illustrate.

This Hypersphere Surface Volume Theory of subatomic particle structure defines quarks as masses occupying simply defined multiples of n-sphere surface volumes. Thus the mass of a quark does not have one specific value, but rather a series of possible values based on the n-sphere surface volume equation defining that quark. In HSV Theory, the base mass of the 'down' quark corresponds to the surface volume of a 3-sphere, and the base mass of the 'strange' quark corresponds to the surface volume of a 4-sphere. (Mass throughout this paper is in units of MeV/c^2.)

Surface volume of a 3-sphere: $S3 = 4 \pi r^2$
Surface volume of a 4-sphere: $S4 = 2\pi^2 r^3$

To get the 3d mass of a meson, the two n-sphere surface volume formulae representing its quark content must be multiplied together, along with 'h' - Planck's constant's coefficient (h = 6.62607015). Also, the r's in the n-sphere surface volume formulae must be set equal to one (r=1). So, to get the 3d base mass of the kaon, set the r's equal to one, then multiply S3, S4, and 'h' together.

$$S3\ S4 = (4\pi r^2)(2\pi^2 r^3) = 8\ \pi^3 r^5$$
$$S3\ S4\ h = (4\pi)(2\pi^2)h = 8\ \pi^3 h$$
$$S3\ S4\ h = 1643.598\ \ MeV/c^2$$

Notice that the surface volume of a 6-sphere is $\pi^3 r^5$, which means that the volume of S3 S4 is exactly eight times bigger.

$S6 = \pi^3 r^5$ → surface volume of a 6-sphere
$S3\ S4 = 8\ \pi^3 r^5$ → base mass of the kaon

Because of this equivalence (S3S4 = 8 S6) , S6 will be used as the factoring unit throughout this paper, rather than S3 S4, because the notation for S6 is more concise, and, because S6 is eight times smaller than S3S4. To get the value of the factoring unit that will be used, set r=1, then multiply S6 by the coefficient of Planck's constant (h = 6.62607015).

$$S6\ h = \pi^3 h$$
$$S6\ h = \underline{205.4497644}\ \ MeV/c^2 = (1/8)(S3\ S4\ h)$$

This is the <u>factoring unit</u> which will be used to construct the mass spectra throughout this paper.

A Note About Factoring Kaon Masses

All kaon masses are multiples of **S6h** (multiples of S3S4h actually, but S6h = (1/8)S3S4h).

Mass of any Kaon = **nS6h**

But **n** is not just any number. It is a fraction of the form **a/b** where '**a**' is a multiple of a power of two and '**b**' is a power of 3 times 100. (The 100 can be folded into the value of 'h' used, and may not actually be a part of 'b'.)

n = a/b

a can be: m(1), m(2), m(4), m(8), m(16), m(32), m(64), m(128), etc (m = positive integer)

b can be: 3^2(100), 3^3(100), 3^4(100), 3^5(100), 3^6(100), 3^7(100), etc.

There is no theoretical reason currently known why kaons factor this way. This finding came from trial and error searches for the correct factoring coefficients.

Here are the divisors needed to factor some kaon masses.
(The list actually shows the smallest divisors necessary to factor those kaon masses. Larger power divisors will factor all kaon masses that are factored by lower power divisors, but at an unnessarily higher resolution.)

Divisor	Kaon
3^2(100)	K(1630)
3^3(100)	K(1270)
3^4(100)	the majority of kaon masses factor using this divisor
3^5(100)	K(700), K(1580), K(1820), K(1830)
3^6(100)	
3^7(100)	
3^8(100)	K(493), K(497)
3^9(100)	K(493), K(497)

2. Mass Spectra of the Experimental Mass Data Associated with Individual Kaons

The following 14 graphs compare the experimental data associated with the 24 kaons listed on the Particle Data Group's website with their theoretical values. As previously mentioned, all kaons factor with S6h divided by a power of three times 100. Many kaons factor with S6h/3^4(100), which is the same as S6h/8100. Of course all kaons will factor with 3^9(100) , but the question is, do the kaons that factor with lower powers of three have a different structure than the kaons that factor with higher powers of three?

Mass Spectrum of K(493) Data

S6h Factoring / **ds** Compatible
Spectrum Range = 0.190388 MeV/c^2
Step Size = 0.0050102 MeV/c^2

n			$\frac{n}{3^8(100)}$ **S6h**	ExpMass	Error	dm	dm / Error
24631(64)	= 98524	(16)	493.625547				
	98525	(16)	493.**630**557	**493.631**	0.007	.000442	6.3%
	98526	(16)	493.**635**567	**493.636**	0.011	.000433	3.9%
	98526.500	(16)	493.**638**072	**493.638**	0.035	.000072	0.2%
	98527	(16)	493.**640**578	**493.640**	0.054	.000577	1.0%
24632(64)	= 98528	(16)	493.645588				
	98529	(16)	493.650598				
	98530	(16)	493.655608				
	98530.333	(16)	493.**657**278	**493.657**	0.020	.000278	1.4%
	98530.500	(16)	493.**658**113	**493.658**	0.019	.000113	0.6%
	98531	(16)	493.660618				
24633(64)	= 98532	(16)	493.665629				
	98533	(16)	493.**670**639	**493.670**	0.029	.000638	2.2%
	98534	(16)	493.**675**649	**493.675**	0.026	.000649	2.5%
	98535	(16)	493.680659				
24634(64)	= 98536	(16)	493.685669				
	98537	(16)	493.**690**680	**493.691**	0.040	.000320	0.8%
	98538	(16)	493.**695**690	**493.696**	0.007	.000310	4.4%
	98539	(16)	493.700700				
24635(64)	= 98540	(16)	493.705710				
	98540.666	(16)	493.**709**050	**493.709**	0.073	.000050	0.06%
	98541	(16)	493.710721				
	98542	(16)	493.715731				
	98543	(16)	493.720741				
24636(64)	= 98544	(16)	493.725751				
	98545	(16)	493.730761				
	98546	(16)	493.735772				
	98547	(16)	493.740782				
	98547.333	(16)	493.**742**451	**493.742**	0.081	.000451	0.5%
24637(64)	= 98548	(16)	493.745792				
	98549	(16)	493.750802				
	98549.500	(16)	493.**753**072	**493.753**	0.042	.000307	0.7%
	98550	(16)	493.755812				
	98551	(16)	493.760823				
24638(64)	= 98552	(16)	493.765833				
	98553	(16)	493.770843				
	98554	(16)	493.775853				
	98555	(16)	493.780863				
24639(64)	= 98556	(16)	493.785874				
	98557	(16)	493.790884				
	98558	(16)	493.795894				
	98559	(16)	493.800904				
24640(64)	= 98560	(16)	493.**805**914	**493.806**	0.095	.000085	0.09%
	98561	(16)	493.810925				
	98562	(16)	493.815935				

Source of ExpMass and Error Data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

Mass Spectrum of K(497) Data

S6h Factoring / **ds** Compatible
Spectrum Range = 0.190388 MeV/c^2
Step Size = 0.0050102 MeV/c^2

n			$\frac{n}{3^8(100)}$ **S6h**	ExpMass	Error	dm	dm / Error
	99310	(16)	497.563569				
	99311	(16)	497.568579				
24828(64) =	99312	(16)	497.573589				
	99313	(16)	497.578600				
	99314	(16)	497.**583**610	**497.583**	0.005	.000609	12.2%
	99315	(16)	497.588620				
24829(64) =	99316	(16)	497.593630				
	99317	(16)	497.598640				
	99318	(16)	497.603651				
	99318.666	(16)	497.**606**990	**497.607**	0.007	.000009	0.1%
	99319	(16)	497.608661				
24830(64) =	99320	(16)	497.613671				
	99321	(16)	497.618681				
	99322	(16)	497.623692				
	99322.250	(16)	497.**624**944	**497.625**	0.001	.000056	5.6%
	99323	(16)	497.628702				
24831(64) =	99324	(16)	497.633712				
	99325	(16)	497.638722				
	99326	(16)	497.643732				
	99327	(16)	497.648743				
24832(64) =	99328	(16)	497.653753				
	99329	(16)	497.658763				
	99329.500	(16)	497.**661**268	**497.661**	0.033	.000268	0.8%
	99330	(16)	497.663773				
	99331	(16)	497.668783				
24833(64) =	99332	(16)	497.673794				
	99333	(16)	497.678804				
	99334	(16)	497.683814				
	99335	(16)	497.688824				
24834(64) =	99336	(16)	497.693834				
	99337	(16)	497.698845				
	99338	(16)	497.703855				
	99339	(16)	497.708865				
24835(64) =	99340	(16)	497.713875				
	99341	(16)	497.718885				
	99342	(16)	497.723896				
	99343	(16)	497.728906				
24836(64) =	99344	(16)	497.733916				
	99345	(16)	497.738926				
	99345.666	(16)	497.**742**266	**497.742**	0.085	.000266	3.1%
	99346	(16)	497.743936				
	99347	(16)	497.748947				
24837(64) =	99348	(16)	497.753957				

Source of ExpMass and Error Data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

Mass Spectrum of K(700) Data

S6h Factoring / **ds** Compatible
Spectrum Range = 259.7 MeV/c^2
Step Size = 4.328 MeV/c^2

	n	n **S6h** $3^5(100)$	ExpMass	Error	dm	dm / Error
19(4096) =	152(512)	657.980	**658**	13	0.020	0.2%
	153(512)	662.309				
	154(512)	666.638				
	155(512)	670.967				
	156(512)	675.296				
	157(512)	679.624				
	158(512)	683.953				
	159(512)	688.282				
20(4096) =	160(512)	692.611	**694**	53	1.389	2.6%
	161(512)	696.940				
	162(512)	701.269				
	163(512)	705.597				
	164(512)	709.926				
	165(512)	714.255				
	166(512)	718.584				
	167(512)	722.913				
21(4096) =	168(512)	727.241	**727**		0.241	
	169(512)	731.570				
	170(512)	735.899				
	171(512)	740.228				
	172(512)	744.557				
	173(512)	748.886				
	174(512)	753.214				
	175(512)	757.543				
22(4096) =	176(512)	761.871				
	706(128)	764.036	**764**	63	0.036	0.1%
	177(512)	766.201				
	178(512)	770.530				
	179(512)	774.858				
	180(512)	779.187				
	181(512)	783.516				
	182(512)	787.845				
	183(512)	792.174				
23(4096) =	184(512)	796.503	**797**	19	0.497	2.6%
	185(512)	800.831				
	186(512)	805.160				
	187(512)	809.489				
	188(512)	813.818				
	189(512)	818.147				
	190(512)	822.475				
	191(512)	826.804	**826**	49	0.804	1.6%
24(4096) =	192(512)	831.133				
	193(512)	835.462				
	194(512)	839.791				
	195(512)	844.120				
	196(512)	848.448				
	197(512)	852.777				
	198(512)	857.106				
	199(512)	861.435				
26(4096) =	208(512)	900.394				
	209(512)	904.723	**905**	65	0.277	0.4%
	210(512)	909.052				
	211(512)	913.381				
	212(512)	917.709				

Source of ExpMass and Error Data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

Mass Spectrum of K(892) Data

S6h Factoring / **ds** Compatible
Spectrum Range = 7.711 MeV/c^2
Step Size = 0.203 MeV/c^2

	n	n 8100 **S6h**	ExpMass	Error	dm	dm / Error
	4388(8)	890.383	**890.4**	0.2	.017	8.5%
	4389(8)	890.586				
	+(4)	890.688	**890.7**	0.9	.012	1.3%
	4390(8)	890.789				
	4391(8)	890.992	**891**	1	.008	0.8%
549(64) =	4392(8)	891.195				
	4393(8)	891.398				
	4394(8)	891.601				
	+(4)	891.702	**891.7**	0.6	.002	0.6%
	4395(8)	891.804				
	+(4)	891.905	**891.9**	0.7	.005	0.7%
	4396(8)	892.007	**892.0**	2.6	.007	0.3%
	4397(8)	892.209	**892.2**	1.5	.009	0.6%
	4398(8)	892.412				
	4399(8)	892.615	**892.6**	0.5	.015	3.0%
550(64) =	4400(8)	892.818	**892.8**	1.6	.018	1.1%
	4401(8)	893.021	**893**	1	.021	2.1%
	+(1)	893.199	**893.2**	0.1	.001	1.0%
	4402(8)	893.224				
	4403(8)	893.427				
	+(4)	893.528	**893.5**	1.1	.028	2.5%
	+(1)	893.605	**893.6**	0.1	.005	5.0%
	4404(8)	893.630				
	4405(8)	893.833				
	4406(8)	894.036	**894.0**	1.3	.036	2.8%
	4407(8)	894.239	**894.2**	2.0	.039	1.9%
551(64) =	4408(8)	894.442				
	+(4)	894.543	**894.52**	0.76	.023	3.0%
	4409(8)	894.644	**894.63**	0.76	.014	1.8%
	4410(8)	894.847	**894.9**	0.5	.053	10.6%
	4411(8)	895.050	**895**	1	.050	5.0%
	4412(8)	895.253				
	+(2)	895.304	**895.3**	0.2	.004	2.0%
	+(2)	895.405	**895.41**	0.32	.004	1.3%
	4413(8)	895.456				
	4414(8)	895.659	**895.6**	0.8	.059	7.3%
	4415(8)	895.862	**895.9**	0.5	.038	7.6%
552(64) =	4416(8)	896.065	**896.0**	0.6	.065	10.8%
	+(2)	896.217	**896.2**	0.3	.017	5.7%
	4417(8)	896.268				
	4418(8)	896.471	**896.4**	0.9	.071	7.9%
	4419(8)	896.674				
	4420(8)	896.876				
	4421(8)	897.079	**897.1**	0.7	.021	3.0%
	4422(8)	897.282				
	4423(8)	897.485				
	+(4)	897.587	**897.6**	0.9	.013	1.4%
553(64) =	4424(8)	897.688				
	4425(8)	897.891				
	4426(8)	898.094	**898.1**	1.0	.006	0.6%

Source of ExpMass and Error Data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

Mass Spectrum of K(1270) Data

S6h Factoring / **ds** Compatible
Spectrum Range = 60.87 MeV/c^2
Step Size = 1.2174 MeV/c^2

	n	n **S6h** 2700	ExpMass	Error	dm	dm / Error
	1018(16)	1239.395				
	1019(16)	1240.612				
	1020(16)	1241.830	**1242**	9/10	0.170	1.9%
	1021(16)	1243.047				
	1022(16)	1244.265				
	1023(16)	1245.482				
2^{14}	= **1024(16)**	1246.700				
	1025(16)	1247.917	**1248.1**	3.3/1.4	0.183	5.5%
	1026(16)	1249.135				
	1027(16)	1250.352				
	1028(16)	1251.570				
	1029(16)	1252.787				
	1030(16)	1254.004	**1254**	33/34	0.004	0.0%
	1031(16)	1255.222				
	1032(16)	1256.439				
	1033(16)	1257.657				
	1034(16)	1258.874				
	1035(16)	1260.092	**1260**			
	1036(16)	1261.309				
	1037(16)	1262.527				
	1038(16)	1263.744				
	1039(16)	1264.962				
	1040(16)	1266.179				
	1041(16)	1267.397				
	1042(16)	1268.614				
	1043(16)	1269.832	**1270**	10	0.168	1.7%
	1044(16)	1271.049				
	1045(16)	1272.267				
	1046(16)	1273.484				
	1047(16)	1274.702	**1275**	10	0.298	3.0%
	1048(16)	1275.919	**1276**	~		
	1049(16)	1277.136				
	1050(16)	1278.354				
	+(8)	1278.962	**1279**	10	0.038	0.4%
	1051(16)	1279.571				
	1052(16)	1280.789				
	1053(16)	1282.006				
	1054(16)	1283.224				
	1055(16)	1284.441				
	1056(16)	1285.658				
	1057(16)	1286.876				
	1058(16)	1288.093				
	1059(16)	1289.311	**1289**	25	0.311	1.2%
	1060(16)	1290.528				
	1061(16)	1291.746				
	1062(16)	1292.963				
	1063(16)	1294.181	**1294**	10	0.181	1.8%
	1064(16)	1295.988				
	1065(16)	1296.616				
	1066(16)	1297.833				
	1067(16)	1299.051				
	1068(16)	1300.269	**1300**	**~**		

Source of ExpMass and Error Data: P.A. Zyla et al.(Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

S6h Factoring / **ds** Compatible
Spectrum Range = 60.87 MeV/c^2
Step Size = 1.2174 MeV/c^2

	n	n / 8100 **S6h**	ExpMass	Error	dm	dm / Error
6.50(8192) =	416(128)	1350.591	**1350**	~		
	417(128)	1353.837				
	418(128)	1357.084				
	419(128)	1360.331				
	420(128)	1363.577				
	421(128)	1366.824	**1367**	54	0.160	0.3%
	3371**(16)**	1368.042	**1368**	18	0.042	0.2%
	423(128)	1373.318	**1373**	14/18	0.318	2.3%
	424(128)	1376.564				
	425(128)	1379.811	**1380**	21/19	0.189	0.9%
	426(128)	1383.057				
	427(128)	1386.303				
	428(128)	1389.550				
	3430**(16)**	1391.986	**1392**	18	0.014	0.1%
	429(128)	1392.797				
	430(128)	1396.044				
	431(128)	1399.290				
6.75(8192) =	3456(16)	1402.537				
	3457(16)	1402.943	**1403**	7	0.057	0.8%
	3458(16)	1403.349				
	3459(16)	1403.755				
	3460(16)	1404.160	**1404**	10	0.160	1.6%
	3461(16)	1404.566				
	3462(16)	1404.972				
	3463(16)	1405.378				
433(128) =	3464(16)	1405.784				
	3465(16)	1406.189	**1406**	29	0.189	0.7%
	3466(16)	1406.595				
	3467(16)	1407.001				
	3468(16)	1407.407				
	3469(16)	1407.813				
	3470(16)	1408.219				
	3471(16)	1408.624				
434(128) =	3472(16)	1409.030				
	3473(16)	1409.436				
	3474(16)	1409.842	**1410**	25	0.159	0.6%
	3475(16)	1410.248				
	3476(16)	1410.654				
	3477(16)	1411.059				
	3478(16)	1411.465				
	3479(16)	1411.871	**1412**	6	0.129	2.2%
435(128) =	3480(16)	1412.277				
	3481(16)	1412.683				
	3482(16)	1413.089				
	3483(16)	1413.494				
	3484(16)	1413.900	**1414**	130	0.100	0.8%
	3485(16)	1414.306				
	3486(16)	1414.712				
	3487(16)	1415.118	**1415**	15	0.118	0.8%
436(128) =	3488(16)	1415.524				
	3489(16)	1415.929	**1416**	10	0.071	0.7%
	3490(16)	1416.335				
	3491(16)	1416.741				
	3492(16)	1417.147				
	3493(16)	1417.553				
	3494(16)	1417.958	**1418**	8	0.042	0.5%
	3495(16)	1418.364				

S6h Factoring / **ds** Compatible
Spectrum Range = 60.87 MeV/c^2
Step Size = 1.2174 MeV/c^2

n	n 8100 **S6h**	ExpMass	Error	dm	dm / Error
437(128) = 3496(16)	1418.770				
3497(16)	1419.176	**1419.1**	3.7	0.076	2.1%
3498(16)	1419.582				
3499(16)	1419.988	**1420.0**	3.1	0.013	0.4%
3500(16)	1420.393				
3501(16)	1420.799				
3502(16)	1421.205	**1421.1**	2.6	0.105	4.0%
3503(16)	1421.611	**1421.6**	4.2	0.010	0.2%
438(128) = 3504(16)	1422.017				
3505(16)	1422.423				
3506(16)	1422.828	**1423**	5	0.172	3.4%
3507(16)	1423.234				
3508(16)	1423.640	**1423.8**	4.6	0.160	3.5%
3509(16)	1424.046				
3510(16)	1424.452				
3511(16)	1424.858	**1425**	8	0.143	1.8%
439(128) = 3512(16)	1425.263				
3513(16)	1425.669				
3514(16)	1426.075	**1426**	8/24	0.075	0.9%
3515(16)	1426.481				
3516(16)	1426.887	**1427**	12	0.113	0.9%
3517(16)	1427.292	**1427.3**	1.5	0.008	0.5%
3518(16)	1427.698				
3519(16)	1428.104	**1428**	3	0.104	3.3%
440(128) = 3520(16)	1428.510	**1428.5**	3.8	0.010	0.3%
3521(16)	1428.916				
3522(16)	1429.322				
3523(16)	1429.727				
3524(16)	1430.133	**1930**	3.2	0.133	4.2%
3525(16)	1430.539				
3526(16)	1430.945				
3527(16)	1431.351	**1431.2**	1.8	0.150	8.3%
441(128) = 3528(16)	1431.757				
3529(16)	1432.162				
3530(16)	1432.568	**1432.7**	0.7	0.132	18.9%
3531(16)	1432.974	**1433**	6/10	0.026	0.4%
3532(16)	1433.380				
3533(16)	1433.786				
3534(16)	1434.192	**1434**	4/6	0.191	4.8%
3535(16)	1434.597				
442(128) = 3536(16)	1435.003	**1435**	6	0.003	0.1%
3537(16)	1435.409				
3538(16)	1435.815	**1436**	8	0.185	2.3%
3539(16)	1436.221				
3540(16)	1436.627				
3541(16)	1437.032	**1437**	8/16	0.032	0.4%
3542(16)	1437.438				
3543(16)	1437.844	**1438**	8/4	0.156	2.0%
443(128) = 3544(16)	1438.250				
3545(16)	1438.656				
3546(16)	1439.061				
3547(16)	1439.467				
3548(16)	1439.873	**1440**	10	0.127	1.3%
3549(16)	1440.279				
3550(16)	1440.685				
3551(16)	1441.091				

Source of ExpMass and Error Data: P.A. Zyla et al.(Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

S6h Factoring / **ds** Compatible
Spectrum Range = 60.87 MeV/c^2
Step Size = 1.2174 MeV/c^2

	n	n / 8100 **S6h**	ExpMass	Error	dm	dm / Error
	444(128)	1441.496				
	445(128)	1444.743				
	446(128)	1447.990				
	447(128)	1451.236				
7.00(8192) =	3584(16)	1454.483	**1455**	20/15	0.517	2.6%
	3585(16)	1454.888				
	3586(16)	1455.294				
	3587(16)	1455.700				
	3588(16)	1456.106				
	3589(16)	1456.512				
	3590(16)	1456.917				
	3591(16)	1457.323				
449(128) =	3592(16)	1457.729				
	3593(16)	1458.135				
	3594(16)	1458.541				
	3595(16)	1458.946	**1459**	9	0.054	0.6%
	3596(16)	1459.352				
	3597(16)	1459.758				
	3598(16)	1460.164				
	3599(16)	1460.570				
450(128) =	3600(16)	1460.976	**1461.0**	4.0/2.1	0.024	0.6%
	3601(16)	1461.382				
	3602(16)	1461.788				
	3603(16)	1462.194				
	3604(16)	1462.599				
	3605(16)	1463.005	**1463**	64/68	0.005	0.01%
	3606(16)	1463.411				
	3607(16)	1463.817				
451(128) =	3608(16)	1464.223				
	3609(16)	1464.629				
	3610(16)	1465.034				
	3611(16)	1465.440				
	3612(16)	1465.846				
	3613(16)	1466.252				
	3614(16)	1466.658	**1466.6**	0.7/3.4	0.002	0.3%
	3615(16)	1467.064				
452(128) =	3616(16)	1467.469				
	453(128)	1470.716	**1471**	12	0.284	2.4%
	454(128)	1473.963	**1474**	25	0.037	0.1%
	455(128)	1477.209				
	456(128)	1480.456				
	457(128)	1483.702				
	458(128)	1486.949				
	459(128)	1490.196				
	460(128)	1493.442				
	461(128)	1496.689				
	462(128)	1499.935	**1500**	30	0.065	0.2%
	463(128)	1503.182				
7.25(8192) =	464(128)	1506.429				

Source of ExpMass and Error Data: P.A. Zyla et al.(Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

K(1580)

S6h Factoring / **ds** Compatible

n	n **S6h** $3^5(100)$	ExpMass	Error	dm	dm / Error
365(512)	1580.018	**1580**	~	0.018	

K(1630)

S6h Factoring / **ds** Compatible

n	n **S6h** $3^2(100)$	ExpMass	Error	dm	dm / Error
223(32)	1628.988	**1629**	7	0.011	0.2%

Source of ExpMass and Error Data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

Mass Spectrum of K(1650), K(1680) Data

S6h Factoring / **ds** Compatible
Spectrum Range = 194.8 MeV/c^2
Step Size = 3.246 MeV/c^2

	n	n **S6h** 8100	ExpMass	Error	dm	dm / Error
	508(128)	1649.279	**1650**	50	0.721	1.4%
	509(128)	1652.526				
	510(128)	1655.772				
	511(128)	1659.019				
64(1024) =	512(128)	1662.266				
	513(128)	1665.513				
	514(128)	1668.759				
	515(128)	1672.006				
	516(128)	1675.253				
	+(64)	1676.875	**1677**	10/32	0.125	1.3%
	517(128)	1678.499	**1678**	64	0.499	0.8%
	518(128)	1681.746				
	519(128)	1684.992				
65(1024) =	520(128)	1688.239				
	521(128)	1691.486				
	522(128)	1694.732				
	523(128)	1697.979				
	524(128)	1701.226				
	525(128)	1704.472				
	526(128)	1707.719				
	527(128)	1710.965				
66(1024) =	528(128)	1714.212				
	529(128)	1717.459				
	530(128)	1720.705				
	+(64)	1722.328	**1722**	20	0.328	1.6%
	531(128)	1723.952				
	532(128)	1727.198				
	533(128)	1730.445				
	534(128)	1733.692				
	+(64)	1735.314	**1735**	10/20	0.314	3.1%
	535(128)	1736.938				
67(1024) =	536(128)	1740.185				
	537(128)	1743.431				
	538(128)	1746.678				
	539(128)	1749.925				
	540(128)	1753.171				
	541(128)	1756.418				
	542(128)	1759.665				
	543(128)	1762.911				
68(1024) =	544(128)	1766.158				
	545(128)	1769.404				
	546(128)	1772.651				
	547(128)	1775.898				
	548(128)	1779.144				
	549(128)	1782.391				
	550(128)	1785.637				
	551(128)	1788.884				
69(1024) =	552(128)	1792.131	**1793**	59	0.869	1.5%
	553(128)	1795.377				
	554(128)	1798.624	**1800**	70	1.376	2.0%
	555(128)	1801.871				
	556(128)	1805.117				
	557(128)	1808.364				
	558(128)	1811.610				
	559(128)	1814.857				
70(1024) =	560(128)	1818.104				
	567(128)	1840.830	**1840**	~	0.830	
71(1024) =	568(128)	1844.077				

Source of ExpMass and Error Data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

Mass Spectrum of K(1770), K(1780) Data

S6h Factoring / **ds** Compatible
Spectrum Range = 103.8 MeV/c^2
Step Size = 3.2466 MeV/c^2

	n	n 8100 **S6h**	ExpMass	Error	dm	dm / Error
8.25(8192) =	528(128)	1714.212				
	529(128)	1717.459				
	530(128)	1720.705				
	531(128)	1723.952				
	532(128)	1727.198				
	533(128)	1730.445	**1730**	~		
	534(128)	1733.692				
	535(128)	1736.938				
	536(128)	1740.185	**1740**	14/15	0.185	1.3%
	537(128)	1743.431	**1743**	15	0.431	2.9%
	+(64)	1745.054	**1745**	20	0.054	0.3%
	538(128)	1746.678				
	539(128)	1749.925				
	540(128)	1753.171				
	541(128)	1756.418				
	542(128)	1759.665	**1760**	15	0.335	2.2%
	543(128)	1762.911				
8.50(8192) =	544(128)	1766.158				
	+(32)	1766.969	**1767**	6	0.031	0.5%
	545(128)	1769.404				
	546(128)	1772.651	**1773**	8	0.349	4.4%
	547(128)	1775.898	**1776**	26	0.102	0.4%
	548(128)	1779.144	**1779**	11	0.144	1.3%
	+(64)	1780.767	**1781**	8/4	0.233	2.9%
	549(128)	1782.391				
	+(64)	1784.014	**1784**	9	0.014	0.2%
	550(128)	1785.637	**1786**	8	0.363	4.5%
	551(128)	1788.884				
	+(64)	1790.507	**1790**	15	0.507	3.4%
	552(128)	1792.131				
	553(128)	1795.377				
	554(128)	1798.624				
	555(128)	1801.871				
	556(128)	1805.117				
	557(128)	1808.364				
	558(128)	1811.610	**1812**	28	0.390	1.4%
	559(128)	1814.857				
8.750(8192) =	560(128)	1818.104				

Source of ExpMass and Error Data: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

Mass Spectrum of K(1820), K(1830) Data

S6h Factoring / **ds** Compatible
Spectrum Range = 73.59 MeV/c^2
Step Size = 2.1644 MeV/c^2

	n	$\frac{n}{3^5(100)}$ **S6h**	ExpMass	Error	dm	dm / Error
	834(256)	1805.117				
	835(256)	1807.281				
106(2048) =	836(256)	1809.445				
	837(256)	1811.610				
	838(256)	1813.775				
	839(256)	1815.939	**1816**	13	0.061	0.5%
	840(256)	1818.104				
	841(256)	1820.268				
	842(256)	1822.432				
	843(256)	1824.597				
	844(256)	1826.761				
	845(256)	1828.926				
423(512) =	846(256)	1831.090	**1830**	~		
	847(256)	1833.254				
	848(256)	1835.419				
	849(256)	1837.583				
	850(256)	1839.748	**1840**	~		
	851(256)	1841.912				
	852(256)	1844.077				
	853(256)	1846.241				
	854(256)	1848.405				
	855(256)	1850.570				
107(2048) =	856(256)	1852.734	**1853**	27	0.266	1.0%
	857(256)	1854.899				
	858(256)	1857.063				
	859(256)	1859.227				
	860(256)	1861.392				
	861(256)	1863.556				
	862(256)	1865.721				
	863(256)	1867.885				
	864(256)	1870.049				
	865(256)	1872.214				
433(512) =	866(256)	1874.378	**1874**	43	0.378	0.9%
	867(256)	1876.543				
	868(256)	1878.707				

Source of ExpMass and Error Data: P.A. Zyla et al.(Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

Mass Spectrum of K(1950), K(1980) Data

S6h Factoring / **ds** Compatible
Spectrum Range = 110.38 MeV/c^2
Step Size = 3.2466 MeV/c^2

	n	n / 8100 **S6h**	ExpMass	Error	dm	dm / Error
	575(128)	1866.802				
	+(48)	1868.020	**1868**	8/40	0.020	0.3%
9.000(8192) =	576(128)	1870.049				
	577(128)	1873.296				
	578(128)	1876.542				
	579(128)	1879.789				
	580(128)	1883.035				
	581(128)	1886.282				
	582(128)	1889.529				
	583(128)	1892.776				
9.125(8192) =	584(128)	1896.022				
	585(128)	1899.269				
	586(128)	1902.516				
	587(128)	1905.762				
	588(128)	1909.009				
	589(128)	1912.255				
	590(128)	1915.502				
	+(64)	1917.125	**1917**	12	0.125	1.0%
	591(128)	1918.749				
9.250(8192) =	592(128)	1921.995				
	593(128)	1925.242				
	594(128)	1928.488				
	595(128)	1931.735				
	596(128)	1934.982				
	597(128)	1938.228				
	598(128)	1941.475				
	599(128)	1944.721	**1945**	10/20	0.279	2.8%
9.375(8192) =	600(128)	1947.968				
	601(128)	1951.215				
	602(128)	1954.461				
	603(128)	1957.708				
	604(128)	1960.955				
	605(128)	1964.201				
	606(128)	1967.448				
	607(128)	1970.694				
	+(96)	1973.129	**1973**	8/25	0.129	1.6%
9.500(8192) =	608(128)	1973.941				
	609(128)	1977.188				

Source of ExpMass and Error Data: P.A. Zyla et al.(Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

Mass Spectrum of K(2045) Data

S6h Factoring / **ds** Compatible
Spectrum Range = 103.89 MeV/c^2
Step Size = 3.2466 MeV/c^2

	n	n **S6h** 8100	ExpMass	Error	dm	dm / Error
9.750(8192) =	624(128)	2025.887				
	625(128)	2029.133				
	626(128)	2032.380				
	627(128)	2035.627				
	628(128)	2038.873	**2039**	10	0.127	1.3%
	629(128)	2042.120				
	630(128)	2045.367				
	631(128)	2048.613				
	632(128)	2051.860				
	633(128)	2055.106				
	634(128)	2058.353				
	635(128)	2061.600	**2062**	14/13	0.400	2.9%
	636(128)	2064.846				
	637(128)	2068.093				
	638(128)	2071.339				
	639(128)	2074.586				
10.000(8192) =	640(128)	2077.833				
	+(64)	2079.455	**2079**	7	0.455	6.5%
	641(128)	2081.079				
	642(128)	2084.326				
	643(128)	2087.573	**2088**	20	0.427	2.1%
	644(128)	2090.819	**2090**	9/11	0.819	9.1%
	645(128)	2094.066				
	646(128)	2097.312				
	647(128)	2100.559				
	648(128)	2103.806				
	649(128)	2107.052				
	650(128)	2110.299				
	651(128)	2113.545	**2115**	46	1.455	3.2%
	652(128)	2116.792				
	653(128)	2120.039				
	654(128)	2123.285				
	655(128)	2126.532				
10.250(8192) =	656(128)	2129.778				

Source of ExpMass and Error Data: P.A. Zyla et al.(Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

Mass Spectrum of K(2250), K(2320), K(2380), K(2500) Data

S6h Factoring / **ds** Compatible
Spectrum Range = 415.5 MeV/c^2
Step Size = 12.986 MeV/c^2

	n	n 8100 **S6h**	ExpMass	Error	dm	dm/Error	Kaon
10(8192) =	160(512)	2077.833					
	161(512)	2090.819					
	162(512)	2103.806					
	163(512)	2116.792					
	164(512)	2129.778					
	165(512)	2142.765					
	166(512)	2155.751					
	167(512)	2168.738					
	168(512)	2181.724					
	169(512)	2194.711					
	170(512)	2207.697					
	171(512)	2220.684					
	172(512)	2233.670	**2235**	50	1.330	2.7%	**K(2250)**
	173(512)	2246.657	**2247**	17	0.343	2.0%	**K(2250)**
	174(512)	2259.643	**2260**	20	0.357	1.8%	**K(2250)**
	175(512)	2272.629					
11(8192) =	176(512)	2285.616					
	177(512)	2298.602					
	178(512)	2311.589					
	179(512)	2324.575	**2324**	24	0.575	2.4%	**K(2320)**
	180(512)	2337.562					
	181(512)	2350.548					
	182(512)	2363.535					
	183(512)	2376.521					
	+(256)	2383.014	**2382**	14/19	1.014	7.2%	**K(2380)**
	184(512)	2389.508					
	185(512)	2402.494					
	186(512)	2415.480					
	187(512)	2428.467					
	188(512)	2441.453					
	189(512)	2454.440					
	190(512)	2467.426					
	191(512)	2480.413					
	767(128)	2490.152	**2490**	20	0.152	0.8%	**K(2500)**
12(8192) =	192(512)	2493.399					

Source of ExpMass and Error Data: P.A. Zyla et al.(Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

Mass Spectrum of K(3100) Data

S6h Factoring / **ds** Compatible
Spectrum Range = 73.04 MeV/c^2
Step Size = 1.6233 MeV/c^2

	n		n **S6h** 8100	ExpMass	Error	dm	dm / Error
	1875	(64)	3043.700				
	1876	(64)	3045.323	**3045**	8/20	0.323	4.0%
	1877	(64)	3046.946				
	1878	(64)	3048.570				
	1879	(64)	3050.193				
	1880	(64)	3051.816	**3052**	8/20	0.184	2.3%
	1881	(64)	3053.440				
	1882	(64)	3055.063	**3055**	7/20	0.063	0.9%
	1882.666	(64)	3056.145	**3056**	7/20	0.145	2.1%
	1883	(64)	3056.686				
	1884	(64)	3058.309				
	1885	(64)	3059.933	**3060**	8/20	0.067	0.8%
	1886	(64)	3061.556				
	1887	(64)	3063.179				
14.75(8192) =	1888	(64)	3064.803				
	1889	(64)	3066.426				
	1889.333	(64)	3066.967	**3067**	6/20	0.033	0.6%
	1890	(64)	3068.049				
	1891	(64)	3069.673				
	1892	(64)	3071.296				
	1893	(64)	3072.919				
	1894	(64)	3074.543				
	1895	(64)	3076.166				
	1896	(64)	3077.789				
	1897	(64)	3079.412				
	1898	(64)	3081.036				
	1899	(64)	3082.659				
	1900	(64)	3084.282				
	1901	(64)	3085.906				
	1902	(64)	3087.529				
	1903	(64)	3089.152				
	1904	(64)	3090.776				
	1905	(64)	3092.399				
	1906	(64)	3094.022				
	1906.666	(64)	3095.104	**3095**	30	0.104	0.3%
	1907	(64)	3095.646				
	1908	(64)	3097.269				
	1909	(64)	3098.892				
	1910	(64)	3100.515				
	1911	(64)	3102.139				
	1912	(64)	3103.762				
	1913	(64)	3105.385	**3105**	30	0.385	1.3%
	1914	(64)	3107.009				
	1915	(64)	3108.632				
	1916	(64)	3110.255				
	1917	(64)	3111.879				
	1918	(64)	3113.502				
	1919	(64)	3115.125	**3115**	30	0.125	0.4%
15.00(8192) =	1920	(64)	3116.749				

Source of ExpMass and Error Data: P.A. Zyla et al.(Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

Mass Spectrum of Kaons Recently Discovered by the BESIII Collaboration

S6h Factoring / **ds** Compatible
Spectrum Range = 428.5 MeV/c^2
Step Size = 12.986 MeV/c^2

	n	n_S6h $3^4(100)$	ExpMass	Error	dm	Reference
	152.000(512)	1973.941				
	153.000(512)	1986.927				
	154.000(512)	1999.913				
	155.000(512)	2012.900				
	156.000(512)	2025.886				
	156.625(512)	2034.003	**2034**	13/9	0.003	arXiv:2009.08099
	157.000(512)	2038.873				
	158.000(512)	2051.859				
	159.000(512)	2064.846				
10(8192) =	160.000(512)	2077.833				
	161.000(512)	2090.819				
	162.000(512)	2103.806				
	162.500(512)	2110.298	**2111**	43/25	0.702	arXiv:2012.07360
	163.000(512)	2116.792				
	163.750(512)	2126.531	**2126.5**	16.8/12.4	0.031	arXiv:2001.04131
	164.000(512)	2129.778				
	165.000(512)	2142.765				
	166.000(512)	2155.751				
	664.666(128)	2157.915	**2158**	30/33	0.085	arXiv:2112.13219
	167.000(512)	2168.738				
	167.750(512)	2178.477	**2179**	21/3	0.523	arXiv:2009.08099
	168.000(512)	2181.724				
	169.000(512)	2194.711				
	170.000(512)	2207.697	**2208**	19/24	0.303	arXiv:2202.06447
	171.000(512)	2220.684				
	684.666(128)	2222.848	**2223**	16/11	0.152	arXiv:2112.15076
	172.000(512)	2233.670				
	173.000(512)	2246.657				
	174.000(512)	2259.643				
	175.000(512)	2272.629				
11(8192) =	176.000(512)	2285.616				
	177.000(512)	2298.602	**2298**	60/44	0.602	arXiv:2112.13219
	178.000(512)	2311.589				
	179.000(512)	2324.575				
	180.000(512)	2337.562				
	181.000(512)	2350.548				
	182.000(512)	2363.535				
	183.000(512)	2376.521				
	184.000(512)	2389.508				
	185.000(512)	2402.494				

'Reference' directs to the source of 'ExpMass' and 'Error' data.

4. Commentaries on Select Mass Spectra

4.1 **K(493)** Mass Spectrum Commentary

Of the 15 experimental mass data points reported for K(493) by PDG, only 14 are plotted in this mass spectrum, because two of the 15 data points are the same (493.640 by LUM and 493.640 by CHENG), which leaves only 14 unique data points.

11 of the 14 experimental masses plotted in this mass spectrum factor with a divisor of $3^8(100)$. The other three factor to integers in the numerator of the factoring fraction if a divisor of $3^9(100)$ is used. These three masses can be identified in the mass spectrum by their **n**'s, which end with either .333 or .666.

The experimental masses of K(493) have the smallest errors of any kaon. They range in size from .007 to .095 with an average of .0385 MeV/c^2. If one assumes that the placements of the experimental masses in this mass spectrum are correct, then by examining the values in the dm/Error column one sees that the experimental errors assigned to the experimental masses are bigger than necessary by anywhere from 16 to 500 times. (Excepting two extreme cases. dm/Error = 0.09% translates to 1111 times too big and dm/Error = 0.06% translates to 1666 times too big.)

The resolution (step size, block size) of this mass spectrum is:

$(16/(3^8(100))$ S6h = 0.0050 approximately.

This is 1.4 to 19 times smaller than the experimental errors, so one might argue that the assignments of the experimental masses to the theoretical masses may be incorrect. But if the experimental errors are on average 50 times too big, then the adjusted experimental errors (errors divided by 50) are 6.5 times smaller on average than 0.0050. (Using the average of the experimental errors, which is 0.0385.) It might be argued that this is a circular argument, but the fact that there are three occurances of two experimental masses plotted sequentially in this mass spectrum (only 0.0050 MeVc^2 apart) that have overlapping error sizes lends credence to the belief that the experimental errors assigned to K(493)'s experimental masses are larger than they should be, i.e. - they do not reflect the true degree of accuracy of the experimenters determinations.

4.2 **K(700)** Mass Spectrum Commentary

Only 7 of the 24 experimental masses reported by PDG for K(700) are plotted in this mass spectrum to emphasize their positions relative to large factor blocks. Four of the plotted masses fall on large factor blocks and three fall very close to large factor blocks.

4.3 **K(892)** Mass Spectrum Commentary

Only 35 of the 65 experimental masses reported by PDG for K(892) are plotted in this mass spectrum, because 17 are redundant, 5 are outlyers, 2 are almost identical to two others, and 6 factor better with a divisor of $3^5(100)$ rather than the $3^4(100)$ divisor used in this mass spectrum.

The experimental mass data associated with K(892) is the third most accurately determined of all kaon mass data. Experimental mass errors for the 35 experimental masses plotted in K(892)'s mass spectrum vary from 0.1 to 2.6 with an average of 0.87 MeV/c2. The base resolution of this mass spectrum is (8/8100)S6h = 0.2029 approximately, so since the resolution of this mass spectrum is 4.29 times smaller than the average error, it could be argued that the assignment of an experimental mass to its closest matching theoretical mass in the mass spectrum could be incorrect. This would be true if the true error size was equal to the reported error size, but there are reasons to believe they are larger on average for the same reasons given in the K(493) mass spectrum commentary.

4.10 **K(1770), K(1780)** Mass Spectrum Commentary

The experimental masses of these kaons seem to be symmetrically arranged around the large block 8.5(8192).

4.16 **K(3100)** Mass Spectrum Commentary

Of the 11 experimental masses reported by PDG for K(3100), 2 are redundant. Of the 9 left that are plotted in this mass spectrum 3 factor with a factor divisor of $3^5(100)$ rather than a divisor of $3^4(100)$, which was used to construct this mass spectrum. Those three can be identified by n's that end in .333 or .666.

4.17 **BESIII Kaons** Mass Spectrum Commentary

Six of the eight kaons plotted in this mass spectrum factor to fractions with integer numerators using a divisor of $3^4(100)$. The other two kaons factor to fractions with integer numerators using a divisor of $3^5(100)$. Those two can be identified as the ones with n's ending in .666.

5. Summary

The good agreement between the values of experimental and theoretical kaon masses shown in the various mass spectra presented in this paper, lends strong support for belief in the idea that kaons are composed of matter that occupies simply defined fractions of 6-sphere surface volumes. Specifically, kaon masses can all be specified by the expression:

$$\frac{n\,(2^{\mathbf{y}})}{3^{\mathbf{x}}(100)}\ S6h$$

Where n, x, and y are integers, S6 = $\boldsymbol{\pi}^3$, and h = 6.62607015.

6. References

[1] P.A. Zyla et al.(Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)

APPENDIX A

Quark Content Possibilities by Factoring Unit Used

Factoring Unit If……. Mass factors with	Quark Content Possibilities Then……. Hadron has one of these Quark Contents
u S2h = (1, 1)	Note: s=du c=dd b=sd=cu t=cd
d S3h = (1 ,2)	
s S4h = (2, 3)	du
c S5h = (2, 4)	dd
b S6h = (3, 5)	ddu ds, uc
t S7h = (3, 6)	ddd dc
v S8h = (4, 7)	dddu dds sc , db, ut
w S9h = (4, 8)	dddd ddc cc , dt
x S10h = (5, 9)	ddddu ddds dsc cb, st
y S11h = (5, 10)	ddddd dddc dcc ct
z S12h = (6, 11)	dddddu dddds ddsc scc bt, cv
S13h = (6, 12)	ddddddd ddddc ddcc ccc t t
S14h = (7, 13)	dddddddu ddddds dddcs dccs bcc tv
S15h = (7, 14)	ddddddd dddddc dddcc dccc tcc tw
S16h = (8, 15)	ddddddddu dddddds ddddcs ddccs cccs btc vw
S17h = (8, 16)	dddddddd ddddddc dddddcc ddccc cccc s t t ww
S18h = (9, 17)	ddddddddu dddddddds ddddddcs dddddcs dcccs cccb bt t wx
S19h = (9, 18)	ddddddddd dddddddc ddddddcc dddddcc dccccc ccct t t t wy
S20h = (10, 19)	dddddddddu ddddddddds dddddddcs ddddddcs ddcccs ccccs cccv t t w xy
S21h = (10, 20)	ddddddddddd dddddddddc ddddddddcc ddddddcc ddccccc ccccc cccw t t x yy

All quark combinations for the factoring units from S4h to S9h are shown. For the factoring units from S10h to S21h not all possible quark combinations are shown, especially for the triquarks (qqq, baryons) and the diquarks (qq, mesons). This was done so the table wouldn't look too complex and potentially confusing.

The parentheses enclosing two integers separated by a comma that is just to the right of the factoring units, such as the (1,2) in the line S3h = (1,2), means the surface volume formula of that factoring unit has the powers 1 and 2 for 'π' and 'r'. In the case of S3h, S3 = $4\pi^1r^2$. 'π' is raised to the power 1, and 'r' is raised to the power 2, that's why it's written S3h = (1,2). Using this parentheses notation for surface volume formula representation makes it easy to determine which factoring unit will factor which quark combinations, or vice versa, which quark combinations can be factored by which factorung unit.

For instance, if you want to know which factoring unit will factor 'ddd', since 'd' = S3 = (1,2), just add the corresponding integers together of the product (1,2)(1,2)(1,2). You are multiplying numbers together ('π' and 'r') that are raised to integer powers, and, powers add, so you get (3,6). Now find the line with (3,6) in it. It is S7h = (3,6). So the factoring unit needed to factor 'ddd' is S7h.

APPENDIX B

Examples of n-Sphere Surface Volume Factoring of Some Hadron Masses Showing a Compatible Quark Content for Each

Subatomic Particle	ExpMass	Error	HSSV Factoring	ThrMass	Compatible QuarkContent
ρ(770)	775.02	0.35	**4.44444 S5h** =	775.071	dd
η	547.865	0.031	**2.66666 S6h** =	547.8660	ds
Δ(1232)	1232.9	1.2	**6.00000 S6h** =	1232.698	ddu
K(1430)	1438	8/4	**7.00000 S6h** =	1438.148	ds
Δ(1700)	1643	6/3	**8.00000 S6h** =	1643.598	ddu
Xi^0	1314.86	0.20	**6.00000 S7h** =	1314.878	ddd
Xi^-	1321.71	0.07	**6.03125 S7h** =	1321.727	ddd
a2(1700)	1721	11/44	**8.00000 S8h** =	1721.172	cs
Ds	1967.0	1.0/1.0	**64/7 S8h** =	1967.053	cs
Ds(2460)	2458.9	1.5	**80/7 S8h** =	2458.817	cs
B2(5747)	5737.2	0.7	**26.66666 S8h =**	5737.239	bd
Ds	1967.0	1.0/1.0	**10.00000 S9h** =	1967.053	cc
Ds(2460)	2458.9	1.5	**12.50000 S9h** =	2458.817	cc
Ds(2700)	2688	4	**13.66666 S9h** =	2688.307	cc
Ds(2700)	2710	2	**13.77777 S9h** =	2710.163	cc
Bj(5732)	5704	4/10	**29.00000 S9h** =	5704.455	cc
Ds(2212)	2112.2	0.4	**12.5000 S10h** =	2112.195	bc
Ω(2250)	2253	13	**13.3333 S10h** =	2253.008	dsc
Ds1(2536)	2534.6	0.3/0.7	**15.0000 S10h** =	2534.634	bc
Ds2(2572)	2572.2	0.3/1.0	**15.2222 S10h** =	2572.185	bc
Ds0(2590)	2591	13	**15.3333 S10h** =	2590.960	bc
Pc(4337)	4337	7/4	**25.6666 S10h** =	4337.041	ddddu
Pc(4457)	4449.8	1.7/2.5	**26.3333 S10h** =	4449.692	ddddu
Y(4500)	4506	11	**26.6666 S10h** =	4506.017	ddddu
b1(1235)	1236	16	**9.0000 S11h** =	1235.936	ddddd
X(2175)	2197.4	4.4	**16.0000 S11h** =	2197.219	ddddd
Z(3985)	3982.5	1.8	**29.0000 S11h** =	3982.461	ddddd
X(4660)	4669	21/3	**34.0000 S11h** =	4669.092	ddddd
Ds(2860)	2866.6(avg)		**27.0000 S12h** =	2866.605	bt
$D(3000)^0$	2971.8	8.7	**28.0000 S12h** =	2972.775	bt
$D(3000)^0$	3008.1	4.0	**28.3333 S12h** =	3008.165	bt
Dsj(3040)	3044	8	**28.6666 S12h** =	3043.555	bt
Λ	1115.59	0.08	**14.2222 S13h** =	1115.599	ccc
Ω	1673.4	1.7	**21.3333 S13h** =	1673.398	ccc
Xi(1950)	1952	11	**24.8888 S13h** =	1952.298	ccc
Xi(2500)	2505	10	**31.9375 S13h** =	2505.195	ccc
fj(2220)	2223.9	2.5	**40.0000 S14h** =	2223.630	vt
Xc0(1P)	3415.5	0.4/0.4	**61.4400 S14h** =	3415.496	ccsd
Xc2(1P)	3557.8	0.2/4	**64.0000 S14h** =	3557.808	ccsd
ηb(1S)	9394.8	2.7/3.1	**169.0000 S14h** =	9394.839	vt
f0(980)	977.3	0.9/3.7	**99.7500 S18h** =	977.298	cccb
f0(980)	982.2	1.0/8.1	**100.2500 S18h** =	982.197	cccb
f0(980)	984.7	0.4/2.4	**100.5000 S18h** =	984.646	cccb

Source of ExpMass and Error data: P.A. Zylaet al.(Particle Data Group), Prog. Theor. Exp. Phys.2020, 083C01 (2020) and 2021 update

APPENDIX C

Hypersphere Surface Volume Formulae

(Dimension 2 - Dimension 21)

Sphere Dimension	Sn	Surface Volume Formula	(π, r) Powers
2	S2 =	$2\ \pi^1 r^1$	(1, 1)
3	S3 =	$4\ \pi^1 r^2$	(1, 2)
4	S4 =	$2\ \pi^2 r^3$	(2, 3)
5	S5 =	$8/3\ \pi^2 r^4$	(2, 4)
6	S6 =	$\pi^3 r^5$	(3, 5)
7	S7 =	$16/15\ \pi^3 r^6$	(3, 6)
8	S8 =	$1/3\ \pi^4 r^7$	(4, 7)
9	S9 =	$32/105\ \pi^4 r^8$	(4, 8)
10	S10 =	$1/12\ \pi^5 r^9$	(5, 9)
11	S11 =	$64 / 945\ \pi^5 r^{10}$	(5, 10)
12	S12 =	$1 / 60\ \pi^6 r^{11}$	(6, 11)
13	S13 =	$128 / 10395\ \pi^6 r^{12}$	(6, 12)
14	S14 =	$1 / 360\ \pi^7 r^{13}$	(7, 13)
15	S15 =	$256 / 135135\ \pi^7 r^{14}$	(7, 14)
16	S16 =	$1 / 2520\ \pi^8 r^{15}$	(8, 15)
17	S17 =	$512 / 2027025\ \pi^8 r^{16}$	(8, 16)
18	S18 =	$1 / 20160\ \pi^9 r^{17}$	(9, 17)
19	S19 =	$1024 / 34459425\ \pi^9 r^{18}$	(9, 18)
20	S20 =	$1 / 181440\ \pi^{10} r^{19}$	(10, 19)
21	S21 =	$2048 / 654729075\ \pi^{10} r^{20}$	(10, 20)

APPENDIX D

Values of Hypersphere Surface Volume Units of Factorization

(Dimension 2 - Dimension 21)

Sphere Dimension	Unit of Factorization		Formula		Value (MeV/c^2)
2	S2h	=	2 $\pi^1 r^1 h$	=	41.63282661
3	S3h	=	4 $\pi^1 r^2 h$	=	83.26565322
4	S4h	=	2 $\pi^2 r^3 h$	=	130.7933822
5	S5h	=	8/3 $\pi^2 r^4 h$	=	174.3911763
6	S6h	=	$\pi^3 r^5 h$	=	205.4497644
7	S7h	=	16/15 $\pi^3 r^6 h$	=	219.1464153
8	S8h	=	1/3 $\pi^4 r^7 h$	=	215.1464901
9	S9h	=	32/105 $\pi^4 r^8 h$	=	196.7053624
10	S10h	=	1/12 $\pi^5 r^9 h$	=	168.9756582
11	S11h	=	64 / 945 $\pi^5 r^{10} h$	=	137.3262492
12	S12h	=	1 / 60 $\pi^6 r^{11} h$	=	106.1705373
13	S13h	=	128 / 10395 $\pi^6 r^{12} h$	=	78.44057013
14	S14h	=	1 / 360 $\pi^7 r^{13} h$	=	55.59076334
15	S15h	=	256 / 135135 $\pi^7 r^{14} h$	=	37.91204905
16	S16h	=	1 / 2520 $\pi^8 r^{15} h$	=	24.94907624
17	S17h	=	512 / 2027025 $\pi^8 r^{16} h$	=	15.88056197
18	S18h	=	1 / 20160 $\pi^9 r^{17} h$	=	9.797479330
19	S19h	=	1024 / 34459425 $\pi^9 r^{18} h$	=	5.869441980
20	S20h	=	1 / 181440 $\pi^{10} r^{19} h$	=	3.419965454
21	S21h	=	2048 / 654729075 $\pi^{10} r^{20} h$	=	1.940989032